Journal of Prime Research in Mathematics Vol. **4***(2008), 132-142*

COMMON FIXED POINT THEOREMS FOR TWO MAPPINGS IN D^*-METRIC SPACES

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Abstract. In this paper, we give some new definitions of D^*-metric spaces and we prove a common fixed point theorem for two mappings under the condition of weakly compatible mappings in complete D^*-metric spaces. We get some improved versions of several fixed point theorems in complete D^*-metric spaces.

Key words : D^*-metric contractive mapping, complete D^*-metric space, common fixed point theorem.
AMS SUBJECT: 54E40, 54E35, 54H25.

1. Introduction and Preliminaries

In 1922, the Polish mathematician, Banach, proved a theorem which ensures, under appropriate conditions, the existence and uniqueness of a fixed point.His result is called Banach's fixed point theorem or the Banach contraction principle. This theorem provides a technique for solving a variety of applied problems in mathematical science and engineering.Many authors have extended, generalized and improved Banach's fixed point theorem in different ways. In [20], Rhoades introduced more generalized commuting mappings, called *compatible* mappings, which are more general than commuting and weakly commuting mappings. This concept has been useful for obtaining more comprehensive fixed point theorems(see, e.g.,([2, 3, 4, 5, 10, 12, 13, 21, 26, 27, 30]). Dhage[6] introduced the notion of generalized metric or D-metric

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spaces and claimed that D-metric convergence defines a Hausdorff topology and that D-metric is sequentially continuous in all the three variables.Many authors have taken these claims for granted and used them in proving fixed point theorems in D-metric spaces.Rhoades [20] generalized Dhage's contractive condition by increasing the number of factors and proved the existence of unique fixed point of a self-map in D-metric space. Recently, motivated by the concept of compatibility for metric space, Singh and Sharma [29] introduced the concept of D-compatibility of maps in D-metric space and proved some fixed point theorems using a contractive condition. Unfortunately, almost all theorems in D-metric spaces are not valid (see [17, 18, 19]).In this paper, we introduce D^*-metric which is a probable modification of the definition of D-metric introduced by Dhage [6] and prove some basic properties in D^*-metric spaces.(also see [25])

In what follows (X, D^*) will denote a D^*-metric space, $\mathbb{N}$ the set of all natural numbers, and $\mathbb{R}^+$ the set of all positive real numbers.

Definition 1. *Let X be a nonempty set. A generalized metric (or D^*-metric) on X is a function: $D^* : X^3 \longrightarrow \mathbb{R}^+$ that satisfies the following conditions for each $x, y, z, a \in X$.*

(1) $D^*(x, y, z) \geq 0$,

(2) $D^*(x, y, z) = 0$ if and only if $x = y = z$,

(3) $D^*(x, y, z) = D^*(p\{x, y, z\})$,(symmetry) where p is a permutation function,

(4) $D^*(x, y, z) \leq D^*(x, y, a) + D^*(a, z, z)$.

The pair (X, D^*) is called a generalized metric (or D^*-metric) space.

Immediate examples of such a function are

(a) $D^*(x, y, z) = \max\{d(x, y), d(y, z), d(z, x)\}$,

(b) $D^*(x, y, z) = d(x, y) + d(y, z) + d(z, x)$.

Here, d is the ordinary metric on X.

(c) If $X = \mathbb{R}^n$ then we define

$$D^*(x, y, z) = (||x - y||^p + ||y - z||^p + ||z - x||^p)^{\frac{1}{p}}$$

for every $p \in \mathbb{R}^+$.

(d) If $X = \mathbb{R}^+$ then we define

$$D^*(x, y, z) = \begin{cases} 0 & \text{if } x = y = z, \\ \max\{x, y, z\} & \text{otherwise} , \end{cases}$$

Remark 1. *In a D^*-metric space, we prove that $D^*(x, x, y) = D^*(x, y, y)$. For*

(i) $D^*(x, x, y) \leq D^*(x, x, x) + D^*(x, y, y) = D^*(x, y, y)$ and similarly

(ii) $D^*(y, y, x) \leq D^*(y, y, y) + D^*(y, x, x) = D^*(y, x, x)$.

Hence by (i),(ii) we get $D^*(x, x, y) = D^*(x, y, y)$.

Let (X, D^*) be a D^*-metric space. For $r > 0$ define

$$B_{D^*}(x, r) = \{y \in X : D^*(x, y, y) < r\}$$

Example 1. *Let $X = \mathbb{R}$. Denote $D^*(x, y, z) = |x - y| + |y - z| + |z - x|$ for all $x, y, z \in \mathbb{R}$. Thus*

$$\begin{aligned} B_{D^*}(1, 2) &= \{y \in \mathbb{R} : D^*(1, y, y) < 2\} \\ &= \{y \in \mathbb{R} : |y - 1| + |y - 1| < 2\} \\ &= \{y \in \mathbb{R} : |y - 1| < 1\} = (0, 2) \end{aligned}$$

Definition 2. *Let (X, D^*) be a D^*-metric space and $A \subset X$.*

(1) If for every $x \in A$ there exist $r > 0$ such that $B_{D^*}(x, r) \subset A$, then subset A is called open subset of X.

(2) Subset A of X is said to be D^*-bounded if there exists $r > 0$ such that $D^*(x, y, y) < r$ for all $x, y \in A$.

(3) A sequence $\{x_n\}$ in X converges to x if and only if $D^*(x_n, x_n, x) = D^*(x, x, x_n) \to 0$ as $n \to \infty$. That is for each $\epsilon > 0$ there exist $n_0 \in \mathbb{N}$ such that

$$\forall n \geq n_0 \Longrightarrow D^*(x, x, x_n) < \epsilon \ (*)$$

This is equivalent with, for each $\epsilon > 0$ there exist $n_0 \in \mathbb{N}$ such that

$$\forall n, m \geq n_0 \Longrightarrow D^*(x, x_n, x_m) < \epsilon \ (**)$$

Indeed, if have $(*)$, then

$$D^*(x_n, x_m, x) = D^*(x_n, x, x_m) \leq D^*(x_n, x, x) + D^*(x, x_m, x_m) < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \varepsilon$$

Conversely, set $m = n$ in $(**)$ we have $D^*(x_n, x_n, x) < \epsilon$.

(4) Sequence $\{x_n\}$ in X is called a Cauchy sequence if for each $\epsilon > 0$, there exits $n_0 \in \mathbb{N}$ such that $D^*(x_n, x_n, x_m) < \epsilon$ for each $n, m \geq n_0$. The D^*-metric space (X, D^*) is said to be complete if every Cauchy sequence is convergent.

Let τ be the set of all $A \subset X$ with $x \in A$ if and only if there exist $r > 0$ such that $B_{D^*}(x, r) \subset A$. Then τ is a topology on X (induced by the D^*-metric D^*).

Lemma 1. *Let (X, D^*) be a D^*-metric space. If $r > 0$, then ball $B_{D^*}(x, r)$ with center $x \in X$ and radius r is open ball.*

Proof. see [25] □

Definition 3. *Let (X, D^*) be a D^*- metric space. D^* is said to be continuous function on $X^3 \times (0, \infty)$ if*

$$\lim_{n \to \infty} D^*(x_n, y_n, z_n) = D^*(x, y, z).$$

Whenever a sequence $\{(x_n, y_n, z_n)\}$ *in* $X^3 \times (0, \infty)$ *converges to a point* $(x, y, z) \in X^3 \times (0, \infty)$ *i.e.*

$$\lim_{n\to\infty} x_n = x, \lim_{n\to\infty} y_n = y, \lim_{n\to\infty} z_n = z$$

Lemma 2. *Let* (X, D^*) *be a* D^**- metric space. Then* D^* *is continuous function on* $X^3 \times (0, \infty)$.

Proof. see [25] □

Lemma 3. *Let* (X, D^*) *be a* D^**-metric space. If sequence* $\{x_n\}$ *in* X *converges to* x*,then* x *is unique.*

Proof. Let $x_n \longrightarrow y$ and $y \neq x$. Since $\{x_n\}$ converges to x and y, for each $\epsilon > 0$ there exist
$n_1 \in \mathbb{N}$ such that for every $n \geq n_1 \Longrightarrow D^*(x, x, x_n) < \frac{\epsilon}{2}$
and
$n_2 \in \mathbb{N}$ such that for every $n \geq n_2 \Longrightarrow D^*(y, y, x_n) < \frac{\epsilon}{2}$.
If set $n_0 = \max\{n_1, n_2\}$, then for every $n \geq n_0$ by triangular inequality we have

$$D^*(x, x, y) \leq D^*(x, x, x_n) + D^*(x_n, y, y) < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \varepsilon.$$

Hence $D^*(x, x, y) = 0$ is a contradiction. So, $x = y$. □

Lemma 4. *Let* (X, D^*) *be a* D^**-metric space. If sequence* $\{x_n\}$ *in* X *is converges to* x *,then sequence* $\{x_n\}$ *is a Cauchy sequence.*

Proof. Since $x_n \longrightarrow x$ for each $\epsilon > 0$ there exists
$n_1 \in \mathbb{N}$ such that for every $n \geq n_1 \Longrightarrow D^*(x_n, x_n, x) < \frac{\epsilon}{2}$
and
$n_2 \in \mathbb{N}$ such that for every $m \geq n_2 \Longrightarrow D^*(x, x_m, x_m) < \frac{\epsilon}{2}$.
If set $n_0 = \max\{n_1, n_2\}$, then for every $n, m \geq n_0$ by triangular inequality we have
$D^*(x_n, x_n, x_m) \leq D^*(x_n, x_n, x) + D^*(x, x_m, x_m) < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$.Hence sequence $\{x_n\}$ is a Cauchy sequence. □

In 1998, Jungck and Rhoades [12] introduced the following concept of weak compatibility.

Definition 4. *Let* A *and* S *be mappings from a* D^**-metric space* (X, D^*) *into itself. Then the mappings are said to be weak compatible if they commute at their coincidence point, that is,* $Ax = Sx$ *implies that* $ASx = SAx$.

Let (X, D^*) be a D^*-metric space, for $A, B, C \subseteq X$, define

$$\delta_{D^*}(A, B, C) = sup\{D^*(a, b, c); a \in A, b \in B, c \in C\}.$$

If A consists of a single point a, we write $\delta_{D^*}(A,B,C) = \delta_{D^*}(a,B,C)$. If B and C also consists of a single point b and c respectively, we write $\delta_{D^*}(A,B,C) = D^*(a,b,c)$.

It follows immediately from the definition that

$$\begin{aligned} \delta_{D^*}(A,B,C) &= 0 \Longleftrightarrow A = B = C = \{a\}, \\ \delta_{D^*}(A,B,C) &= \delta_{D^*}(p\{A,B,C\}) \geq 0, \end{aligned}$$

(symmetry) where p is a permutation function, for all $A,B,C \subseteq X$. In particular for $\emptyset \neq A = B = C \subset X$,

$$\delta_{D^*}(A) = sup\{D^*(a,b,c); a,b,c \in A\}.$$

It follows immediately from the definition that:

(i): If $A \subseteq B$, then $\delta_{D^*}(A) \leq \delta_{D^*}(B)$.

For a sequence $A_n = \{x_n, x_{n+1}, x_{n+2}, \cdots\}$ in D^*- metric space (X, D^*), let $a_n = \delta_{D^*}(A_n)$ for $n \in \mathbb{N}$ .Then

(a): by (i), since $A_n \supseteq A_{n+1}$ hence $a_n \leq a_{n+1}$,

(b): $D^*(x_l, x_m, x_k) \leq \delta_{D^*}(A_n) = a_n$ for every $l, m, k \geq n$,

(c): $0 \leq \delta_{D^*}(A_n) = a_n$ and $a_{n+1} \leq a_n$ for every $n \geq 1$.

Therefore, $\{a_n\}$ is decreasing and bounded for all $n \in \mathbb{N}$, and so there exists an $0 \leq a$ such that $\lim_{n\to\infty} a_n = a$.

Lemma 5. *Let (X, D^*) be a D^*-metric space. If $\lim_{n\to\infty} a_n = 0$, then sequence $\{x_n\}$ is a Cauchy sequence.*

Proof. Since $\lim_{n\to\infty} a_n = 0$, we have that for every $\epsilon > 0$, there exists a $n_0 \in \mathbb{N}$ such that for every $n > n_0$, $|a_n - 0| < \epsilon$. That is $a_n = \delta_{D^*}(A_n) < \epsilon$.Then for $l, m, k \geq n > n_0$ by (b) we have

$$D^*(x_l, x_m, x_k) \leq \sup\{D^*(x_i, x_j, x_p) \mid x_i, x_j, x_p \in A_n\} = a_n < \epsilon.$$

Therefore, $\{x_n\}$ is a Cauchy sequence in X. □

2. Main Results

Now we give our main theorem.

Theorem 6. *Let f and g be self-mappings of a complete D^*-metric space (X, D^*) satisfying:*

(i) *$g(X) \subseteq f(X)$, and $f(X)$ is closed subset of X,*

(ii) *the pair (f, g) is weakly compatible,*

(iii)

$$D^*(gx, gy, gz) \leq \phi(D^*(fx, fy, fz)),$$

for every $x, y, z \in X$,where $\phi : [0,\infty) \longrightarrow [0,\infty)$ is a nondecreasing continuous function with $\phi(t) < t$ for every $t > 0$.

Then f and g have a unique common fixed point in X.

Proof. Let x_0 be an arbitrary point in X. By (i), we can choose a point x_1 in X such that $y_0 = gx_0 = fx_1$ and $y_1 = gx_1 = fx_2$. In general, there exists a sequence $\{y_n\}$ such that, $y_n = gx_n = fx_{n+1}$, for $n = 0, 1, 2, \cdots$. We prove that sequence $\{y_n\}$ is a Cauchy sequence. Let $A_n = \{y_n, y_{n+1}, y_{n+2}, \cdots\}$ and $a_n = \delta_{D^*}(A_n)$, $n \in \mathbb{N}$. Then we know $\lim_{n\to\infty} a_n = a$ for some $a \geq 0$.

Taking $x = x_{n+k}, y = x_{m+k}, z = x_{l+k}$ in (iii) for $k \geq 1$ and $m, n, l \geq 0$, we have

$$\begin{aligned} D^*(y_{n+k}, y_{m+k}, y_{l+k}) &= D^*(gx_{n+k}, gx_{m+k}, gx_{l+k}) \\ &\leq \phi(D^*(fx_{n+k}, fx_{m+k}, fx_{l+k})) \\ &= \phi(D^*(y_{n+k-1}, y_{m+k-1}, y_{l+k-1})) \end{aligned}$$

Since $D^*(y_{n+k-1}, y_{m+k-1}, y_{l+k-1}) \leq a_{k-1}$, for every $n, m, l \geq 0$ and ϕ is increasing in t, we get

$$D^*(y_{n+k}, y_{m+k}, y_{m+k}) \leq \phi(D^*(y_{n+k-1}, y_{m+k-1}, y_{l+k-1})).$$

Hence

$$\sup_{m,n,l\geq 0} \{D^*(y_{n+k}, y_{m+k}, y_{l+k}) \leq \phi(a_{k-1}).$$

Therefore, we have $a_k \leq \phi(a_{k-1})$. Letting $k \to \infty$, we get $a \leq \phi(a)$. If $a \neq 0$, then $a \leq \phi(a) < a$, which is a contradiction.Thus $a = 0$ and hence $\lim_{n\to\infty} a_n = 0$. Thus by Lemma 5 $\{y_n\}$ is a Cauchy sequence in X. By the completeness of X, there exists a $u \in X$ such that

$$\lim_{n\to\infty} y_n = \lim_{n\to\infty} gx_n = \lim_{n\to\infty} fx_{n+1} = u.$$

Let $f(X)$ is closed, there exist $v \in X$ such that $fv = u$. Now we show that $gv = u$. For this it is enough set x_n, x_n, v replacing x, y, z respectively , in inequality (iii) we get

$$D^*(gx_n, gx_n, gv) \leq \phi(D^*(fx_n, fx_n, fv))$$

Taking $n \longrightarrow \infty$, we get

$$D^*(u, u, gv) \leq \phi(D^*(0)) = 0,$$

it implies $gv = u$.

Since the pair (f, g) are weakly compatible , hence we get, $gfv = fgv$. Thus $fu = gu$. Now we prove that $gu = u$. If we substitute x, y and z in (iii) by x_n, x_n and u respectively, we get

$$D^*(gx_n, gx_n, gu) \leq \phi(D^*(fx_n, fx_n, fu))$$

Taking $n \longrightarrow \infty$, we get

$$D^*(u, u, gu) \leq \phi(D^*(u, u, gu))$$

If $gu \neq u$, then $D^*(u,u,gu) < D^*(u,u,gu)$, is contradiction. Therefore,

$$fu = gu = u.$$

For the uniqueness, let u and u' be fixed points of f, g. Taking $x = y = u$ and $z = u'$ in (iii), we have

$$\begin{aligned} D^*(u,u,u') &= D^*(gu,gu,gu') \\ &\leq \phi(D^*(fu,fu,fu')) \\ &= \phi(D^*(u,u,u')) < D^*(u,u,u'), \end{aligned}$$

which is a contradiction. Thus we have $u = u'$. □

Corollary 7. *Let f , g and h be self-mappings of a complete D^*-metric space (X, D^*) satisfying:*

(i) *$g(X) \subseteq fh(X)$, and $fh(X)$ is closed subset of X,*

(ii) *the pair (fh, g) is weakly compatible and $fh = hf$, $gh = hg$*

(iii)

$$D^*(gx,gy,gz) \leq \phi(D^*(fhx,fhy,fhz)),$$

for every $x, y, z \in X$,where $\phi : [0,\infty) \longrightarrow [0,\infty)$ is a nondecreasing continuous function with $\phi(t) < t$ for every $t > 0$.

Then f , g and h have a unique common fixed point in X.

Proof. By Theorem 6 there exist a fixed point $u \in X$ such that $fhu = gu = u$. Now , we prove that $hu = u$. If $hu \neq u$ in (iii), then we have

$$\begin{aligned} D^*(hu,u,u) &= D^*(hgu,gu,gu) \\ &= D^*(ghu,gu,gu) \\ &\leq \phi(D^*(fhhu,fhu,fhu)) = \phi(D^*(hu,u,u)) \\ &< D^*(hu,u,u), \end{aligned}$$

which is a contradiction. Thus we have $hu = u$. Therefore,

$$fu = fhu = u = hu = gu.$$

□

Corollary 8. *Let g be self-mapping of a complete D^*-metric space (X, D^*) satisfying:*

$$D^*(g^nx,g^ny,g^nz) \leq \phi(D^*(x,y,z)),$$

for every $x, y, z \in X$ and $n \in \mathbb{N}$,where $\phi : [0,\infty) \longrightarrow [0,\infty)$ is a nondecreasing continuous function with $\phi(t) < t$ for every $t > 0$.

Then g have a unique common fixed point in X.

Proof. If we define $f = I$ identity map in Theorem 6. Hence all conditions of Theorem 2.1 hold and therefore there exists a unique $u \in X$ such that $g^n u = u$. Thus

$$g^n(gu) = g(g^n u) = gu.$$

Since u is unique, we have $gu = u$. □

Corollary 9. *Let f and g be self-mappings of a complete D^*-metric space (X, D^*) satisfying:*

(i) *$g^n(X) \subseteq f^m(X)$, and $f^m(X)$ is closed subset of X,*

(ii) *the pair (f^m, g^n) is weakly compatible and $f^m g = g f^m$, $g^n f = f g^n$*

(iii)

$$D^*(g^n x, g^n y, g^n z) \leq \phi(D^*(f^m x, f^m y, f^m z)),$$

for every $x, y, z \in X$ and $n, m \in \mathbb{N}$,where $\phi : [0, \infty) \longrightarrow [0, \infty)$ is a nondecreasing continuous function with $\phi(t) < t$ for every $t > 0$.

Then f and g have a unique common fixed point in X.

Proof. By Theorem 6 there exist a fixed point $u \in X$ such that $f^m u = g^n u = u$. On the other hand, we have

$$gu = g(g^n u) = g^n(gu) \text{ and } gu = g(f^m u) = f^m(gu).$$

Since u is unique, we have $gu = u$. Similarly, we have $fu = u$. □

Corollary 10. *Let (X, D^*) be a complete D^*-metric space and let $f_1, f_2, \cdots, f_n, g : X \longrightarrow X$ be maps that satisfy the following conditions:*

(a) $g(X) \subseteq f_1 f_2 \cdots f_n(X)$;

(b) the pair $(f_1 f_2 \cdots f_n, g)$ is weak compatible, $f_1 f_2 \cdots f_n(X)$ is closed subset of X;

(c)

$$D^*(gx, gy, gz) \leq \phi(D^*(f_1 f_2 \cdots f_n(x), f_1 f_2 \cdots f_n(y), f_1 f_2 \cdots f_n(z))),$$

for all $x, y, z \in X$ and $n \in \mathbb{N}$,where $\phi : [0, \infty) \longrightarrow [0, \infty)$ is a nondecreasing continuous function with $\phi(t) < t$ for every $t > 0$;

(d) $g(f_2 \cdots f_n) = (f_2 \cdots f_n)g$,
$g(f_3 \cdots f_n) = (f_3 \cdots f_n)g$,
$\vdots$
$g f_n = f_n g$,
$f_1(f_2 \cdots f_n) = (f_2 \cdots f_n) f_1$,
$f_1 f_2 (f_3 \cdots f_n) = (f_3 \cdots f_n) f_1 f_2$,
$\vdots$
$f_1 \cdots f_{n-1}(f_n) = (f_n) f_1 \cdots f_{n-1}$.

Then $f_1, f_2, \cdots, f_n, g$ have a unique common fixed point.

Proof. By Corollarly7, if set $f_1 f_2 \cdots f_n = f$ then f, g have a unique common fixed point in X. That is, there exists $x \in X$, such that $f_1 f_2 \cdots f_n(x) = g(x) = x$. We prove that $f_i(x) = x$, for $i = 1, 2, \cdots$.From (c), we have

$$D^*(g(f_2 \cdots f_n x), g(x), g(x)) \leq \phi(D^*(f_1 f_2 \cdots f_n(f_2 \cdots f_n x), f_1 f_2 \cdots f_n(x), f_1 f_2 \cdots f_n(x))).$$

By (d), we get

$$\begin{aligned} D^*(f_2 \cdots f_n x, x, x) &\leq \phi(D^*(f_2 \cdots f_n x, x, x)) \\ &< D^*(f_2 \cdots f_n x, x, x). \end{aligned}$$

Hence, $f_2 \cdots f_n(x) = x$. Thus , $f_1(x) = f_1 f_2 \cdots f_n(x) = x$.
Similarly, we have $f_2(x) = \cdots f_n(x) = x$. □

Now, we give one example to validate Theorem 2.1.

Example 2. *Let (X, D^*) be a complete D^*-metric space, where $X = [0, 2]$ and*

$$D^*(x, y, z) = |x - y| + |y - z| + |z - x|.$$

Define self-maps f and g on X as follows: $fx = \frac{x+1}{2}$ and $gx = \frac{x+5}{6}$,for all $x \in X$.

Let $\phi(t) = \frac{1}{2}t$. Then , we have

$$\begin{aligned} D^*(gx, gy, gz) = \frac{1}{6}(|x - y| + |y - z| + |z - x|) \leq &\ \tfrac{1}{4}(|x - y| + |y - z| + |x - z|) \\ &= \phi(D^*(fx, fy, fz). \end{aligned}$$

That is all conditions of Theorem 6 are holds and 1 is the unique common fixed point of f and g.

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