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A COMMON UNIQUE RANDOM FIXED POINT THEOREMS IN S-METRIC SPACES

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Abstract. In this paper, we present some new definitions of S-metric spaces and prove some random fixed point theorem for two random functions in complete S-metric spaces. We get some improved versions of several fixed point theorems in S-metric spaces.

Key words: D^*-metric space, S-metric space, common fixed point theorem.
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1. Introduction

In 1922, the Polish mathematician, Banach, proved a theorem which ensures, under appropriate conditions, the existence and uniqueness of a fixed point. His result is called Banach's fixed point theorem or the Banach contraction principle. This theorem provides a technique for solving a variety of applied problems in mathematical science and engineering. Many authors have extended, generalized and improved Banach's fixed point theorem in different ways. In [8] Jungck introduced more generalized commuting mappings, called *compatible* mappings, which are more general than commuting and weakly commuting mappings. This concept has been useful for obtaining more comprehensive fixed point theorems(see, e.g.,[1, 3, 4, 5, , 9, 11, 16, 19, 20, 22, 23]. One such generalization is generalized metric space or D-metric space initiated by Dhage [6] in 1992. He proved some results on fixed points for a self-map satisfying a contraction for complete and bounded D-metric spaces. Rhoades

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[17] generalized Dhage's contractive condition by increasing the number of factors and proved the existence of unique fixed point of a self-map in D-metric space. Recently, motivated by the concept of compatibility for metric space, Singh and Sharma [22] introduced the concept of D-compatibility of maps in D-metric space and proved some fixed point theorems using a contractive condition.Naidu et.al.[12, 13, 14] observed that almost all fixed point theorems in D-metric space are not valid or of doubtful validity. Also, Sedghi and Shobe [18, 19, 20] introduced D^*-metric space by modifying the tetrahedral inequality in D-metric space and proved some basic result in it. In this paper, we introduce D^*-metric which is a probable modification of the definition of D-metric introduced by Dhage [6] and prove some basic properties in D^*-metric spaces. We also prove a common fixed point theorem for six mappings under the condition of weakly compatible mappings in D^*-metric spaces.

In what follows (X, D^*) will denote a D^*-metric space, $\mathbb{N}$ the set of all natural numbers, and $\mathbb{R}^+$ the set of all positive real numbers.

The definition of D*-metric as follows:

Definition 1. *Let X be a nonempty set. A generalized metric (or D^*-metric) on X is a function: $D^* : X^3 \longrightarrow [0.\infty)$ that satisfies the following conditions for each $x, y, z, a \in X$.*

(1) $D^*(x, y, z) \geq 0$,

(2) $D^*(x, y, z) = 0$ if and only if $x = y = z$,

(3) $D^*(x, y, z) = D^*(p\{x, y, z\})$,(symmetry) where p is a permutation function,

(4) $D^*(x, y, z) \leq D^*(x, y, a) + D^*(a, z, z)$.

The pair (X, D^*) is called a generalized metric (or D^*-metric) space.

In this paper we introduce new concept of a generalized metric space which is more generalized than D*-metric space, that is S- metric space and prove some basic properties and some fixed point theorems in S-metric spaces.

Definition 2. *Let X be a nonempty set. A generalized metric (or S-metric) on X is a function: $S : X^3 \longrightarrow [0, \infty)$ that satisfies the following conditions for each $x, y, z, a \in X$,*

(1) $S(x, y, z) \geq 0$,

(2) $S(x, y, z) = 0$ if and only if $x = y = z$,

(3) $S(x, y, z) \leq S(a, y, z) + S(a, x, x)$.

The pair (X, S) is called a generalized metric (or S-metric) space.

Immediate examples of such a function are

(a) If $X = \mathbb{R}^n$ then we define

$$S(x, y, z) = ||y + x - 2z|| + ||y - z||.$$

(b) $S(x, y, z) = d(x, y) + d(x, z)$ here, d is the ordinary metric on X.

(c) If $X = \mathbb{R}^n$ then we define

$$S(x, y, z) = ||x - z|| + ||y - z||$$

(d) If $X = \mathbb{R}$ then we define

$$S(x, y, z) = |a^{y+z} - a^{2x}| + |y - z|,$$

for every $x, y, x \in \mathbb{R}, a > 0$ *and* $a \neq 1$.

(e)

$$S(x, y, z) = |a^{d(x,y)} - a^{d(y,z)}| + d(y, z),$$

for every $x, y, z \in X, a > 0$ *and* $a \neq 1$. Here, d is an ordinary metric on X.

Remark 1. *In a S-metric space, we prove that $S(x, y, y) = S(y, x, x)$. Because by (3) and (2) of Definition 2 we have:*

(i) $S(x, y, y) \leq S(y, y, y) + S(y, x, x) = S(y, x, x)$ and similarly

(ii) $S(y, x, x) \leq S(x, x, x) + S(x, y, y) = S(x, y, y)$.

Hence by (i),(ii) we get $S(x, y, y) = S(y, x, x)$.

Remark 2. *Let (X, S) be a S-metric space. If we define $f : X^2 \longrightarrow [0, \infty)$ as $f(x, y) = S(x, y, y)$ for all $x, y \in X$ then f is an ordinary metric on X.*

Proof. Clearly $f(x, y) \geq 0$ for all $x, y \in X$ and $f(x, y) = 0$ iff $x = y$.
$f(x, y) = S(x, y, y) = S(y, x, x) = f(y, x)$ from Remark 1.
From Definition 2 we have

$$\begin{aligned} f(x, y) &= S(x, y, y) \\ &\leq S(z, y, y) + S(z, x, x) = f(z, y) + f(z, x). \end{aligned}$$

Thus f is a metric on X. □

Let (X, S) be a S-metric space. For $r > 0$ define

$$B_S(x, r) = \{y \in X : S(x, y, y) < r\}.$$

Example 1. *Let $X = \mathbb{R}$. Denote $S(x, y, z) = |3^{y+z} - 3^{2x}| + |y - z|$ for all $x, y, z \in \mathbb{R}$. Thus*

$$\begin{aligned} B_S(1, 2) &= \{y \in \mathbb{R} : S(1, y, y) < 2\} = \left\{y \in \mathbb{R} : |3^{2y} - 3^2| < 2\right\} \\ &= \left\{y \in \mathbb{R} : \tfrac{\lg_3^7}{2} < y < \tfrac{\lg_3^{11}}{2}\right\} = (\tfrac{\lg_3^7}{2}, \tfrac{\lg_3^{11}}{2}). \end{aligned}$$

Definition 3. *Let (X, S) be a S-metric space and $A \subset X$.*

(1) If for every $x \in A$ there exists $r > 0$ such that $B_S(x, r) \subset A$, then subset A is called open subset of X.

(2) Subset A of X is said to be S-bounded if there exists $r > 0$ such that $S(x, y, y) < r$ for all $x, y \in A$.

(3) A sequence $\{x_n\}$ in X converges to x if and only if $S(x_n, x, x) = S(x, x_n, x_n) \to 0$ as $n \to \infty$. That is for each $\epsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that

$$\forall n \geq n_0 \Longrightarrow S(x, x_n, x_n) < \epsilon.$$

(4) Sequence $\{x_n\}$ *in* X *is called a Cauchy sequence if for each* $\epsilon > 0$ *, there exists* $n_0 \in \mathbb{N}$ *such that* $S(x_n, x_m, x_m) < \epsilon$ *for each* $n, m \geq n_0$*. The* S*-metric space* (X, S) *is said to be complete if every Cauchy sequence is convergent.*

Let τ be the set of all $A \subset X$ with $x \in A$ if and only if there exists $r > 0$ such that $B_S(x, r) \subset A$. Then τ is a topology on X (induced by the S-metric S).

Lemma 1. *Let* (X, S) *be a* S*-metric space. If* $r > 0$*, then ball* $B_S(x, r)$ *with center* $x \in X$ *and radius* r *is open ball.*

Proof. Let $z \in B_S(x, r)$, hence $S(x, z, z) < r$. If set $S(x, z, z) = \delta$ and $r' = r - \delta$ then we prove that $B_S(z, r') \subseteq B_S(x, r)$. Let $y \in B_S(z, r')$, by triangular inequality we have $S(x, y, y) = S(y, x, x) \leq S(z, x, x) + S(z, y, y) < r' + \delta = r$. Hence $B_S(z, r') \subseteq B_S(x, r)$. That is ball $B_S(x, r)$ is open ball. □

Lemma 2. *Let* (X, S) *be a* S*- metric space. If there exist sequences* $\{x_n\}$ *and* $\{y_n\}$ *such that* $x_n \longrightarrow x$ *and* $y_n \longrightarrow y$*, then* $S(x_n, y_n, y_n) \longrightarrow S(x, y, y)$*.*

Proof. Since sequence $\{(x_n, y_n, y_n)\}$ in X^3 converges to a point $(x, y, y) \in X^3$ i.e.

$$\lim_{n\to\infty} x_n = x, \lim_{n\to\infty} y_n = y,$$

for each $\epsilon > 0$ there exist
$n_1 \in \mathbb{N}$ such that for every $n \geq n_1 \Longrightarrow S(x, x_n, x_n) < \frac{\epsilon}{2}$
and
$n_2 \in \mathbb{N}$ such that for every $n \geq n_2 \Longrightarrow S(y, y_n, y_n) < \frac{\epsilon}{2}$.
If $n_0 = \max\{n_1, n_2\}$, then for every $n \geq n_0$ by triangular inequality we have

$$\begin{aligned} S(x_n, y_n, y_n) &\leq S(x, y_n, y_n) + S(x, x_n, x_n) \\ &\leq S(y, y_n, y_n) + S(y, x, x) + S(x, x_n, x_n) \\ &< \frac{\epsilon}{2} + \frac{\epsilon}{2} + S(y, x, x) = S(y, x, x) + \epsilon. \end{aligned}$$

Hence we have

$$S(x_n, y_n, y_n) - S(y, x, x) < \epsilon.$$

On the other hand

$$\begin{aligned} S(y, x, x) &\leq S(x_n, x, x) + S(x_n, y, y) \\ &\leq S(x_n, x, x) + S(y_n, y, y) + S(y_n, x_n, x_n) \\ &< \frac{\epsilon}{2} + \frac{\epsilon}{2} + S(x_n, y_n, y_n) = S(x_n, y_n, y_n) + \epsilon. \end{aligned}$$

That is,

$$S(y, x, x) - S(x_n, y_n, y_n) < \epsilon.$$

Therefore we have $|S(x_n, y_n, y_n) - S(x, y, y)| < \epsilon$, i.e.

$$\lim_{n\to\infty} S(x_n, y_n, y_n) = S(x, y, y)$$

□

Lemma 3. *Let (X, S) be a S-metric space. If sequence $\{x_n\}$ in X converges to x, then x is unique.*

Proof. Let $x_n \longrightarrow y$ and $y \neq x$. Since $\{x_n\}$ converges to x and y, for each $\epsilon > 0$ there exist
$n_1 \in \mathbb{N}$ such that for every $n \geq n_1 \Longrightarrow S(x_n, x, x) < \frac{\epsilon}{2}$
and
$n_2 \in \mathbb{N}$ such that for every $n \geq n_2 \Longrightarrow S(x_n, y, y) < \frac{\epsilon}{2}$.
If $n_0 = \max\{n_1, n_2\}$, then for every $n \geq n_0$ by triangular inequality we have

$$S(x, y, y) \leq S(x_n, x, x) + S(x_n, y, y) < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \varepsilon.$$

Hence $S(x, y, y) = 0$ is a contradiction. So, $x = y$. □

Lemma 4. *Let (X, S) be a S-metric space. If sequence $\{x_n\}$ in X is converges to x, then sequence $\{x_n\}$ is a Cauchy sequence.*

Proof. Since $x_n \longrightarrow x$ for each $\epsilon > 0$ there exists
$n_1 \in \mathbb{N}$ such that for every $n \geq n_1 \Longrightarrow S(x, x_n, x_n) < \frac{\epsilon}{2}$
and
$n_2 \in \mathbb{N}$ such that for every $m \geq n_2 \Longrightarrow S(x, x_m, x_m) < \frac{\epsilon}{2}$.
If $n_0 = \max\{n_1, n_2\}$, then for every $n, m \geq n_0$ by triangular inequality we have

$$S(x_n, x_m, x_m) \leq S(x, x_n, x_n) + S(x, x_m, x_m) < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon.$$

Hence sequence $\{x_n\}$ is a Cauchy sequence. □

2. Main Results

Definition 4. *Let $F : \mathbb{R} \times X \longrightarrow X$ be a function, where X is a nonempty set. Then function $g : \mathbb{R} \longrightarrow X$ is said to be a random fixed point of the function F if $F(t, g(t)) = g(t)$ for all t in $\mathbb{R}$.*

We shall prove the following theorem.

Theorem 5. *Let (X, S) be a complete S- metric space and let $F, G : \mathbb{R} \times X \longrightarrow X$ be two functions satisfying the following condition:*

$$(i) \qquad \begin{aligned} & S(F(t,x), G(t,y), G(t,y)) \\ \leq\ & k_1.S(x, F(t,x), F(t,x)) + k_2.S(y, G(t,y), G(t,y)) + k_3.S(x,y,y), \end{aligned}$$

for every $x, y \in X$, $t \in \mathbb{R}$ where $k_i \geq 0$ for $i = 1, 2, 3$ and $0 < k_1 + k_2 + k_3 < 1$. Then F and G have a unique common random fixed point.

Proof. We define a sequence of functions $\{g_n\}$ as $g_n : \mathbb{R} \longrightarrow X$ is abitrary function for $t \in \mathbb{R}$, and $n = 0, 1, 2, 3, \cdots$

$$g_{2n+1}(t) = F(t, g_{2n}(t)), g_{2n+2}(t) = G(t, g_{2n+1}(t)).$$

If $g_{2n}(t) = g_{2n+1}(t) = g_{2n+2}(t)$ for $t \in \mathbb{R}$, for some n then we set that $g_{2n}(t)$ is a random fixed point of F and G. Therefore, we suppose that no two consecutive terms of sequence $\{g_n\}$ are equal. Now by using (i) for all $t \in \mathbb{R}$ we have

$$\begin{aligned} & S(g_{2n+1}(t), g_{2n+2}(t), g_{2n+2}(t)) \\ = \ & S(F(t, g_{2n}(t)), G(t, g_{2n+1}(t)), G(t, g_{2n+1}(t))) \\ \leq \ & k_1 S(g_{2n}(t), F(t, g_{2n}(t)), F(t, g_{2n}(t))) \\ + \ & k_2 S(g_{2n+1}(t), G(t, g_{2n+1}(t)), G(t, g_{2n+1}(t))) \\ + \ & k_3 S(g_{2n}(t), g_{2n+1}(t), g_{2n+1}(t)). \end{aligned}$$

Therefore,

$$\begin{aligned} S(g_{2n+1}(t), g_{2n+2}(t), g_{2n+2}(t)) & \leq \frac{k_1 + k_3}{1 - k_2} S(g_{2n}(t), g_{2n+1}(t), g_{2n+1}(t)) \\ & \vdots \\ & \leq (\frac{k_1 + k_3}{1 - k_2})^{2n+1} S(g_0(t), g_1(t), g_1(t)). \end{aligned}$$

Similarly we have

$$S(g_{2n}(t), g_{2n+1}(t), g_{2n+1}(t)) \leq (\frac{k_1 + k_3}{1 - k_2})^{2n} S(g_0(t), g_1(t), g_1(t)).$$

Thus for every $n \in \mathbb{N}$ we get,

$$S(g_n(t), g_{n+1}(t), g_{n+1}(t)) \leq (\frac{k_1 + k_3}{1 - k_2})^{n} S(g_0(t), g_1(t), g_1(t)).$$

Now we show that $\{g_n(t)\}$ is Cauchy sequence.

$$\begin{aligned} & S(g_n(t), g_m(t), g_m(t)) \\ \leq \ & S(g_{n+1}(t), g_m(t), g_m(t)) + S(g_{n+1}(t), g_n(t), g_n(t)) \\ \leq \ & S(g_{n+2}(t), g_m(t), g_m(t)) + S(g_{n+2}(t), g_{n+1}(t), g_{n+1}(t)) \\ + \ & S(g_{n+1}(t), g_n(t), g_n(t)) \\ & \vdots \\ \leq \ & S(g_{m-1}(t), g_m(t), g_m(t)) + \cdots + S(g_{n+2}(t), g_{n+1}(t), g_{n+1}(t)) \\ + \ & S(g_{n+1}(t), g_n(t), g_n(t)) \\ = \ & S(g_{m-1}(t), g_m(t), g_m(t)) + \cdots + S(g_{n+1}(t), g_{n+2}(t), g_{n+2}(t)) \\ + \ & S(g_n(t), g_{n+1}(t), g_{n+1}(t)). \end{aligned}$$

If $q = \frac{k_1+k_3}{1-k_2}$ then

$$\begin{aligned} & S(g_n(t), g_m(t), g_m(t)) \\ \leq\ & q^{m-1}S(g_0(t), g_1(t), g_1(t)) + q^{m-2}S(g_0(t), g_1(t), g_1(t)) \\ +\ & \cdots + q^n S(g_0(t), g_1(t), g_1(t)) \\ =\ & \frac{q^n - q^m}{1-q} S(g_0(t), g_1(t), g_1(t)) \\ \leq\ & \frac{q^n}{1-q} S(g_0(t), g_1(t), g_1(t)) \longrightarrow 0. \end{aligned}$$

Thus, $\{g_n(t)\}$ is Cauchy and by the completeness of X, $\{g_n(t)\}$ converges to $g(t)$ in X. Now we prove that $F(t, g(t)) = g(t)$. Replace $x = g(t)$ and $y = g_{2n+1}(t)$ in inequality (i) we have

$$\begin{aligned} & S(F(t, g(t)), G(t, g_{2n}(t)), G(t, g_{2n}(t))) \\ \leq\ & k_1 S(g(t), F(t, g(t)), F(t, g(t))) + k_2 S(g_{2n}(t), G(t, g_{2n}(t)), G(t, g_{2n}(t))) \\ +\ & k_3 S(g(t), g_{2n}(t), g_{2n}(t)). \end{aligned}$$

On making $n \to \infty$ in the above inequality we get

$$\begin{aligned} & S(F(t, g(t)), g(t), g(t)) \\ \leq\ & k_1 S(g(t), F(t, g(t)), F(t, g(t))) + k_2 S(g(t), g(t), g(t)) + k_3 S(g(t), g(t), g(t)) \\ =\ & k_1 S(g(t), F(t, g(t)), F(t, g(t))). \end{aligned}$$

Therefore $S(g(t), F(t, g(t)), F(t, g(t))) = 0$ that is $F(t, g(t)) = g(t)$. Replace $x = g(t)$ and $y = g(t)$ in inequality (i) we have

$$\begin{aligned} & S(F(t, g(t)), G(t, g(t)), G(t, g(t))) \\ \leq\ & k_1 S(g(t), F(t, g(t)), F(t, g(t))) + k_2 S(g(t), G(t, g(t)), G(t, g(t))) \\ +\ & k_3 S(g(t), g(t), g(t)) = k_2 S(g(t), G(t, g(t)), G(t, g(t))). \end{aligned}$$

Therefore $S(F(t, g(t)), G(t, g(t)), G(t, g(t)) = 0$ that is $F(t, g(t)) = G(t, g(t)) = g(t)$ Thus $g(t)$ is a common random fixed point of F and G.

Now to prove uniqueness let if possible $h(t) \neq g(t)$ be another common random fixed point of F and G. Then by inequality (i) we have

$$\begin{aligned} & S(g(t), h(t), h(t)) = S(F(t, g(t)), G(t, h(t)), G(t, h(t)) \\ \leq\ & k_1 S(g(t), F(t, g(t)), F(t, g(t))) + k_2 S(h(t), G(t, h(t)), G(t, h(t))) \\ +\ & k_3 S(g(t), h(t), h(t)) \\ =\ & k_3 S(g(t), h(t), h(t)). \end{aligned}$$

Therefore $S(g(t), h(t), h(t)) = 0$ that is $g(t) = h(t)$. Thus $g(t)$ is a unique common random fixed point of F and G. □

Corollary 6. *Let (X,S) be a complete S- metric space and let $F:\mathbb{R}\times X\longrightarrow X$ be a function satisfying the following condition:*

$$\begin{aligned} & S(F(t,x),F(t,y),F(t,y)) \\ \leq\ & k_1.S(x,F(t,x),F(t,x))+k_2.S(y,F(t,y),F(t,y))+k_3.S(x,y,y), \end{aligned}$$

for every $x,y\in X$, $t\in\mathbb{R}$ where $k_i\geq 0$ for $i=1,2,3$ and $0<k_1+k_2+k_3<1$. Then F have a unique common random fixed point.

Proof. By Theorem 5, it is enough set $F(t,y)=G(t,y)$. □

Corollary 7. *Let (X,S) be a complete S- metric space and let $F:\mathbb{R}\times X\longrightarrow X$ be a function satisfying the following condition:*

$$S(F(t,x),F(t,y),F(t,y))\leq kS(x,y,y),$$

for every $x,y\in X$, $t\in\mathbb{R}$ where $0<k<1$.
Then F have a unique common random fixed point.

Proof. By Corollary 6, it is enough set $k_1=k_2=0$. □

Now we give an example to support our Corollary 7.

Example 2. *Let $X=\mathbb{R}$ and let S be the S-metric on $X\times X\times X$ defined as follows:*

$$S(x,y,z)=|x+y-2z|+|x-z|,$$

for all $x,y,z\in X$. Then (X,S) is a S- metric space. Define $F(t,x)=\frac{x\sin t-1}{4}$. Then

$$S(F(t,x),F(t,y),F(t,y))=\frac{1}{2}|\sin t||x-y|,$$

and

$$S(x,y,y)=2|x-y|.$$

Hence for $\frac{1}{4}\leq k<1$, all the conditions of Corollary 7 are satisfied and $g(t)=\frac{1}{\sin t-4}$ is a common random fixed point of F.

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