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COMMON FIXED POINTS BY ONE STEP ITERATIVE SCHEME WITH ERROR FOR ASYMPTOTICALLY QUASI-NONEXPANSIVE TYPE NONSELF MAPPINGS

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Abstract. The aim of this paper is to define a new one step iterative scheme with error for approximating common fixed points of asymptotically quasi-nonexpansive type non-self mappings in Banach space.

Key words: asymptotically quasi-nonexpansive type mappings, One step iterative scheme, common fixed point.
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1. Introduction & Preliminaries

Let X be a real Banach space and let C be a nonempty subset of X. Further, for a mapping $T : C \to C$, let $\varphi \neq F(T)$ be the set of all fixed points of T. A mapping $T : C \to C$ is said to be asymptotically quasi-nonexpansive if there exists a sequence $\{k_n\}$ of real numbers with $k_n \geq 1$ and $\lim_n k_n = 1$ such that for all $x \in C$,$q \in F(T)$,

$$\|T^n x - q\| \leq k_n \|x - q\| \tag{1}$$

for all $n \geq 1$
T is called asymptotically quasi-nonexpansive type [5] provided T is uniformly continuous and

$$\limsup_n \{\sup_{x \in C}(\|T^n x - q\| - \|x - q\|)\} \leq 0 \tag{2}$$

for all $q \in F(T)$.
The mapping T is called uniformly L-Lipschitzian if there exists a positive constant L such that

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$$\|T^n x - T^n y\| \le L\|x - y\| \tag{3}$$

for all $x, y \in C$ and all $n \ge 1$

The mapping $T : C \to C$ is completely continuous [6] if for any bounded sequence $\{x_n\} \subset C$ there exists a convergent subsequence of $\{Tx_n\}$.

Again a Banach space X is called uniformly convex [4] if for every $0 < \varepsilon \le 2$, there exists $\delta = \delta(\varepsilon) > 0$ such that $\|\frac{x+y}{2}\| \le 1 - \delta$ for every $x, y \in S_X$ and $\|x - y\| \ge \varepsilon$,where $S_X = \{x \in X : \|x\| = 1\}$

A Banach space X is called smooth [8], if $\lim\limits_{t \to 0} \frac{\|x+ty\|-\|x\|}{t}$ exists for all $x, y \in S_X$, $S_X = \{x \in X : \|x\| = 1\}$.

Recently, H. Y. Zhou, Y. J. Cho and S. M. Kang [8] gave a new iterative scheme for approximating common fixed points of two asymptotically nonexpansive nonself mappings with respect to P as follows: For a given $x_1 \in C$, compute the sequence $\{x_n\}$ by the iterative scheme

$$x_{n+1} = \alpha_n x_n + \beta_n (PT_1)^n x_n + \gamma_n (PT_2)^n x_n \tag{4}$$

for all $n \ge 1$

where $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ are real sequences in $[0, 1]$ .They satisfying $\alpha_n + \beta_n + \gamma_n = 1$ and prove some weak and strong convergence theorems for such mappings in a uniformly convex Banach space.

In this paper, we define a new iterative scheme with error and study the approximating common fixed point of two asymptotically quasi- nonexpansive type nonself mappings with respect to P.

Now, we recall the well known concepts to prove our main result.

Definition 1. *A subset C of X is called retract of X if there exists a continuous mapping $P : X \to C$ such that $Px = x$ for all $x \in C$. Every closed convex subset of a uniformly convex Banach space is a retract. A mapping $P : X \to C$ is called retraction if $P^2 = P$. Note that, if a mapping P is a retraction, then $Pz = z$ for all z in the range of P.*

Definition 2. *Let D, E be subsets of a Banach space X. Then, a mapping $P : D \to E$ is said to be sunny if $P(Px + t(x - Px)) = Px$ whenever $Px + t(x - Px) \in D$ for all $x \in D$ and $t \ge 0$.*

Definition 3. *Let C be a subset of a Banach space X. For all $x \in C$ is defined a set $I_C(x)$ by $I_C(x) = \{x + \lambda(y - x) : \lambda > 0, y \in C\}$. A nonself mapping $T : C \to X$ is said to be inward if $Tx \in I_C(x)$ for all $x \in C$, and T is said to be weakly inward if $Tx \in \overline{I_C(x)}$ for all $x \in C$.*

Let X be a real normed linear space and let C be a nonempty closed convex subset of X. Let $P : X \to C$ be the nonexpansive retraction of X onto C and let $T_1 : C \to X$ and $T_2 : C \to X$ be two asymptotically quasi- nonexpansive type nonself mappings.

Algorithm 1: For a given $x_1 \in C$, compute the sequence $\{x_n\}$ by the iterative scheme

$$x_{n+1} = (1 - \alpha_n - \gamma_n)x_n + \alpha_n (PT_1)^n x_n + \gamma_n (PT_2)^n x_n + \delta_n u_n \tag{5}$$

for all $n \geq 1$

where $\{\alpha_n\}, \{\gamma_n\}, \{\delta_n\}$ are real sequences in $[0, 1]$ and satisfying $\alpha_n + \gamma_n + \delta_n = 1$ and $\{u_n\}$ is bounded sequence in C. The iterative scheme (5) is called the one-step iterative scheme with error.

Definition 4. [8] *Let C be a nonempty subset of real normed linear space X. Let $P : X \to C$ be a nonexpansive retraction of X onto C. Then $T : C \to X$ is said to be L-Lipschitzian nonself mapping with respect to P if there exists a constant $L > 0$ such that*

$$\|(PT)^n x - (PT)^n y\| \leq \|x - y\| \tag{6}$$

for all $x, y \in C$,and $n \geq 1$

Lemma 1. [7] *Let $\{a_n\}$ and $\{b_n\}$ be sequences of nonnegative real numbers such that $a_{n+1} \leq a_n + b_n$ for all $n \geq 1$. If $\sum_{n=1}^{\infty} b_n < \infty$, then $\lim_n a_n$ exists.*

Lemma 2. [2] *Let X be a uniformly convex Banach space and $B_r = \{x \in X : \|x\| < r, r > 0\}$. Then there exists a continuous strictly increasing convex function $g : [0, \infty) \to [0, \infty)$ with $g(0) = 0$ such that*

$$\|\lambda x + \mu y + \xi z + \nu w\|^2 \leq \lambda\|x\|^2 + \mu\|y\|^2 + \xi\|z\|^2 + \nu\|w\|^2 - \lambda\mu g(\|x - y\|)$$

for all $x, y, z, w \in B_r$ and $\lambda, \mu, \xi, \nu \in [0, 1]$ with$\lambda + \mu + \xi + \nu = 1$.

Lemma 3. [8] *Let X be a real smooth Banach space and let C be a nonempty closed subset of X with P as a sunny nonexpansive retraction and let $T : C \to X$ be a mapping satisfying weakly inward condition. Then $F(PT) = F(T)$.*

Definition 5. *Let C be a nonempty subset of a real normed linear space X. Let $P : X \to C$ be a nonexpansive retraction of X onto C, and $F(T) \neq \varphi$ be the set of all fixed points of T. Then $T : X \to C$ is called asymptotically quasi- nonexpansive type nonself mapping with respect to P if T is uniformly continuous and*

$$\limsup_{n} \{\sup_{x \in C} (\|(PT)^n x - q\| - \|x - q\|)\} \leq 0 \tag{7}$$

for all $q \in F(T)$

Remark 1. *If T is self mapping, then P become the identity mapping, so that (6) and (7) are reduced to (3) and (4).*

2. Main Results

Lemma 4. *Let X be a uniformly convex Banach space, C a nonempty closed convex subset of X and $T_1, T_2 : C \to X$ be two asymptotically quasi- nonexpansive type nonself mappings with respect to P, a nonexpansive retraction of X onto C. Put*

$$G_n = \max\{0, \sup_{x\in C}(\|(PT_1)^n x - q\| - \|x - q\|)\}$$

and

$$K_n = \max\{0, \sup_{x\in C}(\|(PT_2)^n x - q\| - \|x - q\|)\}$$

such that $\sum_{n=1}^{\infty} G_n < \infty$ and $\sum_{n=1}^{\infty} K_n < \infty$ respectively. Suppose that $\{x_n\}$ is a sequence defined by (5) with $\sum_{n=1}^{\infty} \delta_n < \infty$. If $F(T_1) \cap F(T_2) \neq \varphi$, then $\lim_{n\to\infty} \|x_n - q\|$ exists for any $q \in F(T_1) \cap F(T_2)$.

Proof. For any $q \in F(T_1) \cap F(T_2)$, by (5) we have ,

$$\begin{aligned}
\|x_{n+1} - q\| &= \|[(1-\alpha_n-\gamma_n)x_n + \alpha_n(PT_1)^n x_n + \gamma_n(PT_2)^n x_n + \delta_n u_n] - q\| \\
&\le (1-\alpha_n-\gamma_n)\|x_n - q\| + \alpha_n\|(PT_1)^n x_n - q\| \\
&+\gamma_n\|(PT_2)^n x_n - q\| + \delta_n\|u_n - q\| \\
&\le (1-\alpha_n-\gamma_n)\|x_n - q\| + \alpha_n(\|(PT_1)^n x_n - q\| - \|x_n - q\|) \\
&+\alpha_n\|x_n - q\| + \gamma_n(\|(PT_2)^n x_n - q\| - \|x_n - q\|) + \gamma_n\|x_n - q\| \\
&+\delta_n\|u_n - q\| \\
&\le (1-\alpha_n-\gamma_n+\alpha_n+\gamma_n)\|x_n - q\| \\
&+\alpha_n \sup_{x\in C}(\|(PT_1)^n x_n - q\| - \|x_n - q\|) \\
&+\gamma_n \sup_{x\in C}(\|(PT_2)^n x_n - q\| - \|x_n - q\|) + \delta_n\|u_n - q\| \\
&\le \|x_n - q\| + \alpha_n G_n + \gamma_n K_n + \delta_n\|u_n - q\| \\
&\le \|x_n - q\| + d_n
\end{aligned}$$

where $d_n = \alpha_n G_n + \gamma_n K_n + \delta_n\|u_n - q\|$ Since $\sum_{n=1}^{\infty} G_n < \infty, \sum_{n=1}^{\infty} K_n < \infty \& \sum_{n=1}^{\infty} \delta_n < \infty$ this implies that $\sum_{n=1}^{\infty} d_n < \infty$. Hence it follows from Lemma 1 that $\lim_{n\to\infty} \|x_n - q\|$ exists. This completes the proof of the Lemma. □

Lemma 5. *Let X be a uniformly convex Banach space, C a nonempty closed convex subset of X and $T_1, T_2 : C \to X$ be two asymptotically quasi- nonexpansive type nonself mappings with respect to P, a nonexpansive retraction of X onto C. Put*

$$G_n = \max\{0, \sup_{x\in C}(\|(PT_1)^n x - q\| - \|x - q\|)\}$$

and

$$K_n = \max\{0, \sup_{x\in C}(\|(PT_2)^n x - q\| - \|x - q\|)\}$$

such that $\sum_{n=1}^{\infty} G_n < \infty$ *and* $\sum_{n=1}^{\infty} K_n < \infty$ *respectively. Suppose that* $\{x_n\}$ *is a sequence defined by (5) with* $\sum_{n=1}^{\infty} \delta_n < \infty$ *and the additional assumption that* $0 < \lim_n \inf \alpha_n, 0 < \lim_n \inf \gamma_n$ *and* $0 < \lim_n \inf \delta_n$.
If $F(T_1) \cap F(T_2) \neq \varphi$, *then* $\lim_n \|x_n - (PT_1)x_n\| = \lim_n \|x_n - (PT_2)x_n\| = 0$.

Proof. By Lemma 4, $\lim\limits_{n\to\infty} \|x_n - q\|$ exists for any $q \in F(T_1) \cap F(T_2)$. It follows that $\{x_n - q\}$, $\{(PT_1)^n x_n - q\}$ and $\{(PT_2)^n x_n - q\}$ are bounded. Also, $\{u_n - q\}$is bounded by assumption. We may assume that such sequences belong to B_r, where $B_r = \{x \in X : \|x\| < r, r > 0\}$.
From (5) and Lemma 2, we have

$$\begin{aligned}
\|x_{n+1} - q\|^2 &= \|[(1-\alpha_n-\gamma_n)x_n + \alpha_n(PT_1)^n x_n + \gamma_n(PT_2)^n x_n + \delta_n u_n] - q\|^2 \\
&\leq \|(1-\alpha_n-\gamma_n)(x_n - q) + \alpha_n((PT_1)^n x_n - q) \\
&\quad + \gamma_n((PT_2)^n x_n - q) + \delta_n(u_n - q)\|^2 \\
&\leq (1-\alpha_n-\gamma_n)\|x_n - q\|^2 + \alpha_n\|(PT_1)^n x_n - q\|^2 \\
&\quad + \gamma_n\|(PT_2)^n x_n - q\|^2 + \delta_n\|u_n - q\|^2 \\
&\quad - (1-\alpha_n-\gamma_n)\alpha_n g(\|x_n - (PT_1)^n x_n\|) \\
&\leq (1-\alpha_n-\gamma_n)\|x_n - q\|^2 + \alpha_n(\|(PT_1)^n x_n - q\|^2 - \|x_n - q\|^2) \\
&\quad + \alpha_n\|x_n - q\|^2 + \gamma_n(\|(PT_2)^n x_n - q\|^2 - \|x_n - q\|^2) + \gamma_n\|x_n - q\|^2 \\
&\quad + \delta_n\|u_n - q\|^2 - (1-\alpha_n-\gamma_n)\alpha_n g(\|x_n - (PT_1)^n x_n\|)
\end{aligned} \tag{8}$$

and

$$\begin{aligned}
\|x_{n+1} - q\|^2 &= \|[(1-\alpha_n-\gamma_n)x_n + \alpha_n(PT_1)^n x_n + \gamma_n(PT_2)^n x_n + \delta_n u_n] - q\|^2 \\
&\leq \|(1-\alpha_n-\gamma_n)(x_n - q) + \alpha_n((PT_1)^n x_n - q) \\
&\quad + \gamma_n((PT_2)^n x_n - q) + \delta_n(u_n - q)\|^2 \\
&\leq (1-\alpha_n-\gamma_n)\|x_n - q\|^2 + \alpha_n\|(PT_1)^n x_n - q\|^2 \\
&\quad + \gamma_n\|(PT_2)^n x_n - q\|^2 + \delta_n\|u_n - q\|^2 \\
&\quad - (1-\alpha_n-\gamma_n)\gamma_n g(\|x_n - (PT_2)^n x_n\|) \\
&\leq (1-\alpha_n-\gamma_n)\|x_n - q\|^2 + \alpha_n(\|(PT_1)^n x_n - q\|^2 - \|x_n - q\|^2) \\
&\quad + \alpha_n\|x_n - q\|^2 + \gamma_n(\|(PT_2)^n x_n - q\|^2 - \|x_n - q\|^2) + \gamma_n\|x_n - q\|^2 \\
&\quad + \delta_n\|u_n - q\|^2 - (1-\alpha_n-\gamma_n)\gamma_n g(\|x_n - (PT_2)^n x_n\|)
\end{aligned} \tag{9}$$

From 8,

$$\begin{aligned}
\|x_{n+1} - q\|^2 &\leq (1-\alpha_n-\gamma_n)\|x_n - q\|^2 \\
&\quad + \alpha_n(\|(PT_1)^n x_n - q\| - \|x_n - q\|)(\|(PT_1)^n x_n - q\| + \|x_n - q\|) \\
&\quad + \gamma_n(\|(PT_2)^n x_n - q\| - \|x_n - q\|)(\|(PT_2)^n x_n - q\| + \|x_n - q\|) \\
&\quad + \alpha_n\|x_n - q\|^2 + \gamma_n\|x_n - q\|^2 + \delta_n\|u_n - q\|^2 \\
&\quad - (1-\alpha_n-\gamma_n)\alpha_n g(\|x_n - (PT_1)^n x_n\|) \\
&\leq \|x_n - q\|^2
\end{aligned}$$

$$
\begin{aligned}
&+\alpha_n(\|(PT_1)^n x_n - q\| + \|x_n - q\|)\sup_{x\in C}(\|(PT_1)^n x_n - q\| - \|x_n - q\|)\\
&+\gamma_n(\|(PT_2)^n x_n - q\| + \|x_n - q\|)\sup_{x\in C}(\|(PT_2)^n x_n - q\| - \|x_n - q\|)\\
&+\delta_n\|u_n - q\|^2 - (1-\alpha_n-\gamma_n)\alpha_n g(\|x_n - (PT_1)^n x_n\|)\\
&\le \|x_n - q\|^2 + \alpha_n(\|(PT_1)^n x_n - q\| + \|x_n - q\|)G_n\\
&+\gamma_n(\|(PT_2)^n x_n - q\| + \|x_n - q\|)K_n + \delta_n\|u_n - q\|^2\\
&+\delta_n\|u_n - q\|^2 - (1-\alpha_n-\gamma_n)\alpha_n g(\|x_n - (PT_1)^n x_n\|)
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\|x_{n+1} - q\|^2 \le \|x_n - q\|^2 + \omega_n G_n + \nu_n K_n + \delta_n\|u_n - q\|^2\\
-(1-\alpha_n-\gamma_n)\alpha_n g(\|x_n - (PT_1)^n x_n\|)
\end{aligned}
$$

where $\omega_n = \alpha_n(\|(PT_1)^n x_n - q\| + \|x_n - q\|)$ and $\nu_n = \gamma_n(\|(PT_2)^n x_n - q\| + \|x_n - q\|$.
Then,
$(1-\alpha_n-\gamma_n)\alpha_n g(\|x_n - (PT_1)^n x_n\|) \le$

$$
\|x_n - q\|^2 - \|x_{n+1} - q\|^2 + \omega_n G_n + \nu_n K_n + \delta_n\|u_n - q\|^2
$$

Since $\sum_{n=1}^{\infty} G_n < \infty, \sum_{n=1}^{\infty} K_n < \infty \& \sum_{n=1}^{\infty}\delta_n < \infty$ and $\lim_{n\to\infty}\|x_n - q\|$ exists. It follows that

$$
\limsup_n (1-\alpha_n-\gamma_n)\alpha_n g(\|x_n - (PT_1)^n x_n\|) = 0
$$

Since g is continuous strictly increasing with $g(0) = 0$ and $0 < \lim_n \inf \alpha_n, 0 < \lim_n \inf \gamma_n$, we have $\lim_n \|x_n - (PT_1)^n x_n\| = 0$.

By using a similar method together with (9), it can be show that $\lim_n \|x_n - (PT_2)^n x_n\| = 0$.
Next, we show that $\lim_n \|x_n - (PT_1)x_n\| = 0$ and $\lim_n \|x_n - (PT_2)x_n\| = 0$.
Using (5), we have
$\|x_{n+1} - x_n\| \le$

$$
(1-\alpha_n-\gamma_n)\|x_n - x_n\| + \alpha_n\|(PT_1)^n x_n - x_n\| + \gamma_n\|(PT_2)^n x_n - x_n\| + \delta_n\|u_n - x_n\|
$$

Since $\sum_{n=1}^{\infty}\delta_n < \infty$, $\|(PT_1)^n x_n - x_n\| \to 0$ and $\|(PT_2)^n x_n - x_n\| \to 0$ as $n \to \infty$. Then $\|x_{n+1} - x_n\| \to 0$ as $n \to \infty$.
Now we consider,

$$
\begin{aligned}
\|x_n - (PT_1)x_n\| &\le \|x_n - x_{n+1}\| + \|x_{n+1} - (PT_1)^{n+1}x_{n+1}\|\\
&+\|(PT_1)^{n+1}x_{n+1} - (PT_1)^{n+1}x_n\| + \|(PT_1)^{n+1}x_n - (PT_1)x_n\|\\
&\le \|x_n - x_{n+1}\| + \|x_{n+1} - (PT_1)^{n+1}x_{n+1}\|\\
&+L\|x_{n+1} - x_n\| + L\|(PT_1)^n x_n - x_n\|
\end{aligned} \tag{10}
$$

for some $L \in R^+$

$$\begin{aligned}\|x_n - (PT_2)x_n\| &\leq \|x_n - x_{n+1}\| + \|x_{n+1} - (PT_2)^{n+1}x_{n+1}\| \\ &+\|(PT_2)^{n+1}x_{n+1} - (PT_2)^{n+1}x_n\| + \|(PT_2)^{n+1}x_n - (PT_2)x_n\| \\ &\leq \|x_n - x_{n+1}\| + \|x_{n+1} - (PT_2)^{n+1}x_{n+1}\| \\ &+L\|x_{n+1} - x_n\| + L\|(PT_2)^n x_n - x_n\|\end{aligned} \tag{11}$$

for some $L \in R^+$

Since $\|(PT_1)^n x_n - x_n\| \to 0$, $\|(PT_2)^n x_n - x_n\| \to 0$,and $\|x_{n+1} - x_n\| \to 0$ as $n \to \infty$ and uniform continuity of T_1, T_2 together with (10) and (11) it can be shown that $\|(PT_1)x_n - x_n\| \to 0$ and $\|(PT_2)x_n - x_n\| \to 0$ as $n \to \infty$, respectively. □

Remark 2. *Expressions (10) and (11) mean that the assumption T_1 and T_2 are uniformly L-Lipschitzian nonself mappings with respect to P is the same as to assume that the mappings PT_1 and PT_2 are uniformly L-Lipschitzian.*

Theorem 6. *Let C be a nonempty closed convex subset of a real smooth and uniformly convex Banach space X. Let $T_1, T_2 : C \to X$ be two weakly inward and asymptotically quasi- nonexpansive type nonself mappings with respect to P, as a sunny nonexpansive retraction of X onto C and T_1, T_2 be uniformly L-Lipschitzian nonself mappings with respect to P. Put*

$$G_n = \max\{0, \sup_{x\in C}(\|(PT_1)^n x - q\| - \|x - q\|)\}$$

and

$$K_n = \max\{0, \sup_{x\in C}(\|(PT_2)^n x - q\| - \|x - q\|)\}$$

such that $\sum_{n=1}^{\infty} G_n < \infty$ and $\sum_{n=1}^{\infty} K_n < \infty$ respectively. Suppose that $\{x_n\}$ is the sequence defined by (5) with $\sum_{n=1}^{\infty} \delta_n < \infty$ and the additional assumption that $0 < \lim_n \inf \alpha_n, 0 < \lim_n \inf \gamma_n$ and $0 < \lim_n \inf \delta_n$.

If one of T_1 or T_2 is completely continuous and $F(T_1) \cap F(T_2) \neq \varphi$, then $\{x_n\}$ converges strongly to a common fixed point of T_1 and T_2.

Proof. From Lemma 4, for any $q \in F(T_1) \cap F(T_2)$, $\lim_n \|x_n - q\|$ exists, therefore $\{x_n\}$ is bounded. Again by Lemma 5, we have

$$\lim_{n\to\infty} \|(PT_1)x_n - x_n\| = 0, \text{ and } \lim_{n\to\infty} \|(PT_2)x_n - x_n\| = 0 \tag{12}$$

Now if T_1 is completely continuous and $\{x_n\}$ is bounded , we conclude that there exists sub sequences $\{PT_1 x_{n_j}\}$ of $\{PT_1 x_n\}$ such that $\{PT_1 x_{n_j}\}$converges.

Therefore, from (12), $\{x_{n_j}\}$ is convergent.

Let $x_{n_j} \to r$, as $j \to \infty$. By the continuity of P, T_1, T_2 and (12) we have $r = PT_1 r = PT_2 r$. Since $F(PT_1) = F(T_1)$ and $F(PT_2) = F(T_2)$ and by Lemma 3, we have $r = T_1 r = T_2 r$. Thus $\{x_n\}$ converges strongly to a common fixed point r of T_1 and T_2. This completes the proof of the theorem. □

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References

[1] S. S. Chang, J. K. Kim, and S. M. Kang, Approximating fixed points of asymptotically quasi-nonexpansive type mappings by the Ishikawa iterative sequences with mixed errors, Dynamic Systems and Applications, **13**(2004), 179-186.

[2] W. Nilsrakoo and S. Saejung, A new three-step fixed point iteration scheme for asymptotically nonexpansive mapping, Applied Mathematics and Computation, **181**(2006), 1026-1034.

[3] J. Quan, S. S. Chang, and X. J. Long, Approximation common fixed point of asymptotically quasi-nonexpansive type mappings by the finite steps iterative sequences, Fixed Point Theory and Applications, **2006**(2006), 8 pages.

[4] Robert E. Megginson, An Introduction to Banach Space Theory, Springer-Verlag New York, 1998.

[5] D. R. Sahu and J. S. Jung, Fixed point iteration processes for non-Lipschitzian mappings of asymptotically quasi-nonexpansive type, International Journal of Mathematics and Mathematical Sciences, **2003**(2003), 2075-2081.

[6] J. Schu, Iterative construction of fixed point of asymptotically nonexpansive mappings, Journal of Mathematical Analysis and Applications, **158**(1991), 407-413.

[7] K. K. Tan and H. K. Xu, Approximating fixed points of nonexpansive mapping by the Ishikawa iteration process, Journal of Mathematical Analysis and Applications, **178**(1993), 301-308.

[8] H. Y. Zhou, Y. J. Cho and S. M. Kang, A new iterative algorithm for approximating common fixed points for asymptotically nonexpansive mappings, Fixed Point Theory and Applications, **2007**(2007), 10 pages.