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SOME RESULTS OF ACCRETIVE OPERATORS AND CONVEX SETS IN 2-PROBABILISTIC NORMED SPACE

P. K. HARIKRISHNAN[1], K. T. RAVINDRAN[2]

Abstract. In this paper we introduce the concept of accretive operators, discuss some properties of resolvents of an accretive operator in 2-probabilistic normed spaces and focusing on the results of convex sets in 2-probabilistic normed spaces.

Key words: linear 2-normed spaces, probabilistic normed spaces, accretive operators.
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1. Introduction

An interesting and important generalisation of the notion of metric space was introduced by K.Menger [6] under the name of statistical metric space, which is now called Probabilistic Metric Space. In the same way, in K.Menger [6] proposed the probabilistic concept of distance by replacing the number $d(p, q)$, as the distance between points p, q, by a distribution function $F_{p,q}$. This idea led to development of probabilistic analysis. An important family of probabilistic metric spaces is probabilistic normed spaces (briefly, PN-spaces).The concept of probabilistic normed spaces was introduced by Sertnev A. N.in 1963 [8] . The theory of probabilistic normed spaces is important as a generalization of deterministic results of linear normed spaces and also in the study of random operator equations. PN spaces may also provide us a tool to study the geometry of nuclear physics and have applications in quantum particle physics particularly in string and $\varepsilon^{(\infty)}$ theory. The concept of 2-probabilistic normed spaces is introduced by the authors I. Golet [5] and Kouroush Nourouzi, Fatemeh Lael [2] independently, extended many results in best approximations and compact operators in linear 2 - normed spaces to

[1]Department of Mathematics, Manipal Institute of Technology, Manipal University, Manipal, Karnataka, India. Email: *pkharikrishnans@gmail.com*.
[2]P. G. Department and Research Centre in Mathematics, Payyanur College, Payyanur, Kerala, India. Email: *drktravindran@gmail.com*.

2 - probabilistic normed spaces. In this paper we are discussing the properties of accretive operators and convex sets in 2 - probabilistic normed spaces.

2. Preliminaries

Definition 1. [7] *A function $f : \mathbb{R} \to [0, 1)$ is said to be a distribution function if it is non decreasing and right continuous with $\inf_{t\in\mathbb{R}} f(t) = 0$ and $\sup_{t\in\mathbb{R}} f(t) = 1$. The set of all distribution functions is denoted by $\mathcal{D}$.*

Definition 2. [2] *Define the distribution function $H(t)$ by,*

$$H(t) = \begin{cases} 1, & \text{if } t > 0 \\ 0, & \text{if } t \leq 0 \end{cases}$$

Definition 3. [2] *A pair (X, N) is called a Menger's 2- Probabilistic Normed space (briefly Menger's 2-PN space) if X is a real vector space of $dimX > 1$, N is a mapping from $X \times X$ into $\mathcal{D}$(for each $x \in \mathcal{D}$, the distribution function $N(x, y)$ is denoted by $N_{x,y}$ and $N_{x,y}(t)$ is the value of $N_{x,y}$ at $t \in \mathbb{R}$) satisfying conditions*

A1: $N_{x,y}(0) = 0$ for all $x, y \in X$

A2: $N_{x,y}(t) = 1$ for all $t > 0$ iff x and y are linearly dependent.

A3: $N_{x,y}(t) = N_{y,x}(t)$ for all $x, y \in X$

A4: $N_{\alpha x,y}(t) = N_{x,y}(\frac{t}{|\alpha|})$ for all $\alpha \in \mathbb{R} - \{0\}$ and for all $x, y \in X$

A5: $N_{x+y,z}(s + t) \geq Min\{N_{x,y}(s), N_{y,x}(t)\}$ for all $x, y, z \in X$ and $s, t \in \mathbb{R}$.

We call the mapping $(x, y) \to N_{x,y}$ a 2-probabilistic norm on X.

From the axioms (A1) and (A2) of the above definition, it is clear that

$$N_{x,y}(t) = H(t) \text{ iff } x \text{ and } y \text{ are linearly dependent.}$$

Example 1. [2] *Let $(X, \|., .\|)$ be a 2-normed space. Every 2-norm induces a 2-PN norm on X as follows:*

$$N_{x,y}(t)) = \begin{cases} \frac{1}{\|x,y\|}, & \text{if } t > 0 \\ 0, & \text{if } t < 0 \end{cases}$$

This 2-probabilistic norm is called the standard 2-PN norm.

Example 2. [2] *Let $(X, \|., .\|)$ be a 2-normed space. Define*

$$N_{x,y}(t) = \begin{cases} 0, & \text{if } t \leq \|x, y\| \\ 1, & \text{if } \|x, y\| \end{cases}$$

where $x, y \in X$ and $t \in \mathbb{R}$ then (X, N) is a 2-PN space.

Definition 4. [2] *Let (X, N) be a 2-PN space, and $\{x_n\}$ be a sequence of X. Then the sequence $\{x_n\}$ is said to be convergent to x if $\lim_{n\to\infty} N_{x_n-x,z}(t) = 1$ for all $z \in X$ and $t > 0$.*

Definition 5. [2] *Let* (X, N) *be a 2-PN space then a sequence* $\{x_n\}$ *in*X *is said to be a Cauchy sequence if* $\lim_{n,m\to\infty} N_{x_m - x_n, z}(t) = 1$ *for all* $z \in X, t > 0$ *and* $m > n$.

Definition 6. [2] *A 2-PN space is said to be complete if every Cauchy sequence in X is convergent to a point of X.*

A Complete 2-PN space is called 2-Probabilistic Banach space.

Definition 7. [2] *Let* (X, N) *be a 2-PN space, E be a subset of X then the closure of E is* $\overline{E} = \{x \in X;$ *there is a sequence* $\{x_n\}$ *of E such that* $x_n \to x\}$.

We say, E is sequentially closed if $E = \overline{E}$.

Definition 8. [1] *Let E be a subset of a real vector space X then E is said to be a convex set if* $\lambda x + (1 - \lambda)y \in E$ *for all* $x, y \in E$ *and* $0 < \lambda < 1$.

Definition 9. [2] *Let* (X, N) *be a 2-PN space, for* $e, x \in X, \alpha \in (0, 1)$ *and* $r > 0$ *we define the locally ball by,*

$$B_{e,\alpha}[x, r] = \{y \in X : N_{x-y,e}(r) \geq \alpha\}$$

Definition 10. [2] *Let* (X, N) *and* (Y, N') *be two 2-PN spaces,* $T : X \to Y$ *then a mapping is said to be sequentially continuous if* $x_n \to x$ *implies* $T(x_n) \to T(x)$.

3. Main Results

3.1. **Accretive Operators in 2-PN space.** Let (X, N) be a 2-PN space and $A : D(A) \subset X \to X$ be an operator with domain $D(A) = \{x \in X; Ax \neq 0\}$ and range $R(A) = \cup\{Ax; x \in D(A)\}$. We may identify A with its graph and the closure of A with the closure of its graph.

Definition 11. *: Let* (X, N) *be a 2-PN space. An operator* $A : D(A) \subset X \to X$ *is said to be accretive if for every* $z \in D(A)$

$$N_{x-y,z}(t) \geq N_{(x-y)+\lambda(Ax-Ay),z}(t) \text{ for all } x, y \in D(A) \text{ and } \lambda > 0.$$

Throughout this article $[x, y] \in A$ means $x, y \in X$ such that $y = Ax$.

Let A be an accretive operator in a 2-PN space (X, N). Define the resolvent of A by $J_\lambda = (1 + \lambda A)^{-1}$ and the Yosida approximation $A_\lambda = \frac{1}{\lambda}(I - J_\lambda)$ for every $\lambda > 0$. Then $D(J_\lambda) = R(I + \lambda A), R(J_\lambda) = D(A), D(A_\lambda) = D(J_\lambda)$ for $t > 0$.

Next we have some properties of J_λ.

Lemma 1. *Let A be an accretive operator in a 2-PN space* (X, N) *,*J_λ *is single valued and*

(1) $N_{J_\lambda(x)-J_\lambda(y),z}(t) \geq N_{x-y,z}(t)$
(2) $N_{\frac{1}{n}[J_\lambda^n(x)-x],z}(t) \geq N_{J_\lambda(x)-x,z}(t)$

for all $x, y \in D(J_\lambda), \lambda > 0, z \in X$

Proof. Let $x, y \in D(J_\lambda), \lambda > 0, t \in R$. Suppose $y_1 = J_\lambda(x), y_2 = J_\lambda(x)$

Since A is accretive,

$$\begin{aligned} N_{y_1-y_2,z}(t) &\geq N_{(y_1-y_2)+\lambda[\frac{1}{\lambda}(x-y_1)-\frac{1}{\lambda}(x-y_2)],z}(t) \\ &= N_{0,z}(t) = H(t) \text{ for all } z \in X \end{aligned}$$

implies $y_1 - y_2, z$ are linearly independent for every $z \in X$

implies $y_1 - y_2 = 0$ implies $y_1 = y_2$.

Therefore, there exists $[x_1, y_1], [x_2, y_2] \in A$ such that $x_1 + \lambda y_1 = x_2 + \lambda y_2$ then $J_\lambda(x) = x_1, J_\lambda(y) = x_2$.

(i) Since A is accretive,

$$\begin{aligned} N_{J_\lambda(x)-J_\lambda(y),z}(t) &= N_{x_1-x_2,z}(t) \\ &\geq N_{[(x_1-x_2)+\lambda(y_1-y_2)],z}(t) \\ &= N_{[(x_1+\lambda y_1)-(x_2+\lambda y_2)],z}(t) \\ &= N_{x-y,z}(t) \text{ for every } z \in X \end{aligned}$$

(ii)We have,

$$\begin{aligned} &N_{\frac{1}{n}[J_\lambda^n(x)-x],z}(t) \\ &= N_{J_\lambda^n(x)-x,z}(nt) \qquad (1) \\ &= N_{[J_\lambda^n(x)-J_\lambda^{n-1}(x)+J_\lambda^{n-1}(x)-x],z}[t+(n-1)t] \\ &= Min\{N_{[J_\lambda^n(x)-J_\lambda^{n-1}(x)],z}(t), N_{[J_\lambda^{n-1}(x)-x],z}[(n-1)t]\} \\ &\geq Min\{N_{[J_\lambda^n(x)-J_\lambda^{n-1}(x)],z}(t), Min\{N_{[J_\lambda^{n-1}(x)-J_\lambda^{n-2}(x)],z}(t), N_{[J_\lambda^{n-2}(x)-x],z}[(n-2)t]\}\} \\ &\geq Min\{N_{[J_\lambda(x)-x,z}(t), Min\{N_{[J_\lambda(x)-x],z}(t)...\}Min\{N_{[J_\lambda(x)-x],z}(t)...\}\}\} \text{ [by (i)]} \\ &\geq N_{J_\lambda(x)-x,z}(t) \text{ for every } z \in X \end{aligned}$$

□

Definition 12. *Let (X, N) be a 2-PN space. An operator $A : D(A) \subset X \to X$ is said to be m-accretive if $R(I + \lambda A) = X$ for $\lambda > 0$.*

An operator $A : D(A) \subset X \to X$ and $B : D(B) \subset X \to X$ be two operators then B is said to be an extension of A if $D(A) \subset D(B)$ and $Ax = Bx$ for every $x \in D(A)$, denote it by $A \subset B$.

Definition 13. *Let (X, N) be a 2-PN space. An operator $A : D(A) \subset X \to X$ is said to be a maximal accretive operator in X if A is an accretive operator in X and for every accretive operator B of X with $A \subset B$ then $A = B$.*

Theorem 1. *Let (X, N) be a 2-PN space. If A is an m-accretive operator in X then A is a maximal accretive operator.*

Proof. Let B be an accretive operator with $A \subset B$. Let $\lambda > 0$ and $[x, y] \in B$.

Since A is m- accretive we have $x + \lambda y \in R(I + \lambda A)$ implies there exists $[x_1, y_1] \in A$ such that $x + \lambda y = x_1 + \lambda y_1$

Since B is accretive and $[x_1, y_1] \in B$ we have,

$$\begin{aligned} N_{x-x_1,z}(t) &\geq N_{(x-x_1)+\lambda(Bx-Bx_1),z}(t) \\ &= N_{(x-x_1)+\lambda(y-y_1),z}(t) \\ &= N_{(x+\lambda y)-(x_1+\lambda y_1),z}(t) = N_{0,z}(t) \text{ for every } z \in X \\ &= H(t) \text{ for every } z \in X \end{aligned}$$

implies $x - x_1 = 0$ implies $x = x_1$

Therefore $y = y_1$ implies $[x, y] \in A$. So A=B.

Hence A is a maximal accretive operator. □

Lemma 2. *Let A be an accretive operator in a 2-PN space (X, N) and let $(u, v) \in X \times X$ then A is maximal accretive in X iff $N_{(x-u),z}(t) \geq N_{(x-u)+\lambda(y-v),z}(t)$ for every $[x, y] \in A, z \in X$ and $\lambda > 0$ implies $[u, v] \in A$*

Proof. Let A be a maximal accretive operator in X. Put $T = A \cup [u, v]$

Suppose $N_{(x-u),z}(t) \geq N_{(x-u)+\lambda(y-v),z}(t)$ for every $[x, y] \in A, z \in X$ and $\lambda > 0$

then T is accretive in X and $A \subset T$ implies $[u, v] \in A$

Conversly, Suppose that if A is accretive operator in X and $N_{(x-u),z}(t) \geq N_{(x-u)+\lambda(y-v),z}(t)$ for every $[x, y] \in A, z \in X$ and $\lambda > 0$ implies $[u, v] \in A$

Let B be accretive in X with $A \subset B$ and $[x_1, y_1] \in B$

Since Bis accretive in X, we have for every $[x, y] \in A, z \in X$ and $\lambda > 0$ with $N_{x-x_1,z}(t) \geq N_{(x-x_1)+\lambda(Bx-Bx_1),z}(t) = N_{(x-x_1)+\lambda(y-y_1),z}$

implies $[x_1, y_1] \in A$. Therefore $B \subset A$. So $A = B$.

Hence A is maximal accretive in X. □

Theorem 2. *Let A be an accretive operator in a 2-PN space (X, N) then the closure $\overline{A}$ of A is accretive.*

Proof. Let $[x_1, y_1], [x_2, y_2] \in \overline{A}$ then there exists sequences $\{[x_n, y_n]\}$, $\{[x_m, y_m]\}$ in A such that $x_n \to x_1; y_n \to y_1; x_m \to x_2; y_m \to y_2$ and $\lambda > 0$.

Since A is accretive we have,

$$\begin{aligned} N_{x_n-x_m,z}(t) &\geq N_{(x_n-x_m)+\lambda(Ax_n-Ax_m),z}(t) \text{ for every } z \in X \\ &= N_{(x_n-x_m)+\lambda(y_n-y_m),z}(t) \text{ for every } z \in X \end{aligned}$$

as $n \to \infty$, $N_{x_1-x_2,z}(t) \geq N_{(x_1-x_2)+\lambda(y_1-y_2),z}(t)$ for every $z \in X$

implies $\overline{A}$ is accretive in X. □

Theorem 3. *Let A be a maximal accretive operator in a 2-PN space (X, N) then A is sequentially closed.*

Proof. For all $x_n, y_n \in D(A)$, Let $\{[x_n, y_n]\}$ in A such that $x_n \to u, y_n \to v$ and $\lambda > 0$

Since A is accretive in X and $[x, y] \in A$

implies $N_{x-x_n,z}(t) \geq N_{(x-x_n)+\lambda(y-y_n),z}(t)$ for every $z \in X$

as $n \to \infty$ we have $N_{x-u,z}(t) \geq N_{(x-u)+\lambda(y-v),z}(t)$ for every $z \in X$

Therefore,by Lemma (2) $[u, v] \in A$. Hence A is sequentially closed. □

Corollary 1. *If A is an m-maximal accretive operator in a 2-PN space (X, N) then A is sequentially closed.*

Proof. We have an m-accretive operator A in X is a maximal accretive operator in X. Hence by Theorem (3), A is sequentially closed. □

Theorem 4. *Let (X, N) be a complete 2-PN space. Let A be a sequentially continuous accretive operator on X. If A is closed then $R(I + \lambda A)$ is closed for $\lambda > 0$.*

Proof. Let $\{z_n\}$ be a sequence in $R(I + \lambda A)$ with $z_n \to z^{'}$ in X then $\{z_n\}$ is a Cauchy sequence in X.

$\{z_n\} \in R(I + \lambda A)$ implies there exists $[x_n, y_n] \in A$ such that $z_n = x_n + \lambda y_n$ implies $J_\lambda(z_n) = x_n$

Therefore for every $t \in R$ and $z \in X$,

$N_{x_n-x_m,z}(t) = N_{J_\lambda(z_n)-J_\lambda(z_m),z}(t) \geq N_{z_n-z_m,z}(t)$

implies $\underline{\lim}_{n,m\to\infty} N_{x_n-x_m,z}(t) = H(t) = 1$ for every $t > 0$

implies $\lim\limits_{n,m\to\infty} N_{x_n-x_m,z}(t) = 1$ for every $t > 0$ and $z \in X$

Therefore $\{x_n\}$ is a Cauchy sequence in X.

Since X is complete, there exists $x \in X$ such that $x_n \to x$ and $y_n = \frac{1}{\lambda}(z_n - x_n)$

implies $y_n \to \frac{1}{\lambda}(z^{'} - x)$ as $n \to \infty$

Since $Ax_n = y_n$ and A is sequentially continuous, $Ax = \frac{1}{\lambda}(z^{'} - x)$

implies $z^{'} = x + \lambda Ax \in R(I + \lambda A)$. Hence $R(I + \lambda A)$ is closed for $\lambda > 0$. □

3.2. **Convex sets in 2-PN space.** In this section we are discussing about some properties of convex sets in 2-PN spaces.

Theorem 5. *Every open ball in a 2-PN space (X, N) is Convex.*

Proof. We have the locally ball is $B_{e,\alpha}[x, r] = \{y \in X : N_{x-y,e}(r) \geq \alpha\}$

Let $x \in X, r \in (0, 1), e \in X$

Choose $z, y \in$ and $0 \leq \lambda \leq 1$ then $N_{x-z,e}(r) \geq \alpha, N_{x-y,e}(r) \geq \alpha$

We have,

$$\begin{aligned} N_{x-[\lambda y+(1-\lambda)z],e}(r) &= N_{x-[\lambda y+(1-\lambda)z],e}([\lambda+(1-\lambda)]r) \\ &= N_{\lambda(x-y)+(1-\lambda)(x-z),e}([\lambda+(1-\lambda)]r) \\ &\geq Min\{N_{\lambda(x-y),e}(\lambda r), N_{(1-\lambda)(x-z),e}((1-\lambda)r)\} \\ &= Min\{N_{(x-y),e}(r), N_{(x-z),e}(r)\} \\ &\geq Min\{\alpha,\alpha\} = \alpha \end{aligned}$$

Therefore $\lambda y+(1-\lambda)z \in B_{e,\alpha}[x,r]$ for all $z, y \in B_{e,\alpha}[x,r]$

So, every locally ball in a 2-PN space is Convex □

Theorem 6. *The closure of a closed convex set in a 2-PN space (X, N) is convex*

Proof. Let E be a closed convex set in a 2-PN space (X, N) then we have to prove that $\overline{E}$ is convex.

Let $x, y \in \overline{E}$ then there exists sequences $\{x_n\}, \{y_n\} \in E$ such that $x_n \longrightarrow x$ and $y_n \longrightarrow y$

Since, $\{x_n\}, \{y_n\} \in E$ and E is convex implies $\lambda x_n + (1-\lambda)y_n \in E$ for all $0 < \lambda < 1$

as $n \longrightarrow \infty$ we get $\lambda x + (1-\lambda)y \in E$. The facts $\lambda x + (1-\lambda)y \in E$ and $E = \overline{E}$ imply $\overline{E}$ is convex. □

Definition 14. *Let E be a subset of a 2-PN space (X, N) then an element $x \in E$ is called a interior point of E if there are $r > 0, e \in X$ such that $B_{e,\alpha}[x,r] \subseteq E$.*

The set of all interior points of E is denoted by $int(E)$.

Definition 15. *A subset E of a 2-PN space (X, N) is said to be open if $E = int(E)$.*

For any two points x,y in the real vector space X denote,

$$(x, y) = \{\lambda x + (1-\lambda)y; \lambda \in (0,1)\}$$

Theorem 7. *Let E be a convex subset of a 2-PN space (X, N) . Let $a \in E$ and x is an interior point of E then every point in $(a, x) = \{\lambda a+(1-\lambda)x; \lambda \in (0,1)\}$ is an interior point of E.*

Proof. Let $u \in (a, x)$ then $u = \lambda x + (1-\lambda)a$ for some $\lambda \in (0,1)$

Since x is an interior point of E then there exists $r_0 > 0, e \in X$ and $\alpha \in (0,1)$ such that $B_{e,\alpha}[x, r_0] \subseteq E$.

So it is enough to show that $B_{e,\alpha}[u, \lambda r_0] \subseteq E$ for $\lambda r_0 \in (0,1)$.

Let $y \in B_{e,\alpha}[u, \lambda r_0]$ then $N_{(u-y),e}(\lambda r_0) \geq \alpha$

Therefore, $N_{\lambda^{-1}(y-u),e}(r_0) = N_{(u-y),e}(\lambda r_0) \geq \alpha$

implies $x + \lambda^{-1}(y-u) \in B_{e,\alpha}[x, r_0]$

Let $w = x + \lambda^{-1}(y-u)$ then $\lambda w = \lambda x + (y-u)$
implies $y = \lambda(w-x) + u$
implies $y = \lambda(w-x) + \lambda x + (1-\lambda)a$ impies $y = (1-\lambda)a + w$ with $w, a \in E$
Since E is convex, $y = (1-\lambda)a + w \in E$
Hence any point in (a, x) is an interior point of E. □

Corollary 2. *Let E be a convex subset of a 2-PN space (X, N) . Let x be an interior point of E and $y \in \overline{E}$ then $(x, y) \subseteq int(E)$.*

Proof. Suppose x is an interior point of E and $y \in \overline{E}$ then there exists a sequence $\{y_n\} \in E$ such that $y_n \longrightarrow y$.
Let $z \in (x, y)$ then $z = \lambda x + (1-\lambda)y$ for some $\lambda \in (0,1)$
Define $z_n = \lambda x + (1-\lambda)y_n$
Since x is an interior point of E then there exists $r_0 > 0, e \in X$ and $\alpha \in (0,1)$ such that $B_{e,\alpha}[x, r_0] \subseteq E$.
By Theorem (7), $B_{e,\alpha}[z_n, \lambda r_0] \subseteq E$ for $\lambda r_0 \in (0,1)$ and $z_n \longrightarrow z$.
Since N is continuous for the first component,$z_n \longrightarrow z$ means that $lim_{n\to\infty} N_{z_n - z, y}(t) = 1$ for $y \in X$ and $t > 0$
ie; there exists $n_0 \in \mathbf{N}$ such that $z_n \in B_{e,\alpha}[z_n, \lambda r_0]$ for every $n \geq n_0$
Now $N_{z_n - z, y}(t) = N_{z - z_n, y}(t)$ and we can say that $z \in B_{e,\alpha}[z_n, \lambda r_0] \subseteq E$.
Hence $(x, y) \subseteq int(E)$. □

Corollary 3. *Let E be a non empty convex subset of a 2-PN space (X, N) then $\overline{int(E)} = \overline{E}$.*

Proof. It is obvious that, $\overline{int(E)} \subseteq \overline{E}$.
Let $y \in \overline{E}$ and take $x \in int(E)$ then by Corollary (2), $(x, y) \subseteq int(E)$.
If $\lambda_n \in (0,1)$ with $\lambda_n \to 0$ then $\{\lambda_n x + (1-\lambda_n)y\}$ is a sequence in (x, y)
Then $N_{[\lambda_n x + (1-\lambda_n)y] - y, z}(t) = N_{\lambda_n(x-y), z}(t) = N_{0,z}(t)$for every $z \in X$, as $\lambda_n \to 0$
ie; $N_{[\lambda_n x + (1-\lambda_n)y] - y, z}(t) = H(t)$ implies $N_{[\lambda_n x + (1-\lambda_n)y] - y, z}(t) = 1$ for $t > 0$
implies $\lambda_n x + (1-\lambda_n)y \longrightarrow y$ as $n \to \infty$
So,$y \in \overline{int(E)}$ implies $\overline{E} \subseteq \overline{int(E)}$
Hence $\overline{int(E)} = \overline{E}$. □

References

[1] Erwin Kreyszi: *Introductory Functional Analysis with Applications*, Wiley India(P) Ltd, New Delhi, 2007.
[2] Fatemeh Lael Kourosh Nourouzi: *Compact Operators Defined on 2-Normed and 2-Probabilistic Normed Spaces*,Hindawi Publishing Corporation,Mathematical Problems in Engineering, Volume 2009 (2009), Article ID 950234, 17 pages.

[3] P. K. Harikrishnan, Bernardo Lafuerza Guillen, K. T. Ravindran: *Accretive operators and Banach Alogolu Theorem in Linear 2-normed spaces*,Proyeccions Journal of Mathematics, Vol 30, $N^0 3$, (2011), 319-327.

[4] P. K. Harikrishnan, K. T. Ravindran: *Some Properties of Accretive operators in Linear 2-normed spaces*,International Mathematical Forum, Vol. 6, No. 59, (2011),2941 - 2947.

[5] Ioan Golet: *Approximation Theorems in probabilistic normed spaces*, NOVI SAD J. MATH. Vol.38, No.3, (2008), 73-79.

[6] K. Menger:*Statistical metrics*, Proc. Nat. Acad. Sci., USA, 28, (1942), 535-537.

[7] Schweizer, B. Sklar, A: *Probabilistic metric spaces*, New York, Amsterdam, Oxford: North Holland (1983).

[8] Sertnev, A. N: *On the notion of a random normed space*, Dokl. Akad. Nauk SSSR. 149 (2) 280283; English translation in Soviet Math. Dkl., 4 ,(1963), 388-390.

[9] Shih sen Chang, Yeol Je Cho, Shin Min Kang: *Nonlinear operator theory in Probabilistic Metric spaces*,Nova Science publishers, Inc, Newyork,(2001).