Journal of Prime Research in Mathematics Vol. **10***(2015), 45-58*

MAGNETOHYDRODYNAMICS OF ROTATING FRACTIONAL SECOND GRADE FLUID IN POROUS MEDIUM

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Abstract. Exact solution for the unsteady flow of a fractional second grade fluid through the porous medium under the influence of magnetic field in the direction normal to the flow has been investigated using the integral transforms. Expressions for dimensionless velocity have been obtained and are presented in terms of Fox's H–function. The influence of the fractional parameter on the fluid motion is studied and a comparison between velocity of the fractional and classical fluid is made.

Key words : Second grade fluid, fractional calculus, magnetohydrodynamics, velocity field.
AMS SUBJECT : (2010). 76A05, 76A10.

1. Introduction

Non-Newtonian fluids play an active and significant role in comparison with Newtonian fluids in a large number of industries and the branches of knowledge concerned with applied sciences. The dynamics of non-Newtonian fluids is much more complex and delicate enough to analyze [3]. The distinctive character of the response of non-Newtonian fluids cannot be studied by the classical Navier-Stokes equations but a huge count of models are in agreement to justify the rheological response of non-Newtonian fluids [8], [38]. A number of investigators in the field, propose/quotes a large number of applications of non-Newtonian fluids in rheological problems, geophysics, petroleum, chemical industries and biological sciences [50].

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Generally, the non-Newtonian fluids are categorized as differential, rate and integral type fluids. Amongst them, the fluids of differential type have received special attention [5], [6], [9], [11], [12], [13], [14], [15], [39], [45]. The second grade fluids form a subclass of non-Newtonian fluids and is the simplest subclass of differential type fluids which can show the normal stress effects. The flow of electrically conductive fluids in the presence of magnetic field is known as magnetohydrodynamics (MHD). The subject MHD seems to be initiated by H. Alfven [1]. The principle of MHD is helpful in stabilizing a flow against the transition from laminar to turbulent. In recent years, the utility of MHD led the researchers to renew their investigation of hydrodynamics in the setting of MHD. The dynamics of non-Newtonian fluids with and without influence of magnetic field has a lot of applications e.g. boundary layer control in aerodynamics [28], the handling of molten metals, biological fluids, polymers and fossil fuels etc. For further studies regarding MHD we refer [32].

Over the last few years, fractional calculus has attracted much attention in the mathematical modeling of dynamical systems. It also has been employed with success in the constitutive modeling of certain non-Newtonian fluid models. The major reason is that, a fractional model has the ability to explain with simplicity the complex response of viscoelastic fluid. Few major contributions in the discussion of viscoelastic fluids flows with a fractional calculus approach can be seen in [16], [17], [18], [22], [24], [26], [30], [33], [37], [42], [43], [44], [46], [47], [48], [49]. Several researchers have discussed flows of second grade fluid in different configurations, and there are on hand few attempts which include the effects of rotation and MHD (for instance, see studies in [4], [7], [20], [21], [27], [29], [31], [34], [36], [40], [41] and references therein). Recently, Imran et al. [25] analyzed the unsteady MHD flow of a rotating second grade fluid in a porous medium over an oscillating plate.

To the best of authors knowledge, so far no study has been reported to analyze the unsteady MHD flow of a rotating fractional second grade fluid in a porous medium past an oscillating plate. Therefore, it is here proposed to make such an attempt. The main aim of the current study is to establish analytic solutions for the velocity field of a fractional second grade fluid which flows in the presence of an applied magnetic field. The fluid occupies a half porous space bounded by a rigid and nonconducting plate and, the whole system is in a rigid-rotating motion. The boundary plate also has a secondary motion, namely, a translation in its plane with the time dependent velocity of the form $V_o \sin(\omega t)$ or $V_o \cos(\omega t)$ or $V_o(a + be^{-\omega t})$. The closed-form of the complex velocity field is obtained by means of Laplace transform method and is expressed in an elegant form using the Fox functions [19]. The Fox function, also referred as the Fox H-function, generalizes the Mellin-Barnes function.

The importance of the Fox function lies in the fact that it includes nearly all special functions occurring in applied mathematics and statistics as special cases. From our general solutions the flow of an ordinary fluid as well as the hydrodynamic flow appear as limiting cases of the fractional model.

2. Statement of the problem

Consider an incompressible rotating fractional second grade fluid bounded by a rigid plate at $z = 0$. The fluid is electrically conducting and fills the porous region $z > 0$. The z–axis is taken to be normal to the plate. Initially, the fluid and plate are at rest. At time $t = 0^+$, the fluid and plate start rotating around the z–axis as a solid body with a constant angular velocity Ω. The plate is also subject to translation motion in its plane along the x-axis. A uniform transverse magnetic field of strength B_o is applied parallel to the axis of rotation. It is assumed that the induced magnetic field and the electric field due to polarization of charges are negligible. The motion of the second grade fluid is governed by the following partial differential equation [20], [27].

$$\frac{\partial F}{\partial t} + (2i\Omega + \frac{\sigma B_o^2}{\rho})F = \nu \frac{\partial^2 F}{\partial z^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 F}{\partial z^2 \partial t} - \frac{\nu\phi}{k}(1 + \frac{\alpha_1}{\mu}\frac{\partial}{\partial t})F, \tag{1}$$

where $F(z,t) = u(z,t) + iv(z,t)$ is the complex velocity of the fluid, u and v are the velocity components along x and y axis respectively, ρ is the fluid density, μ is the dynamic viscosity, α_1 the normal stress module, ν is the kinematic viscosity, σ is the electrical conductivity of the fluid, $\phi (0 < \phi < 1)$ is the porosity and $k > 0$ is the permeability of the porous medium.

The governing equation corresponding to such a motion of fractional second grade fluids is [46], [48]

$$\frac{\partial F}{\partial t} + (2i\Omega + \frac{\sigma B_o^2}{\rho})F = \nu \frac{\partial^2 F}{\partial z^2} + \eta_\beta D_t^\beta \frac{\partial^2 F}{\partial z^2} - \frac{\nu\phi}{k}(1 + \frac{\alpha_1}{\mu}\frac{\partial}{\partial t})F, \tag{2}$$

where $D_t^\beta f$ is the Caputo fractional derivative operator defined as [24],

$$D_t^\beta f(t) = \begin{cases} \frac{1}{\Gamma(1-\beta)} \int\limits_0^t \frac{f'(\tau)}{(t-\tau)^\beta} d\tau, & 0 < \beta < 1, \\ \frac{df(t)}{dt}, & \beta = 1. \end{cases}$$

Into Eq. (2), $\eta_\beta [m^2 s^{\beta-1}]$ is a material coefficient and $\eta_1 = \frac{\alpha_1}{\rho}$.

The appropriate initial and boundary conditions are

$$F(z,0) = 0; \quad z \geq 0, \tag{3}$$

$$F(0,t) = V_o \sin(\omega t) \text{ or } V_o \cos(\omega t) \text{ or } V_o(a + be^{-\omega t}); \quad t > 0, \tag{4}$$

$$F(z,t) \to 0 \text{ as } z \to \infty, \tag{5}$$

where V_o and ω are constants, $\omega > 0$.

Introducing the dimensionless variables

$$z^* = z\sqrt{\frac{\omega}{\nu}}, t^* = \omega t, F^* = \frac{F}{V_o}, \Omega^* = \frac{\Omega}{\omega}, \eta_\beta^* = \frac{\eta_\beta \omega^\beta}{\nu} \tag{6}$$

into Eqs.(2)-(5) and dropping the asteriks we get,

$$\frac{\partial F}{\partial t} + \frac{1}{\omega}(2i\Omega + \frac{\sigma B_o^2}{\rho})F = \frac{\partial^2 F}{\partial z^2} + \eta_\beta D_t^\beta \frac{\partial^2 F}{\partial z^2} - \frac{\nu\phi}{k\omega}(1 + \frac{\alpha_1\omega}{\mu}\frac{\partial}{\partial t})F, \tag{7}$$

or

$$\frac{\partial^2 F}{\partial z^2} + \eta_\beta D_t^\beta \frac{\partial^2 F}{\partial z^2} - a_o\frac{\partial F}{\partial t} - b_o F = 0, \tag{8}$$

subject to

$$F(z, 0) = 0; \quad z > 0, \tag{9}$$

$$F(0, t) = \sin t \;\; or \cos t \;\; or \;\; a + be^{-t}; \quad t > 0$$

$$F(z, t) \to 0 \;\; as \;\; z \to \infty, \tag{10}$$

where $a_o = 1 + \frac{\alpha}{K}$, $\alpha = \frac{\alpha_1\omega}{\rho\nu}$, $b_o = \frac{1}{K} + 2i\Omega + M^2$, $\frac{1}{K} = \frac{\phi\nu}{k\omega}$ and $M^2 = \frac{\sigma B_o^2}{\rho\omega}$

3. Calculation of the velocity field

Let us denote by $F_s(z,t)$, $F_c(z,t)$ and $F_e(z,t)$ the solutions of Eq. (8) related to the boundary conditions $(10)_1$, $(10)_2$ and $(10)_3$, respectively. In the following, in order to avoid repetition, we shall present calculi for $F_s(z,t)$ only. Applying the Laplace transform to Eq. (8), and having the initial conditions (9) in mind we find that

$$\frac{\partial^2 \overline{F_s}(z,q)}{\partial z^2} = \frac{a_o q + b_o}{1 + \eta_\beta q^\beta}\overline{F_s}(z,q) \tag{11}$$

with the boundary conditions

$$\overline{F}_s(0,q) = \frac{1}{1+q^2}, \quad \overline{F}_s(z,q) \to 0 \text{ as } z \to \infty, \tag{12}$$

where $\overline{F}(z,q)$ is the Laplace transform of $F(z,t)$ and q is transform parameter. Solution of the equation (11) satisfying (12) is

$$\overline{F_s}(z,q) = \frac{1}{q^2+1} \cdot \exp\left(- z\sqrt{\frac{a_o q + b_o}{1 + \eta_\beta q^\beta}}\right) \tag{13}$$

Application of inverse Laplace transform to Eq. (13), leads to

$$F_s(z,t) = (F_1 * F_2)(t) = \int_0^t F_1(t-s)F_2(z,s)ds, \tag{14}$$

where $F_1(z,t)$ and $F_2(z,t)$ are inverse Laplace transforms of $\overline{F}_1(z,q) = \frac{q}{q^2+1}$ and
$\overline{F}_2(z,q) = \frac{1}{q}\exp\left(-z\sqrt{\frac{a_oq+b_o}{1+\eta_\beta q^\beta}}\right)$ respectively. Now, we write

$$\overline{F_2}(z,q) = \frac{1}{q}\exp\left(-z\sqrt{\frac{a_oq+b_o}{1+\eta_\beta q^\beta}}\right)$$

$$= \sum_{k=0}^{\infty}\frac{\left(-za_o^{\frac{1}{2}}\eta_\beta^{\frac{-1}{2}}\right)^k}{k!}\sum_{m=0}^{\infty}\frac{\left(\frac{-b_o}{a_o}\right)^m}{m!}\sum_{n=0}^{\infty}\frac{-\eta_\beta^{-n}}{n!}\frac{\Gamma(n+\frac{k}{2})\Gamma(m-\frac{k}{2})}{\Gamma(\frac{k}{2})\Gamma(-\frac{k}{2})}\frac{1}{q^{(\beta-1)\frac{k}{2}+\beta n+m+1}}. \quad (15)$$

Applying the inverse Laplace transform, we obtain

$$F_2(z,t) = \sum_{k=0}^{\infty}\frac{\left(-za_o^{\frac{1}{2}}\eta_\beta^{\frac{-1}{2}}\right)^k}{k!}\sum_{m=0}^{\infty}\frac{\left(\frac{-b_o}{a_o}\right)^m}{m!}\times$$
$$\times\sum_{n=0}^{\infty}\frac{\eta_\beta^{-n}}{n!}\frac{\Gamma(n+\frac{k}{2})\Gamma(m-\frac{k}{2})}{\Gamma(\frac{k}{2})\Gamma(-\frac{k}{2})\Gamma((\beta-1)\frac{k}{2}+\beta n+m+1)}t^{(\beta-1)\frac{k}{2}+\beta n+m} \quad (16)$$

or

$$F_2(z,t) = \sum_{k=0}^{\infty}\frac{\left(-za_o^{\frac{1}{2}}\eta_\beta^{\frac{-1}{2}}\right)^k}{k!}\sum_{m=0}^{\infty}\frac{\left(\frac{-b_o}{a_o}\right)^m}{m!}t^{(\beta-1)\frac{k}{2}+m}\times$$
$$\times H_{2\ 4}^{1\ 2}\left[\eta_\beta^{-1}t^\beta\left|\begin{matrix}(1-\frac{k}{2},1),(1-m+\frac{k}{2},0)\\ (0,1),(1-\frac{k}{2},0),(1+\frac{k}{2},0),(\frac{(1-\beta)k}{2}-m,\beta)\end{matrix}\right.\right] \quad (17)$$

where $H_{2\ 4}^{1\ 2}$ is the Fox functions [19]. Finally, as $F_1(z,t) = \cos t$, it results

$$F_s(z,t) = \int_0^t\cos(t-s)\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^{k+m}}{k!m!}\left(z\sqrt{\frac{a_os^{\beta-1}}{\eta_\beta}}\right)^k\left(\frac{b_os}{a_o}\right)^m\times$$
$$\times H_{2\ 4}^{1\ 2}\left[\eta_\beta^{-1}s^\beta\left|\begin{matrix}(1-\frac{k}{2},1),(1-m+\frac{k}{2},0)\\ (0,1),(1-\frac{k}{2},0),(1+\frac{k}{2},0),(\frac{(1-\beta)k}{2}-m,\beta)\end{matrix}\right.\right]ds \quad (18)$$

To avoid repetition, we present the final form for $F_c(z,t)$ and $F_e(z,t)$

$$F_c(z,t)=\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^{k+m}}{k!m!}\big(z\sqrt{\frac{a_o}{\eta_\beta}}\big)^k\big(\frac{b_o}{a_o}\big)^m t^{(\beta-1)\frac{k}{2}+m}\times$$
$$\times H_{2\ 4}^{1\ 2}\left[\eta_\beta^{-1}t^\beta\left|\begin{matrix}(1-\frac{k}{2},1),(1-m+\frac{k}{2},0)\\(0,1),(1-\frac{k}{2},0),(1+\frac{k}{2},0),(\frac{(1-\beta)k}{2}-m,\beta)\end{matrix}\right.\right]-$$
$$-\int_0^t\sin(t-s)\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^{k+m}}{k!m!}\big(z\sqrt{\frac{a_os^{\beta-1}}{\eta_\beta}}\big)^k\big(\frac{b_os}{a_o}\big)^m\times$$
$$\times H_{2\ 4}^{1\ 2}\left[\eta_\beta^{-1}s^\beta\left|\begin{matrix}(1-\frac{k}{2},1),(1-m+\frac{k}{2},0)\\(0,1),(1-\frac{k}{2},0),(1+\frac{k}{2},0),(\frac{(1-\beta)k}{2}-m,\beta)\end{matrix}\right.\right]ds \qquad (19)$$

$$F_e(z,t)=(a+b)\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^{k+m}}{k!m!}\big(z\sqrt{\frac{a_o}{\eta_\beta}}\big)^k\big(\frac{b_o}{a_o}\big)^m t^{(\beta-1)\frac{k}{2}+m}\times$$
$$\times H_{2\ 4}^{1\ 2}\left[\eta_\beta^{-1}t^\beta\left|\begin{matrix}(1-\frac{k}{2},1),(1-m+\frac{k}{2},0)\\(0,1),(1-\frac{k}{2},0),(1+\frac{k}{2},0),(\frac{(1-\beta)k}{2}-m,\beta)\end{matrix}\right.\right]-$$
$$-b\int_0^t e^{-(t-s)}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^{k+m}}{k!m!}\big(z\sqrt{\frac{a_os^{\beta-1}}{\eta_\beta}}\big)^k\big(\frac{b_os}{a_o}\big)^m\times$$
$$\times H_{2\ 4}^{1\ 2}\left[\eta_\beta^{-1}s^\beta\left|\begin{matrix}(1-\frac{k}{2},1),(1-m+\frac{k}{2},0)\\(0,1),(1-\frac{k}{2},0),(1+\frac{k}{2},0),(\frac{(1-\beta)k}{2}-m,\beta)\end{matrix}\right.\right]ds. \qquad (20)$$

4. Conclusions and discussion

In this paper the unsteady flow of a second grade fluid in a rotating frame is studied. By introducing a complex field of velocity, the flow problem is formulated in a non-dimensional, suitable form. Solutions of studied problems are determined by means of the Laplace transform method and are expressed by Fox functions. Some existing results can be obtained as particular cases from our general solutions. Also it is worth to note that, by customizing our solutions for Newtonian fluids we obtained solutions for many problems that could be formulated for these fluids. Some of the results are known and others are novel in literature. So, by making $\beta\to 1$ into Eqs. (18) and (19) we recover the solutions obtained by Imran et al. [[25] , Eq(17) and (20)] as it results from Figs. 1 and 2. For $\Omega\to 0$ and $b\to 0$ into Eq. (20) we recover the similar solutions obtained in [[10], Eq. (26)]. By making $\Omega\to 0$ and $\beta\to 1$ in Eqs. (18) and (20) we recover the solutions obtained by Ali et al. [[2] ,Eq(15) and (16)]. Further by making $M=0$ (hydrodynamic fluid), $\frac{1}{K}=0$ (non porous space) and $\Omega \to 0$ into Eqs. (18) and (19), we get

$$F_s(z,t) = \int_0^t \cos(t-s) \sum_{k=0}^{\infty} \frac{(-1)^k z^k \eta_\beta^{\frac{-k}{2}}}{k!} s^{(\beta-1)\frac{k}{2}} \times$$
$$\times H_{2\ 4}^{1\ 2}\left[\eta_\beta^{-1} s^\beta \middle|_{(0,1),(1-\frac{k}{2},0),(1+\frac{k}{2},0),(\frac{(1-\beta)k}{2},\beta)}^{(1-\frac{k}{2},1),(1+\frac{k}{2},0)}\right] ds \tag{21}$$

$$F_c(z,t) = \sum_{k=0}^{\infty} \frac{(-1)^k z^k \eta_\beta^{\frac{-k}{2}}}{k!} t^{(\beta-1)\frac{k}{2}} H_{2\ 4}^{1\ 2}\left[\eta_\beta^{-1} t^\beta \middle|_{(0,1),(1-\frac{k}{2},0),(1+\frac{k}{2},0),(\frac{(1-\beta)k}{2},\beta)}^{(1-\frac{k}{2},1),(1+\frac{k}{2},0)}\right] -$$
$$- \int_0^t \sin(t-s) \sum_{k=0}^{\infty} \frac{(-1)^k z^k \eta_\beta^{\frac{-k}{2}}}{k!} s^{(\beta-1)\frac{k}{2}} H_{2\ 4}^{1\ 2}\left[\eta_\beta^{-1} s^\beta \middle|_{(0,1),(1-\frac{k}{2},0),(1+\frac{k}{2},0),(\frac{(1-\beta)k}{2},\beta)}^{(1-\frac{k}{2},1),(1+\frac{k}{2},0)}\right] ds \tag{22}$$

Now for $\beta \to 1$ in Eqs. (21) and (22) we get similar results obtained by Nazar et al. [[35] , Eqs. (13) and (14)].

Making $\eta_\beta = 0$ into Eq. (13) we find the velocity field corresponding to Newtonian fluids. Firstly, $\overline{F}_{sN}(z,q) = \frac{1}{q^2+1} e^{-z\sqrt{a_o}\sqrt{q+c_o}}$, $c_o = b_o/a_o$, or, the equivalent form $\overline{F}_{sN}(z,q) = \frac{1}{2i}\left[\frac{1}{q-i} - \frac{1}{q+1}\right] e^{-z\sqrt{a_o}\sqrt{q+c_o}}$, with the original function [23],

$$F_{sN}(z,t) = i[A_1(z,t) - A_2(z,t)], \tag{23}$$

where

$$A_1(z,t) = \frac{e^{-it}}{4}\Bigg[e^{-z\sqrt{b_o-ia_o}} efrc\left(\frac{z\sqrt{a_o}}{2\sqrt{t}} - \sqrt{\frac{b_o-ia_o}{a_o}t}\right) +$$
$$+ e^{z\sqrt{b_o-ia_o}} efrc\left(\frac{z\sqrt{a_o}}{2\sqrt{t}} + \sqrt{\frac{b_o-ia_o}{a_o}t}\right)\Bigg] \tag{24}$$

and

$$A_2(z,t) = \frac{e^{it}}{4}\Bigg[e^{-z\sqrt{b_o+ia_o}} efrc\left(\frac{z\sqrt{a_o}}{2\sqrt{t}} - \sqrt{\frac{b_o+ia_o}{a_o}t}\right) +$$
$$e^{z\sqrt{b_o+ia_o}} efrc\left(\frac{z\sqrt{a_o}}{2\sqrt{t}} + \sqrt{\frac{b_o+ia_o}{a_o}t}\right)\Bigg]. \tag{25}$$

Similarly,

$$F_{cN}(z,t) = A_1(z,t) + A_2(z,t), \tag{26}$$

or

$$F_{eN}(z,t)=\frac{a}{2}\left[e^{-z\sqrt{b_o}}efrc\left(\frac{z\sqrt{a_o}}{2\sqrt{t}}-\sqrt{\frac{b_ot}{a_o}}\right)+e^{z\sqrt{b_o}}efrc\left(\frac{z\sqrt{a_o}}{2\sqrt{t}}+\sqrt{\frac{b_ot}{a_o}}\right)\right]+$$
$$\frac{b}{2}e^{-t}\left[e^{-z\sqrt{b_o-a_o}}efrc\left(\frac{z\sqrt{a_o}}{2\sqrt{t}}-\sqrt{\frac{b_o-a_o}{a_o}t}\right)+e^{z\sqrt{b_o-a_o}}efrc\left(\frac{z\sqrt{a_o}}{2\sqrt{t}}+\sqrt{\frac{b_o-a_o}{a_o}t}\right)\right].(27)$$

It is worth pointing out that the above solutions contain as particular cases much problems regarding to Newtonian fluids. For example;

Classical first problem of Stokes, by setting $\frac{1}{K}=0$, $M=0$, $\Omega=0$,$b=0$, $a_o=1$, $b_o=0$ into Eq. (27)

$$F_{eN}(z,t)=\frac{a}{2}\left[erfc\left(\frac{z}{2\sqrt{t}}\right)+erfc\left(\frac{z}{2\sqrt{t}}\right)\right]=aerfc\left(\frac{z}{2\sqrt{t}}\right) \qquad (28)$$

and the classical second Stokes' problem, $(sin(\omega t))$, by setting $\frac{1}{k}=0$, $M=0$, $\Omega=0$, $a_o=1$, $b_o=0$ into Eq. (23)

$$F_{sN}(z,t)=\frac{ie^{-it}}{4}\left[e^{-z\sqrt{-i}}erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{-it}\right)+e^{z\sqrt{-i}}erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{-it}\right)\right]-$$
$$-\frac{ie^{it}}{4}\left[e^{z\sqrt{i}}erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{it}\right)+e^{z\sqrt{i}}erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{it}\right)\right],(29)$$

etc.

In order to analyze the influence of the fractional parameter β on the velocity components, the figures 3 and 4 were plotted. From these figures it is observed that, for high values of the fractional parameter β, the fluid flows more slowly for cosine oscillations and faster for the sine oscillations of the plate. Influence of fractional parameter is significant in the vicinity of the plate and far from the plate the motion of the fluid is mitigated. There, the fluid velocity approaches to zero. In figures 5 and 6 are presented comparisons between the classical fluid ($\beta=1$) and the fractional fluid ($\beta=0.4$), for two different values of the strength of the magnetic field. For both types of the oscillations, the increasing of strength of the magnetic field leads to a decreasing of the absolute values of velocity.

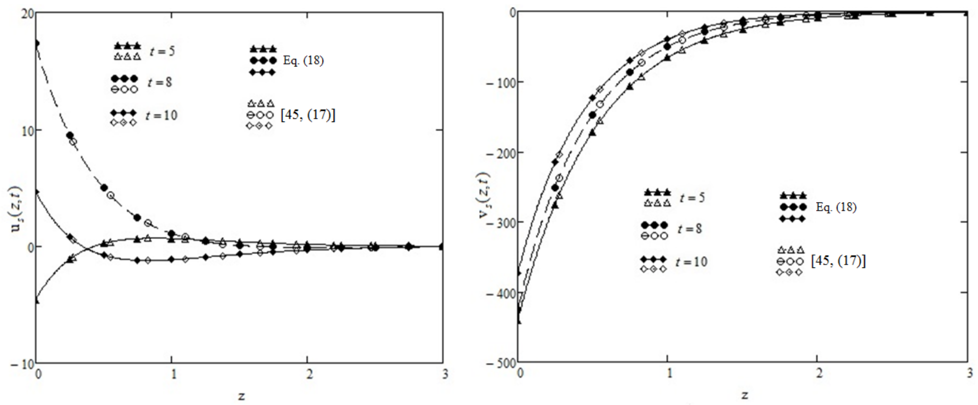

FIGURE 1. . Profile of the components $u(z,t)$ and $v(z,t)$ of the complex field $F_s(z,t)$ given by Eq. (18) and [45, (17)] with $M = 2.5$, $K = 2$, $\Omega = 0.05$, $\omega = 2$ and $\alpha = 1.2$ at different times.

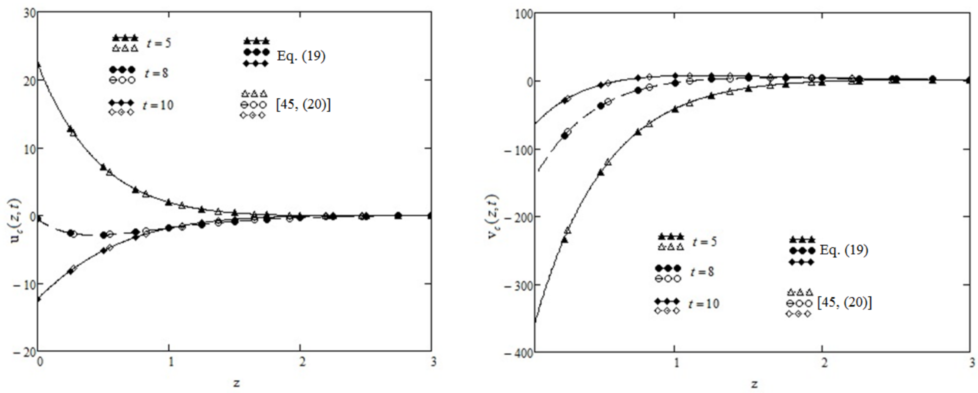

FIGURE 2. . Profile of the components $u(z,t)$ and $v(z,t)$ of the complex field $F_c(z,t)$ given by Eq. (19) and [45, (20)] with $M = 2.5$, $K = 2$, $\Omega = 0.05$, $\omega = 2$ and $\alpha = 1.2$ at different times.

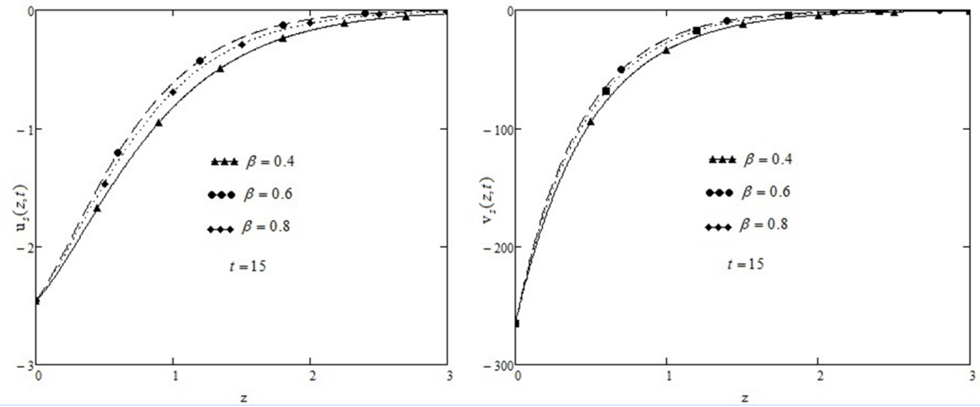

FIGURE 3. . Profile of the components $u(z,t)$ and $v(z,t)$ of the complex field $F_s(z,t)$ given by Eq. (18) with $M = 2.5$, $K = 2$, $\Omega = 0.05$, $\omega = 2$ and $\alpha = 1.2$ at $t = 15$ and different values of β.

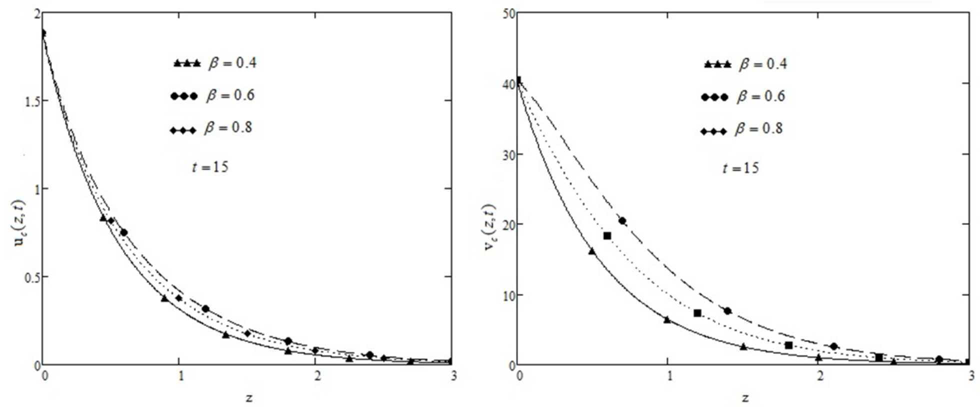

FIGURE 4. . Profile of the components $u(z,t)$ and $v(z,t)$ of the complex field $F_c(z,t)$ given by Eq. (19) with $M = 2.5$, $K = 2$, $\Omega = 0.05$, $\omega = 2$ and $\alpha = 1.2$ at $t = 15$ and different values of β.

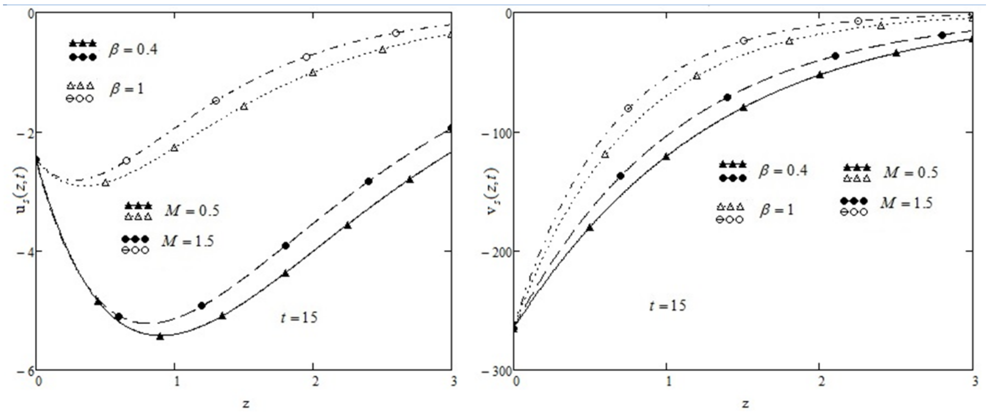

FIGURE 5. . Profile of the components $u(z,t)$ and $v(z,t)$ of the complex field $F_s(z,t)$ given by Eq. (18) with $K=2$, $\Omega=0.05$, $\omega=2$ and $\alpha=1.2$ at $t=15$ and different values of β and M.

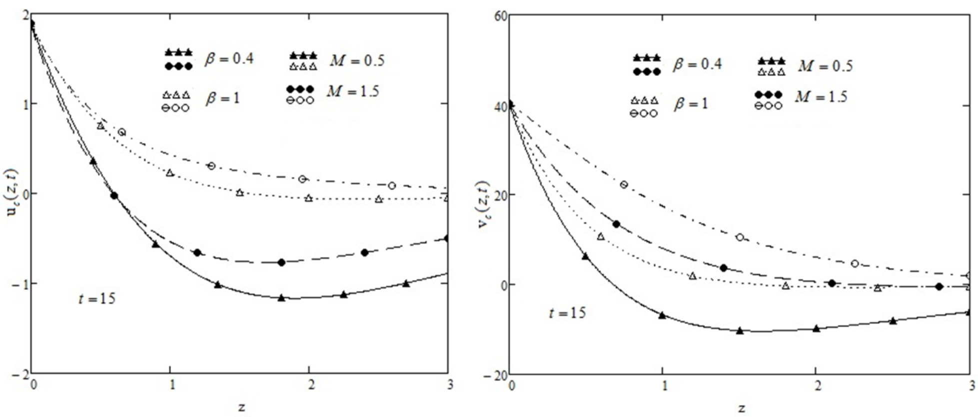

FIGURE 6. . Profile of the components $u(z,t)$ and $v(z,t)$ of the complex field $F_c(z,t)$ given by Eq. (19) with $K=2$, $\Omega=0.05$, $\omega=2$ and $\alpha=1.2$ at $t=15$ and different values of β and M.

Acknowledgement: The authors are highly thankful and grateful to the Abdus Salam School of Mathematical Sciences, GC University Lahore, Pakistan and also Higher Education Commission of Pakistan for generous supporting and facilitating this research work.

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