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COMBINED EFFECT OF SLIP AND RADIATION ON MHD FLOW PAST A CONSTANTLY MOVING VERTICAL PLATE WITH VARIABLE TEMPERATURE

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Abstract. The unsteady free convection of an MHD flow of a viscous fluid passing a vertical plate which is constantly moving with variable temperature is analyzed by taking slip and radiation into consideration. The dimensionless governing equations for temperature and velocity fields are solved using Laplace transform technique. The radiative and slip effects are taken into consideration and the whole system is rotating as a rigid body with a constant angular velocity about the z-axis. Exact solutions are obtained for the two components of velocity. Some known solutions from the literature are obtained as a limiting case. The obtained solutions satisfy the initial and boundary conditions. Some physical aspects of flow parameters on the fluid motion are graphically presented.

Key words : Slip effect, Viscous fluid, MHD flow, Rotating frame, Free convection, Vertical plate.
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1. Nomenclature

C_p-Specific heat at constant pressure
$Ek-$ Ekman number
$g-$ Gravitational acceleration
$Gr-$ Grashof number
$Im-$ Imaginary part of a complex number
$k-$ Thermal conductivity
$Pr-$ Prandtl number
$Re-$ Real part of a complex number

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$s-$ Laplace transform parameter
$T-$ Fluid temperature
$Tw-$ Wall temperature
$T_\infty-$ Temperature far away from the plate
$u, v-$Velocity components along x and y direction
$\mu-$Dynamic viscosity
$\nu-$ Kinematic viscosity
$\Omega-$ Angular velocity of the frame
$\theta-$ Non-dimensional temperature
B_o- External magnetic field
$\sigma-$ Stefan- Boltzmann constant
q_r- Radiative heat flux in the z-direction
$\eta-$ Slip Parameter
$H(t)-$Heaviside unit step function

2. Introduction

The study of free or forced convection flow past a vertical plate has drawn attention of many researchers considering different sets of thermal conditions at the boundary plate, due to its industrial and technological applications. Gupta et. all [1], studied free convection on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Chandran, Sacheti and Singh [2] investigated the natural convection near the vertical plate with ramped wall temperature. In recent years, the problems of magnetohydrodynamic free and forced convection flow in porous and non-porous media is investigated by a number of researchers due to generation, MHD pump, flow meters and accelerators, plasma studies, nuclear reactors using liquid metal coolant and geothermal energy extraction etc.

Among the most interesting results in this direction we remember here the work [3-6], all investigated MHD effects on free convection and mass transfer and references therein. Like magnetohydrodynamic convective flows, radiative convective flows also play important role in countless industrial and environmental processes e.g fossil fuel combustion energy processes, heating and cooling chambers, astrophysical flows, solar power technology and space vehicle re-entry. It also has numerous applications in engineering like nuclear power plants, various propulsion devices for air craft, missiles and satellites etc. Keeping in view this fact many researchers [7-14] discussed the impact of radiations on free or forced convection flow. For the study of the fluid flow many investigations are made by considering a large variety of the thermal and mechanical boundary conditions.

Usually, for the velocity of the fluid the well known boundary condition is non-slip condition. However, more experiments proved that flows with slip at the boundary often appear into practical application [15-20]. In the present paper we extend the results of [21], only for velocity field with combined effect of slip and radiation on MHD flow past a constantly moving vertical plate with variable temperature.

3. Problem Formulation

Let us consider the unsteady MHD flow of an incompressible electrically conducting fluid past a constantly moving vertical plate with variable temperature when the fluid and the plate rotate as a rigid body with the uniform angular velocity Ω about z-axis in the presence of an imposed uniform magnetic field β_o normal to the plate. Initially, the temperature of the plate is assumed to be T_∞. At $t > 0$, the fluid and plate are in the state of rigid body rotation with a constant angular velocity $\overrightarrow{\Omega} = \Omega \overrightarrow{k}$, $\overrightarrow{k}$ being a unit vector parallel with z-axis. The plate starts moving with a velocity U_0 in its own plane and the temperature from the plate is of the form $T_\infty + (T_w - T_\infty)\frac{t}{t_o}$. The fluid occupies the half-space $z \geq 0$, the plate starts constantly moving in its plane along the x- axis and slip condition on the plate is considered. Since, the plate is represented by the (x, y) - plane, all physical variables are functions of z and t only. It is assumed that the induced magnetic field is negligible so that $\overrightarrow{\beta}_0 = (0, 0, \beta_0)$. Under the usual Boussinesq's approximation of the temperature gradient the governing equations of the flow are [21].

$$\frac{\partial u(z,t)}{\partial t} - 2\Omega v(z,t) = g\alpha(T - T_\infty) + \nu\frac{\partial^2 u(z,t)}{\partial z^2} - \frac{\beta_o^2\sigma}{\rho}u(z,t) \tag{1}$$

$$\frac{\partial v(z,t)}{\partial t} + 2\Omega u(z,t) = \nu\frac{\partial^2 v(z,t)}{\partial z^2} - \frac{\beta_o^2\sigma}{\rho}v(z,t) \tag{2}$$

$$\rho C_P\frac{\partial T(z,t)}{\partial t} = k\frac{\partial^2 T(z,t)}{\partial z^2} - \frac{\partial q_r}{\partial z}, \tag{3}$$

where $(u(z,t),\, v(z,t))$ are the velocity components along the x-axis and y-axis respectively, g the gravitational acceleration, α coefficient of volume expansion, ν kinematic viscosity, ρ the density, β_o - external magnetic field, σ is the electrical conductivity and is known as Stefan- Boltzmann constant, q_r radiative heat flux in the z-direction. The local radiant for the case of an optically thin gray gas is expressed by,

$$\frac{\partial q_r}{\partial z} = -4a^*\sigma(T_\infty^4 - T^4), \tag{4}$$

where a^* is absorption constant. Considering the temperature difference within the flow sufficiently small, T^4 can be expressed as the linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms

$$T^4 = 4T_\infty^3 T - 3T_\infty^3, \tag{5}$$

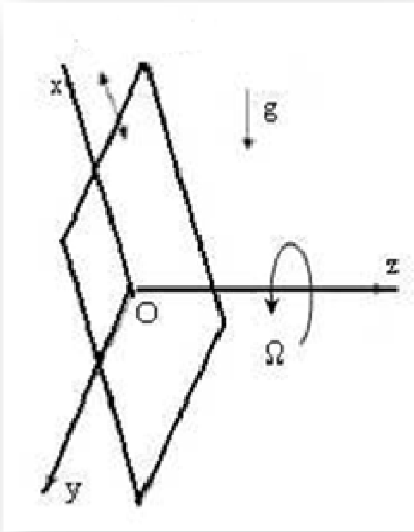

FIGURE 1. Geometry of the Problem

Using eq. (4), and eq. (5) equation (3) becomes

$$\rho C_P \frac{\partial T(z,t)}{\partial t} = k \frac{\partial^2 T(z,t)}{\partial z^2} + 16a^* \sigma T_\infty^3 (T_\infty - T), \qquad (6)$$

Cp the specific heat at constant pressure, k the coefficient of thermal conductivity and $T(z,t)$ is the temperature of the fluid. Initial and boundary conditions are:

$$u(z,0)=0,\ \ v(z,0)=0,\ \ T(z,0)=T_{\infty},\ \ for\ \ all\, z\geq 0, \tag{7}$$

$$u(0,t)-\eta\frac{\partial u(0,t)}{\partial z}=U_0H(t),\ \ v(0,t)=0,$$
$$T(0,t)=T_{\infty}+(T_w-T_{\infty})\frac{t}{t_o}\ \ at\ z=0,\ t>0, \tag{8}$$

$$u(z,t)\to 0,\ \ v(z,t)\to 0,\ \ T(z,t)\to T_{\infty},\ \ as\ z\to\infty,\ t>0, \tag{9}$$

where η is the slip co-efficient and $H(t)$ is a Heaviside unit step function. We use the following set of non-dimensional variables and functions:

$$u^{'}=\frac{u}{U_0},\ \ v^{'}=\frac{v}{U_0},\ \ t^{'}=\frac{t{U_0}^2}{\nu}\ \ z^{'}=\frac{zU_0}{\nu},\ Gr=\frac{g\alpha\nu(T_w-T_{\infty})}{U_o^3},$$
$$Pr=\frac{\mu C_p}{K},\ Ek=\frac{\Omega\nu}{U_o^2},\ \theta=\frac{T-T_{\infty}}{T_w-T_{\infty}},$$
$$R=\frac{16a^{\star}\nu^2\sigma T_{\infty}^3}{kU_o^2},\ M=\frac{\sigma\beta_o^2\nu}{\rho U_o^2}\ and\ \gamma=\frac{\eta U_o}{\nu} \tag{10}$$

where Pr is the Prandtl number, Gr is the Grashof number, θ dimensionless temperature, Ek is the Ekman number, R radiation parameter and M magnetic field parameter. Dropping prime notations, the set of non-dimensional partial differential equations is

$$\frac{\partial u(z,t)}{\partial t}-2Ekv(z,t)=Gr\theta+\frac{\partial^2 u(z,t)}{\partial z^2}-Mu(z,t), \tag{11}$$

$$\frac{\partial v(z,t)}{\partial t}+2Eku(z,t)=\frac{\partial^2 v(z,t)}{\partial z^2}-Mv(z,t), \tag{12}$$

$$\frac{\partial\theta(z,t)}{\partial t}=\frac{1}{Pr}\frac{\partial^2\theta(z,t)}{\partial z^2}-\frac{R}{Pr}\theta(z,t), \tag{13}$$

and the associated initial and boundary conditions are:

$$u(z,0)=0,\ \ v(z,0)=0,\ \ \theta(z,0)=0,\ \ for\ \ all\, z\geq 0, \tag{14}$$

$$u(0,t)-\gamma\frac{\partial u(0,t)}{\partial z}=H(t),\ \ v(0,t)=0,\ \ \ \theta(0,t)=t,\ \ t>0, \tag{15}$$

$$u(z,t)\to 0,\ \ \ v(z,t)\to 0,\ \ \theta(z,t)\to 0\ \ \ as\ \ z\to\infty. \tag{16}$$

Now, we determine solution of the problem (11)-(16) by using the Laplace transform method (R.B. Hetnarski et al (1975))[22]. Applying Laplace transform to the problem (13), $(14)_3$, $(15)_3$, $(16)_3$, we obtain the known expressions of the fluid temperature

$$\theta(z,t) = \theta_1(z,t)e^{-z\sqrt{R}}ercf\left(\frac{z\sqrt{Pr}}{2\sqrt{t}} - \sqrt{at}\right) +$$
$$+\theta_2(z,t)e^{z\sqrt{R}}ercf\left(\frac{z\sqrt{Pr}}{2\sqrt{t}} + \sqrt{at}\right), \tag{17}$$

where $\theta_1(z,t) = \left(\frac{t}{2} - \frac{zPr}{4\sqrt{R}}\right)$ and $\theta_2(z,t) = \left(\frac{t}{2} + \frac{zPr}{4\sqrt{R}}\right)$.
Introducing the complex velocity field $q(z,t) = u(z,t) + iv(z,t)$ and with the notation $m = M + 2iEk$ then applying Laplace transform to the resulting equations, we obtain the transformed problem:

$$\frac{\partial^2 \overline{q}(z,s)}{\partial z^2} - (s+m)\overline{q}(z,s) = -Gr\frac{e^{-c\sqrt{s+a}}}{s^2}, \tag{18}$$

$$\overline{q}(0,s) - \gamma\frac{\partial \overline{q}(0,s)}{\partial z} = \frac{1}{s}, \quad \overline{q}(z,s) \to 0, \quad as \quad z \to \infty. \tag{19}$$

The problem given by eqs. (18) – (19) has the solution

$$\overline{q}(z,s) = \frac{\beta}{s(\beta + \sqrt{s+m})}e^{-z\sqrt{s+m}} + \frac{G_1(\sqrt{Prs+R} + \beta)}{s^2(s-b)(\beta + \sqrt{s+m})}e^{-z\sqrt{s+m}} -$$
$$-\frac{G_1}{s^2(s-b)}e^{-z\sqrt{Prs+R}}, \tag{20}$$

In order to find the inverse Laplace transform eq. (20) can be written in suitable form

$$\overline{q}(z,s) = \frac{\beta}{\beta^2 - m}\left[\frac{\beta}{s} - \frac{\beta}{s-(\beta^2-m)} + \frac{\sqrt{s+m}}{s-(\beta^2-m)} - \frac{\sqrt{s+m}}{s}\right]e^{-z\sqrt{s+m}} +$$
$$+G_1\beta A\frac{\sqrt{s+m}}{s}e^{-z\sqrt{s+m}} + G_1\beta B\frac{\sqrt{s+m}}{s^2}e^{-z\sqrt{s+m}} - G_1\beta C\frac{\sqrt{s+m}}{s-b}e^{-z\sqrt{s+m}} +$$
$$+G_1\beta D\frac{\sqrt{s+m}}{s-(\beta^2-m)}e^{-z\sqrt{s+m}} - \frac{\sqrt{Prs+R}}{s^2(s-b)(\beta+\sqrt{s+m})}e^{-z\sqrt{s+m}}, \tag{21}$$

where $\beta = \frac{1}{\gamma}$, $G_1 = \frac{Gr}{1-Pr}$, $b = \frac{R-m}{1-Pr}$, $Pr \neq 1$, $A = \frac{b+(\beta^2-m)}{b^2(\beta^2-m)^2}$, $B = \frac{1}{b(\beta^2-m)}$, $C = \frac{1}{b^2[b-(\beta^2-m)]}$, $D = -\frac{1}{(\beta^2-m)^2[b-(\beta^2-m)]}$. Using the technique to find the inverse Laplace transform of exponential form [22], we have the solution of eq. (21)

$$q(z,t) = \beta_1 e^{-z\sqrt{m}} erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{mt}\right) + \beta_2 e^{z\sqrt{m}} erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{mt}\right) +$$
$$+\beta_3 e^{z\beta} erfc\left(\frac{z}{2\sqrt{t}} + \beta\sqrt{t}\right) - p_7(z,t) e^{z\sqrt{R}} erfc\left(\frac{z\sqrt{Pr}}{2\sqrt{t}} + \sqrt{at}\right) +$$
$$-p_8(z,t) e^{-z\sqrt{R}} erfc\left(\frac{z\sqrt{Pr}}{2\sqrt{t}} - \sqrt{at}\right) - \frac{G_1 e^{bt}}{2b^2}\left[e^{-c\sqrt{a+b}} erfc\left(\frac{z\sqrt{Pr}}{2t} - \sqrt{(a+b)t}\right) +\right.$$
$$\left.+e^{c\sqrt{a+b}} erfc\left(\frac{z\sqrt{Pr}}{2t} + \sqrt{(a+b)t}\right)\right] + G1\beta K(t) - \frac{G1\sqrt{Pr}}{2\sqrt{\pi}} \int_0^t \frac{e^{-a\tau}}{\tau^{3/2}} K(t-\tau) d\tau, \quad (22)$$

where $a = \frac{R}{Pr}$ and $c = z\sqrt{Pr}$, $b = \frac{R-m}{1-Pr}$, $\beta_1 = \frac{\beta}{2(\beta+\sqrt{m})}$, $\beta_2 = \frac{\beta}{2(\beta-\sqrt{m})}$
$\beta_3 = -\frac{\beta^2 e^(\beta^2-m)t}{\beta^2-m)}$,
$K(t) = p_1(t) e^{-z\sqrt{m}} erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{mt}\right) + p_2(t) e^{z\sqrt{m}} erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{mt}\right) +$
$+ p_3(t) e^{z\beta} erfc\left(\frac{z}{2\sqrt{t}} + \beta\sqrt{t}\right) + p_4(t) e^{-z\sqrt{m+b}} erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{(m+b)t}\right) +$
$+ p_5(t) e^{z\sqrt{m+b}} erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{(m+b)t}\right) + p_6(t) e^{-\frac{z^2}{4t} - mt}$,
$p_1(t) = 1/2\left\{(\sqrt{m} - \beta)A + B[\frac{1}{(2\sqrt{m})} + (\sqrt{m} - \beta)t] - \frac{z}{2}B(1 - \frac{\beta}{\sqrt{m}}\right\}$,
$p_2(t) = -1/2\left\{(\sqrt{m} + \beta)A + B[\frac{1}{(2\sqrt{m})} + (\sqrt{m} + \beta)t] + \frac{z}{2}B(1 + \frac{\beta}{\sqrt{m}}\right\}$,
$p_3(t) = -D\beta e^{(\beta^2-m)t}$,
$p_4(t) = \frac{C}{2}(\sqrt{m+b} - \beta)e^{bt}$,
$p_5(t) = -\frac{C}{2}(\sqrt{m+b} + \beta)e^{bt}$,
$p_6(t) = \left[(A + C + D)\frac{1}{\pi t} + B\sqrt{\frac{t}{\pi}}\right]$,
$p_7(z,t) = -\frac{G_1}{b}\left[\left(\frac{t}{2} + \frac{zPr}{4\sqrt{R}}\right) + \frac{1}{2b}\right]$, $p_8(z,t) = \frac{G_1}{b}\left[\left(\frac{t}{2} - \frac{zPr}{4\sqrt{R}}\right) + \frac{1}{2b}\right]$,
The components of the velocity field $u(z,t)$ and $v(z,t)$ are given by

$$u(z,t) = Re[q(z,t)], \quad v(z,t) = Im[q(z,t)]. \quad (23)$$

4. Limiting cases

a :) The solution in the absence of slip effect i.e. $\gamma = 0$.

$$q(z,t) = p_1 e^{-z\sqrt{m}} erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{mt}\right) + p_2 e^{z\sqrt{m}} erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{mt}\right) -$$
$$-p_3(t) e^{z\sqrt{R}} erfc\left(\frac{z\sqrt{Pr}}{2\sqrt{t}} + \sqrt{at}\right) - p_4(t) e^{-z\sqrt{R}} erfc\left(\frac{z\sqrt{Pr}}{2\sqrt{t}} - \sqrt{at}\right) -$$
$$-\frac{G_1 e^{bt}}{2b^2}\left[e^{-c\sqrt{a+b}} erfc\left(\frac{z\sqrt{Pr}}{2t} - \sqrt{(a+b)t}\right) + e^{c\sqrt{a+b}} erfc\left(\frac{z\sqrt{Pr}}{2t} + \sqrt{(a+b)t}\right)\right]$$
$$+\frac{G_1 e^{bt}}{2b^2}\left[e^{-c\sqrt{Pr}} erfc\left(\frac{z\sqrt{Pr}}{2t} - \sqrt{ct}\right) + e^{c\sqrt{Pr}} erfc\left(\frac{z\sqrt{Pr}}{2t} + \sqrt{ct}\right)\right].$$

where $c = z\sqrt{Pr}$, which is identical to the solution obtained in [21, Eq. (15)].
b :) In the absence of thermal effects, the velocity is reduced to its mechanical part only with $\gamma \neq 0$, $Gr = 0$ in eq. (22)

$$q(z,t) = \beta_1 e^{-z\sqrt{m}} erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{mt}\right) +$$
$$+\beta_2 e^{z\sqrt{m}} erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{mt}\right) + \beta_3 e^{z\beta} erfc\left(\frac{z}{2\sqrt{t}} + \beta\sqrt{t}\right). \quad (25)$$

c :) The solution obtained in the absence of magnetic field with $\gamma \neq 0$, $M = 0$

$$q(z,t) = \beta_1 e^{-z\sqrt{2iEk}} erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{2iEkt}\right) + \beta_2 e^{z\sqrt{2iEk}} erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{2iEkt}\right) +$$
$$+\beta_3 e^{z\beta} erfc\left(\frac{z}{2\sqrt{t}} + \beta\sqrt{t}\right) - p_7(z,t) e^{z\sqrt{R}} erfc\left(\frac{z\sqrt{Pr}}{2\sqrt{t}} + \sqrt{at}\right) +$$
$$-p_8(z,t) e^{-z\sqrt{R}} erfc\left(\frac{z\sqrt{Pr}}{2\sqrt{t}} - \sqrt{at}\right) - \frac{G_1 e^{bt}}{2b^2}\left[e^{-c\sqrt{a+b}} erfc\left(\frac{z\sqrt{Pr}}{2t} - \sqrt{(a+b)t}\right) +\right.$$
$$\left.+e^{c\sqrt{a+b}} erfc\left(\frac{z\sqrt{Pr}}{2t} + \sqrt{(a+b)t}\right)\right] + G1\beta K(t) - \frac{G1\sqrt{Pr}}{2\sqrt{\pi}} \int_0^t \frac{e^{-a\tau}}{\tau^{3/2}} K(t-\tau) d\tau,$$

The components of the velocity field $u(z,t)$ and $v(z,t)$ are given by

$$u(z,t) = Re[q(z,t)], \quad v(z,t) = Im[q(z,t)]. \quad (27)$$

d :) The solution obtained in the non-rotating frame, $\gamma \neq 0$, $Ek = 0$

$$q(z,t) = \beta_1 e^{-z\sqrt{M}} erfc\left(\frac{z}{2\sqrt{t}} - \sqrt{Mt}\right) + \beta_2 e^{z\sqrt{M}} erfc\left(\frac{z}{2\sqrt{t}} + \sqrt{Mt}\right) +$$
$$+\beta_3 e^{z\beta} erfc\left(\frac{z}{2\sqrt{t}} + \beta\sqrt{t}\right) - p_7(z,t) e^{z\sqrt{R}} erfc\left(\frac{z\sqrt{Pr}}{2\sqrt{t}} + \sqrt{at}\right) +$$
$$-p_8(z,t) e^{-z\sqrt{R}} erfc\left(\frac{z\sqrt{Pr}}{2\sqrt{t}} - \sqrt{at}\right) - \frac{G_1 e^{bt}}{2b^2}\left[e^{-c\sqrt{a+b}} erfc\left(\frac{z\sqrt{Pr}}{2t} - \sqrt{(a+b)t}\right) +\right.$$
$$\left. + e^{c\sqrt{a+b}} erfc\left(\frac{z\sqrt{Pr}}{2t} + \sqrt{(a+b)t}\right)\right] + G1\beta K(t) - \frac{G1\sqrt{Pr}}{2\sqrt{\pi}} \int_0^t \frac{e^{-a\tau}}{\tau^{3/2}} K(t-\tau) d\tau, \quad (28)$$

The components of the velocity field $u(z,t)$ and $v(z,t)$ are given by

$$u(z,t) = Re[q(z,t)], \quad v(z,t) = 0. \quad (29)$$

5. Numerical discussion and results

In this paper free convection of unsteady MHD flow of a viscous fluid passing by a constantly moving vertical plate with variable temperature is analyzed by taking slip and radiation into consideration. The dimensionless governing equations for temperature and velocity fields have been solved using Laplace transform technique. The radiative effects are taken into consideration and the whole system is rotating as a rigid body with a constant angular velocity about the z-axis. Exact solutions are obtained for the two components of velocity. Some physical aspects of flow parameters on the fluid motion are graphically presented. The graphical results of our solution are as follows:

In figure 2, the velocity field given by equation (22) is plotted against z at different values of time by fixing the other parameters like $Gr = 0.15$, $Pr = 0.7$, $M = 0.5$ and $\gamma = 0.9$. It is observed that by increasing the value of time the fluid's velocity also increases. In figure 3, the influence of radiation parameter R is examined for particular values of $t = 0.15$, $Pr = 0.7$, $Gr = 0.2$, $M = 0.5$ and $\gamma = 0.8$, the velocity seems to be decreasing with increasing value of R . Figure 4 illustrates that the fluid's velocity decreases with increasing values of Grashof number Gr when plotted by setting the other parameters constant like $t = 0.15$, $Pr = 0.1$, $R = 3$, $M = 0.9$ and $\gamma = 0.8$.

Fluid started flowing with greater velocity as magnetic parameter M increases as shown in figure 5, when graph was plotted against particular values of fluid parameters like $t = 0.15$, $R = 7$, $Gr = 0.015$ and $\gamma = 0.9$. In figure 6, the velocity field is plotted against different values of $t = 0.2$, $Gr = 0.15$, $Pr = 0.7$, $R = 7$ and $M = 0.5$. It can be seen that with decreasing values of γ the fluid's velocity decreases. The velocity field is plotted against different values of Prandtl number Pr are replicated in figure 7. Pr is plotted by keeping the other parameters constant like $t = 0.15$, $Gr = 0.2$, $R = 7$, $M = 0.5$ and $\gamma = 0.9$. It can be seen clearly that by increasing the values of Pr the fluid velocity decreases.

6. Conclusion

In this paper a theoretical analysis has been done to study the influence of the combined effects of slip and radiation on MHD flow of fluid passing a vertical plate which is constantly moving with variable temperature. The dimensionless governing equations along with imposed initial and boundary conditions are solved by using Laplace transform technique. The radiative and slip effects are taken into consideration and the whole system is rotating as a rigid body with a constant angular velocity about the z-axis. Exact solutions are obtained for the two components of velocity. Some conclusions of the study are as below

Real part of the velocity increases with the increase in time t as shown in Fig. 2.

Figs. 3, 4 and 5 depicted for different values of R, Gr and M respectively which show that the velocity of the fluid increases with the increase in R, Gr and M.

Figs. 6 and 7 show that the Real part of the velocity is a decreasing function of Prandtl number Pr and slip parameter γ.

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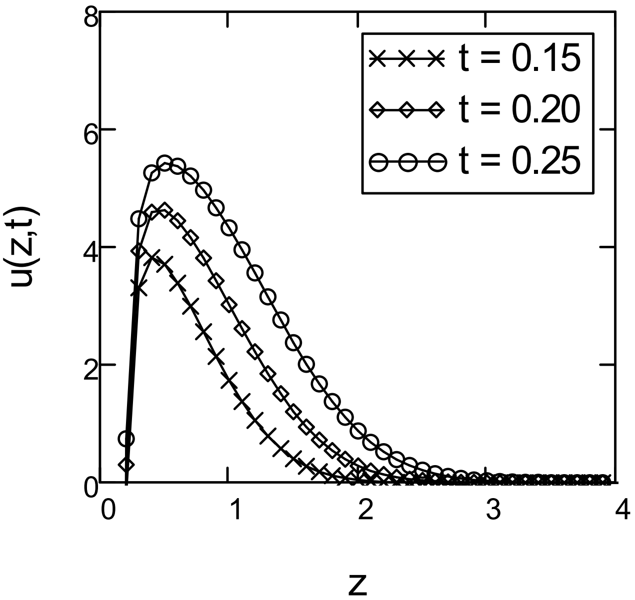

FIGURE 2. Profiles of the velocity field $u(z,t)$ for $Gr = 0.15$, $Pr = 0.7$, $R = 7$, $M = 0.5$, $\gamma = 0.9$ and different values of t

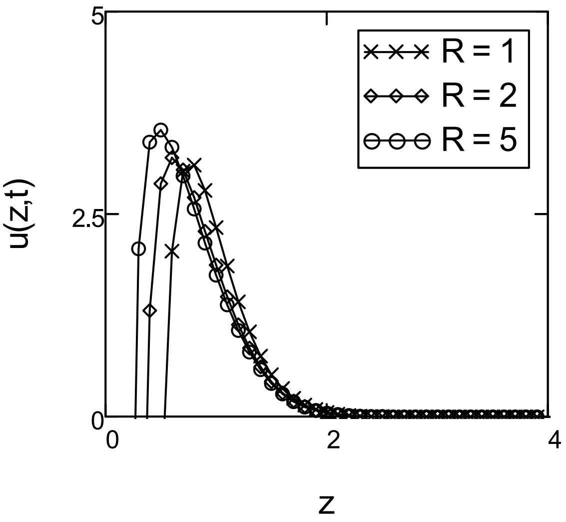

FIGURE 3. Profiles of the velocity field for $t = 0.15$, $Pr = 0.7$, $Gr = 0.2$, $M = 0.5$, $\gamma = 0.8$ and different values of R

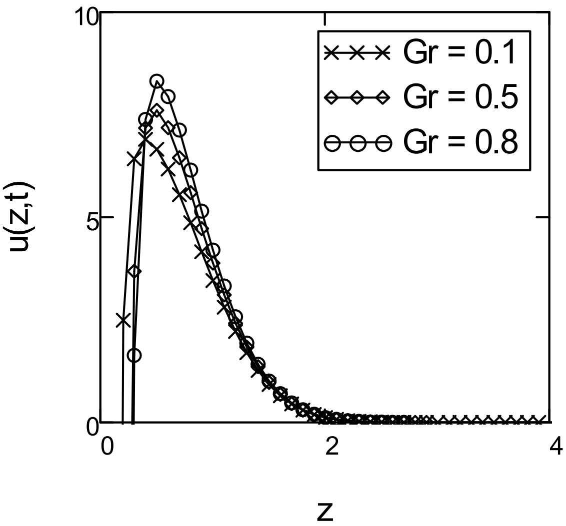

FIGURE 4. Profiles of the velocity field for $t = 0.15$, $Pr = 0.1$, $R = 3$, $M = 0.9$, $\gamma = 0.8$ and different values of Gr

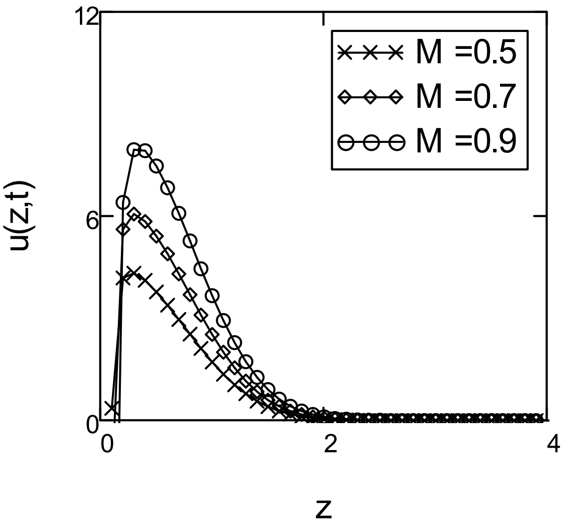

FIGURE 5. Profiles of the velocity field for $t = 0.15$, $Pr = 0.7$, $R = 7$, $Gr = 0.015$, $\gamma = 0.9$ and different values of M

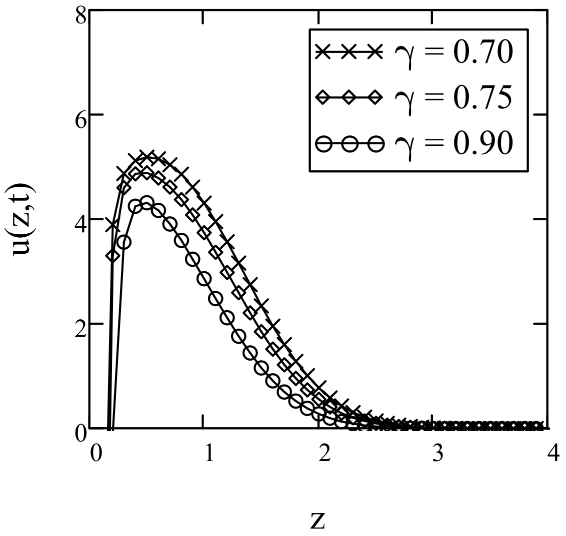

FIGURE 6. Profiles of the velocity field for $t = 0.2$ $Gr = 0.15$, $Pr = 0.7$, $R = 7$, $M = 0.5$, and different values of γ

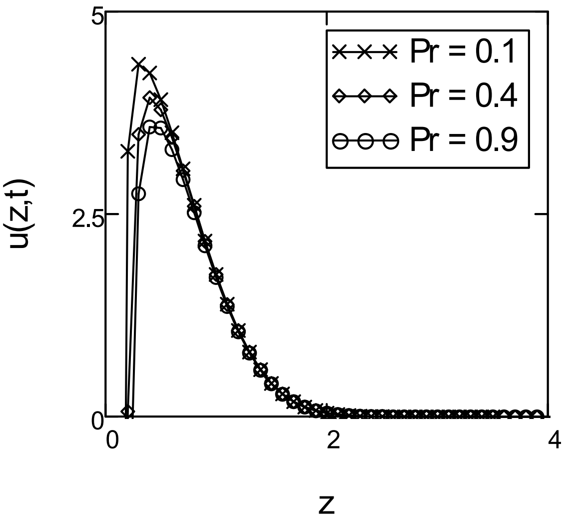

FIGURE 7. Profiles of the velocity field for $t = 0.15$, $Gr = 0.2$, $R = 7$, $M = 0.5$, $\gamma = 0.9$ and different values of Pr