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HEAT AND MASS TRANSFER IN MHD MAXWELL FLUID OVER AN INFINITE VERTICAL PLATE

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Abstract. The main focus of this article is to investigate the exact solution for MHD flow of Maxwell fluid over an infinite vertical plate with ramped temperature and constant concentration. Plate is moving along a straight line with arbitrary velocity which depends on time. Laplace transform and convolution theorem are used to acquire solutions for temperature, concentration and velocity. Moreover, results already present in literature are acquired as limiting case from these general results. *Key words* : Maxwell fluid, Laplace transform, Magnetic effect, Concentration, Ramped temperature, Free convection.
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1. introduction

The Maxwell fluid model is one of the most important model among researchers as compared to other fluid models because it is simplest rate-type fluid model [1]-[5]. Maxwell fluids play important role in polymeric industry. However, they some restrictions. In a simple shear flow this model does not explain the link between shear stress and shear rate [6]-[8]. Maxwell fluids have been considered in many research articles due to its simplicity and has great importance in momentum transfer [9]-[15]. Fetecau et al. [16] investigated exact solution for Maxwell fluid over a moving plate. Numerical solution of Maxwell fluid over a shrinking sheet was explored by Motsa et al. [17]. Khan et al. [18] proposed exact solution of MHD Maxwell fluid in porous medium. Exact solutions of Maxwell fluid on an infinite plate and oscillating plane were

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considered by Abro et al. [19]-[21]. Maxwell nanofluid with ramped temperature was observed by Aman et al. [22]. Karra et al. [23] investigated solution of generalized Maxwell fluid with pressure dependent material. Riaz et al. [24] discovered exact solution of generalized Maxwell fluid. Generalized Maxwell fluid over a moving plate having slip effect was considered by Liu and Guo [25]. Imran et al. [26] investigated analytical solution of Maxwell fluid in the presence of Newtonian heating and slip effect. Comparative analysis for Maxwell fluid with Newtonian heating was investigated by Raza et al. [27].
The phenomena of heat and mass transfer in MHD boundary layer flow have gained attention in chemical engineering, and geophysical environments etc. Different models of fluid with heat and mass transfer have been studies by taking various conditions on temperature and concentration. Singh and Kumar [28] explored the heat and mass transfer in viscous fluid with slip condition. Further chemical reaction and thermal radiation have been considered. Tahir et al. [29] observed heat transfer in Maxwell fluid with wall slip on oscillating plate. Exact solution of transient free convective mass transfer flow under ramped wall temperature and plate velocity has investigated by Ahmed and Dutta [30].
Ghara et al. [31] considered Laplace transformation in order to investigate exact solution of MHD free convection flow under the influence of ramped wall temperature on a moving plate. Seth et al. [32] obsereved heat and mass transfer in viscous fluid on a moving plate. Soret and Hall effects are also considered. Heat and mass transfer in incompressible fluid along with oscillatory suction velocity was observed by Reddy [33]. Second grade fluid having ramped wall temperature under magnetic field was considered by Ahmad et. al. [34]. Narahari and Debnath [35] explored free convection flow with heat generation/absorption on a vertical plate. Furthermore, some significant results see [36]-[38].
In present paper, we consider Maxwell fluid under the influence of ramped temperature and constant concentration over an infinite vertical plate. We considered the generalized boundary condition on velocity. Exact solutions are attained via Laplace transform method. Results for velocity, temperature and concentration are discussed. Previous results in literature can be obtained by the general results in this paper.

2. Mathematical Model

Let incompressible magnetohydrodynamic flow of Maxwell fluid along an infinite plate. Motion of the plate is rectilinear. A magnetic field acting on the plate having strength B_0. By considering very small Reynolds number external electric and induced magnetic field is negligible. Initially system is at rest. After some time plate starts moving with velocity $U_0 f(t)$. Mathematical modeling of the problem is given below [39]:

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial u\left(y,t\right)}{\partial t}=\upsilon\frac{\partial^{2}u\left(y,t\right)}{\partial y^{2}}+g\beta_{T}\left(T\left(y,t\right)-T_{\infty}\right)+g\beta_{C}\left(C\left(y,t\right)-C_{\infty}\right)$$
$$-\frac{\sigma B_{0}^{2}}{\rho}\left(1+\lambda\frac{\partial}{\partial t}\right)u\left(y,t\right), \tag{2.1}$$

$$\rho C_{p}\frac{\partial T\left(y,t\right)}{\partial t}=k\frac{\partial^{2}T\left(y,t\right)}{\partial y^{2}}-Q\left(T\left(y,t\right)-T_{\infty}\right), \tag{2.2}$$

$$\frac{\partial C\left(y,t\right)}{\partial t}=D_{m}\frac{\partial^{2}C\left(y,t\right)}{\partial y^{2}}-R\left(C\left(y,t\right)-C_{\infty}\right). \tag{2.3}$$

The appropriate initial and boundary conditions are

$$u\left(y,0\right)=0,\ T\left(y,0\right)=T_{\infty},\ C\left(y,0\right)=C_{\infty},\ y\geq 0, \tag{2.4}$$

$$u\left(0,t\right)=U_{0}f\left(t\right),\ T\left(0,t\right)=\begin{cases}T_{\infty}+\left(T_{w}-T_{\infty}\right)\frac{t}{t_{0}}, & 0<t\leq t_{0};\\ T(0,t)=T_{w}, & t>t_{0}\end{cases},\ C\left(0,t\right)=C_{w}, \tag{2.5}$$

$$u\left(y,t\right)<\infty,\ T\left(y,t\right)\rightarrow\infty,\ C\left(y,t\right)\rightarrow\infty\ as\ y\rightarrow\infty. \tag{2.6}$$

Following dimensionless variables are used to form the problem free from geometric regime

$$y^{*}=\frac{y}{\sqrt{\upsilon t_{0}}},\ t^{*}=\frac{t}{t_{0}},\ u^{*}=\frac{u}{U_{0}},\ T^{*}=\frac{T-T_{\infty}}{T_{w}-T_{\infty}},\ C^{*}=\frac{C-C_{\infty}}{C_{w}-C_{\infty}},$$
$$G_{r}=\frac{g\beta_{T}t_{0}\left(T-T_{\infty}\right)}{U_{0}},\ G_{m}=\frac{g\beta_{C}t_{0}\left(C-C_{\infty}\right)}{U_{0}},\ M=\sqrt{\upsilon t_{0}}B_{0}\sqrt{\frac{\sigma}{\mu}},$$
$$Pr=\frac{\upsilon C_{p}}{k},\ Q^{*}=\frac{Qt_{0}}{\rho C_{p}},\ Sc=\frac{\upsilon}{D_{m}},\ R^{*}=Rt_{0},\ f^{*}(t^{*})=f(t_{0}t^{*}), \tag{2.7}$$

and dimensionless set of governing equations are:

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial u\left(y,t\right)}{\partial t}=\frac{\partial^{2}u\left(y,t\right)}{\partial y^{2}}+G_{r}T\left(y,t\right)+G_{m}C\left(y,t\right)-M^{2}\left(1+\lambda\frac{\partial}{\partial t}\right)u(y,t), \tag{2.8}$$

$$\frac{\partial T\left(y,t\right)}{\partial t}=\frac{1}{Pr}\frac{\partial^{2}T\left(y,t\right)}{\partial y^{2}}-QT\left(y,t\right), \tag{2.9}$$

$$\frac{\partial C\left(y,t\right)}{\partial t}=\frac{1}{Sc}\frac{\partial^{2}C\left(y,t\right)}{\partial y^{2}}-RC\left(y,t\right). \tag{2.10}$$

and corresponding set of initial and boundary conditions are:

$$u\left(y,0\right)=0,\ T\left(y,0\right)=0,\ C\left(y,0\right)=0, \tag{2.11}$$

$$u\left(0,t\right)=f(t),\ T\left(0,t\right)=\begin{cases}t, & 0<t\leq 1; = tH\left(t\right)-\left(t-1\right)H\left(t-1\right)\\ 1, & t>1\end{cases},\ C(0,t)=1, \tag{2.12}$$

$$u\left(y,t\right)\to 0,\ T\left(y,t\right)\to 0,\ C\left(y,t\right)\to 0\ as\ y\to\infty. \tag{2.13}$$

3. Solution

We use integral transformation in order to get solution of the equations (2.8) – (2.10) by applying initial and boundary conditions (2.11) – (2.13). In order to get solution of velocity we have to find solutions of temperature and concentration first.

3.1. **Temperature Field.** By taking Laplace transform of Eq. (2.9) with suitable given initial condition on temperature, we get

$$Pr\left(q+Q\right)\overline{T}\left(y,q\right)=\frac{\partial^2\overline{T}\left(y,q\right)}{\partial y^2}. \tag{3.1}$$

solution of above differential equation is given below:

$$\overline{T}\left(y,q\right)=\left(\frac{1-e^{-q}}{q^2}\right)e^{-y\sqrt{Pr}\sqrt{q+Q}}. \tag{3.2}$$

Now for the complete solution of the Eq. (3.2) with the help of inverse integral transformation is given by

$$T\left(y,t\right)=\psi\left(y,t,0,Q,Pr\right)-u\left(t-1\right)\psi\left(y,t-1,0,Q,Pr\right). \tag{3.3}$$

where

$$L^{-1}\left(e^{-at}F\left(S\right)\right)=u\left(t-a\right)f\left(t-a\right), \tag{3.4}$$

3.2. **Concentration Field.** By applying Laplace transformation to Eq. (2.10) with the help of initial condition on concentration, we get

$$Sc\left(q+R\right)\overline{C}\left(y,q\right)=\frac{\partial^2\overline{C}\left(y,q\right)}{\partial y^2}, \tag{3.5}$$

and solution is given as

$$\overline{C}\left(y,q\right)=\frac{1}{q}e^{-y\sqrt{Sc}\sqrt{q+R}}. \tag{3.6}$$

Laplace inverse of Eq. (3.6) is given by

$$C\left(y,t\right)=\psi\left(y,t,0,R,Sc\right). \tag{3.7}$$

where

$$\psi(y,t,a,b,c) = L^{-1}\left(\frac{e^{-y\sqrt{c}\sqrt{s+b}}}{s-a}\right) = \frac{e^{at}}{2}\left(e^{-y\sqrt{c}\sqrt{a+b}}erfc\left(\frac{y\sqrt{c}}{2\sqrt{t}} - \sqrt{(a+b)\,t}\right)\right)$$
$$+ \frac{e^{at}}{2}\left(e^{-y\sqrt{c}\sqrt{a+b}}erfc\left(\frac{y\sqrt{c}}{2\sqrt{t}} + \sqrt{(a+b)\,t}\right)\right). \tag{3.8}$$

3.3. **Fluid velocity.** Solving Eq. (2.8) with given condition on velocity at time zero, we get

$$\frac{\partial^2\overline{u}}{\partial y^2} - \left(\lambda q^2 + \left(1+\lambda M^2\right)q + M^2\right)\overline{u} = -Gr\overline{T} - Gm\overline{C}, \tag{3.9}$$

for the solution of differential Eq. (3.9) first we used $\overline{T}(y,q)$ and $\overline{C}(y,q)$ from equation (3.2) and (3.6) in above Eq. (3.9), we get

$$\overline{u}(y,q) = F(q)\,e^{-y\sqrt{\lambda q^2+(1+\lambda M^2)q+M^2}}$$
$$+ \frac{G_r\left(1-e^{-q}\right)}{-\lambda q^4 + \left(Pr - \left(\lambda M^2+1\right)\right)q^3 + \left(QPr - M^2\right)q^2}\times$$
$$\left(e^{-y\sqrt{\lambda q^2+(1+\lambda M^2)q+M^2}} - e^{-y\sqrt{Pr}\sqrt{q+Q}}\right)$$
$$+ \frac{G_m}{-\lambda q^3 + \left(Sc - \left(\lambda M^2+1\right)\right)q^2 + \left(RSc - M^2\right)q}\times$$
$$\left(e^{-y\sqrt{\lambda q^2+(1+\lambda M^2)q+M^2}} - e^{-y\sqrt{Sc}\sqrt{q+Q}}\right). \tag{3.10}$$

Following is the solution of Eq. (3.10)

$$\overline{u}(y,q) = F(q)\,e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} + \left(\frac{2G_r}{\lambda A_2} - \frac{2G_r}{\lambda A_1}\right)\left(\frac{1-e^{-q}}{q}\right)\times$$
$$\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Pr}\sqrt{q+Q}}\right)$$
$$+ \left(\frac{2G_r}{\lambda A_1}\left(\frac{1-e^{-q}}{q-A_4}\right) - \frac{4G_r}{\lambda A_3}\left(\frac{1-e^{-q}}{q^2}\right) - \frac{2G_r}{\lambda A_2}\left(\frac{1-e^{-q}}{q-A_5}\right)\right)\times$$
$$\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Pr}\sqrt{q+Q}}\right)$$
$$+ \left(\frac{4G_m}{\lambda B_1 q} - \frac{G_m}{\lambda B_2\left(q-B_4\right)} - \frac{G_m}{\lambda B_3\left(q-B_5\right)}\right)\times$$
$$\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Sc}\sqrt{q+R}}\right), \tag{3.11}$$

where

$$\alpha_1 = \frac{\lambda M^2 + 1}{2\lambda},\ \alpha_3 = \sqrt{\frac{(\lambda M^2 + 1)^2 - 4\lambda M^2}{4\lambda^2}},\ A_1 = H\left(a_1 - 2H\right)^2,$$
$$A_2 = H\left(a_1 + 2H\right)^2,\ A_3 = (a_1)^2 - 4H^2,\ A_4 = \frac{a_1}{2} - H,\ A_5 = \frac{a_1}{2} + H,$$
$$H = \sqrt{\frac{(a_1)^2 + 4a_2}{4}},\ a_1 = \frac{Pr - \left(M^2 + 1\right)}{\lambda},\ a_2 = \frac{PrQ - M^2}{\lambda},\ B_1 = (b_1)^2 - 4Z^2,$$
$$B_2 = 2Z^2 - Zb_1,\ B_3 = 2Z^2 + Zb_1,\ B_4 = \frac{b_1}{2} - Z,\ B_5 = \frac{b_1}{2} + Z. \tag{3.12}$$

Applying Laplace inverse to Eq. (3.11), it gives

$$u\left(y,t\right) = u_m\left(y,t\right) - u_T\left(y,t\right) - u_C\left(y,t\right), \tag{3.13}$$

$$u_m\left(y,t\right) = L^{-1}\left(P_1\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right), \tag{3.14}$$

where

$$\begin{aligned} L^{-1}\left(P_1\left(y,q\right)\right) &= L^{-1}\left(F\left(q\right)e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}}\right) \\ &= \int_0^t e^{-\alpha_1\tau}\left(\alpha_3 I_1\left(\alpha_3\tau\right) + \delta\left(\tau\right)\right)\left(f'\left(t-\tau\right) + \alpha_1 f\left(t-\tau\right)\right)d\tau \\ &- \alpha_3^2\int_0^t f\left(t-\tau\right)e^{-\alpha_1\tau}I_0\left(\alpha_3\tau\right)d\tau, \end{aligned} \tag{3.15}$$

$$L^{-1}\left(P_2\left(y,q\right)\right) = L^{-1}\left(\frac{e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}}}{\sqrt{\left(q+\alpha_1\right)^2 - \alpha_3^2}}\right) = \begin{cases} 0, & 0 < t < y\sqrt{\lambda}; \\ I_0(\alpha_3\sqrt{t^2 - y^2\lambda}), & t > y\sqrt{\lambda} \end{cases} \tag{3.16}$$

$$
\begin{aligned}
u_T(y,t) = &\left(\frac{G_r}{\lambda} - \frac{2Gr}{\lambda A} + \frac{2Gr}{\lambda A_2}\right)\left(L^{-1}\left(E_1\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right)\right. \\
&- \psi\left(y,t,0,Q,Pr\right) + u\left(t-1\right)\psi\left(y,t-1,0,Q,Pr\right)) \\
&- \left(\frac{4G_r}{\lambda A_3}\right)\left(L^{-1}\left(D_1\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right) - \psi\left(y,t,0,Q,Pr\right)\right) \\
&+ \left(\frac{G_r}{\lambda A_1}\right)\left(L^{-1}\left(C_2\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right) - u\left(t-1\right) * L^{-1}\left(C_2\left(y,q\right)\right)\right. \\
&* L^{-1}\left(P_2\left(y,t-1\right)\right) - \psi\left(y,t,A_4,Q,Pr\right) + u\left(t-1\right)\psi\left(y,t-1,A_4,Q,Pr\right)) \\
&+ \left(\frac{2G_r}{\lambda A_2}\right)\left(L^{-1}\left(C_4\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right) - u\left(t-1\right) * L^{-1}\left(C_4\left(y,q\right)\right)\right. \\
&* L^{-1}\left(P_2\left(y,t-1\right)\right) - \psi\left(y,t,A_5,Q,Pr\right) + u\left(t-1\right)\psi\left(y,t-1,A_5,Q,Pr\right)),
\end{aligned} \tag{3.17}
$$

where

$$
\begin{aligned}
L^{-1}\left(E_1\left(y,q\right)\right) &= L^{-1}\left(\left(\frac{1-e^{-q}}{q}\right)\sqrt{\left(q+\alpha_1\right)^2-\alpha_3^2}\right) \\
&= e^{-\alpha_1\tau}\left(\alpha_3 I_1\left(\alpha_3\tau\right) + \delta\left(\tau\right)\right) \\
&+ \alpha_1\int_0^t H\left(t-\tau\right)e^{-\alpha_1\tau}\left(\alpha_3 I_1\left(\alpha_3\tau\right) + \delta\left(\tau\right)\right)d\tau \\
&- \alpha_3^2\int_0^t H\left(t-\tau\right)e^{-\alpha_1\tau}I_0\left(\alpha_3\tau\right)d\tau \\
&- \left(t-1\right)e^{-\alpha_1 t-1}\left(\alpha_3 I_1\left(\alpha_3\left(t-1\right)\right) + \delta\left(t-1\right)\right)d\tau \\
&- \alpha_1\int_0^t H\left(t-1-\tau\right)e^{-\alpha_1\tau}\left(\alpha_3 I_1\left(\alpha_3\tau\right) + \delta(\tau)\right)d\tau \\
&- \alpha_3^2\int_0^t H\left(t-1-\tau\right)e^{-\alpha_1\tau}I_0\left(\alpha_3\tau\right)d\tau,
\end{aligned} \tag{3.18}
$$

$$
\begin{aligned}
L^{-1}\left(D_1\left(y,q\right)\right) = L^{-1}\quad \left.\frac{\sqrt{(q+\alpha_1)^2-\alpha_3^2}}{q}\right) &= e^{-\alpha_1\tau}\left(\alpha_3 I_1\left(\alpha_3\tau\right) + \delta\left(\tau\right)\right) \\
+ \alpha_1\int_0^t H\left(t-\tau\right)e^{-\alpha_1\tau}\left(\alpha_3 I_1\left(\alpha_3\tau\right) + \delta\left(\tau\right)\right)d\tau& \\
- \alpha_3^2\int_0^t H\left(t-\tau\right)e^{-\alpha_1\tau}I_0\left(\alpha_3\tau\right)d\tau,&
\end{aligned} \tag{3.19}
$$

$$L^{-1}\left(C_2\left(y,q\right)\right)=L^{-1}\left(\frac{\sqrt{\left(q+\alpha_1\right)^2-\alpha_3^2}}{q-A_4}\right)=e^{-\alpha_1\tau}\left(\alpha_3 I_1\left(\alpha_3\tau\right)+\delta\left(\tau\right)\right)$$
$$+\left(A_4+\alpha_1\right)\int_0^t e^{A_4(t-\tau)}e^{-\alpha_1\tau}\left(\alpha_3 I_1\left(\alpha_3\tau\right)+\delta\left(\tau\right)\right)d\tau$$
$$-\alpha_3^2\int_0^t e^{A_4(t-\tau)}e^{-\alpha_1\tau}I_0\left(\alpha_3\tau\right)d\tau, \tag{3.20}$$

$$L^{-1}\left(C_4\left(y,q\right)\right)=L^{-1}\left.\frac{\sqrt{(q+\alpha_1)^2-\alpha_3^2}}{q-A_5}\right)=e^{-\alpha_1\tau}\left(\alpha_3 I_1\left(\alpha_3\tau\right)+\delta\left(\tau\right)\right)$$
$$+\left(A_5+\alpha_1\right)\int_0^t e^{A_5(t-\tau)}e^{-\alpha_1\tau}\left(\alpha_3 I_1\left(\alpha_3\tau\right)+\delta\left(\tau\right)\right)d\tau$$
$$-\alpha_3^2\int_0^t e^{A_5(t-\tau)}e^{-\alpha_1\tau}I_0\left(\alpha_3\tau\right)d\tau, \tag{3.21}$$

$$u_C\left(y,t\right)=\left(\frac{4G_m}{\lambda B_1}\right)\big(L^{-1}\left(D_1\left(y,q\right)\right)*L^{-1}\left(P_2\left(y,q\right)\right)-\psi\left(y,t,0,R,Sc\right)\big)$$
$$+\left(\frac{G_m}{\lambda B_2}\right)\big(L^{-1}\left(C_1\left(y,q\right)\right)*L^{-1}\left(P_2\left(y,q\right)\right)-\psi\left(y,t,B_4,R,Sc\right)\big)$$
$$+\left(\frac{G_m}{\lambda B_3}\right)\big(L^{-1}\left(C_3\left(y,q\right)\right)*L^{-1}\left(P_2\left(y,q\right)\right)-\psi\left(y,t,B_5,R,Sc\right)\big), \tag{3.22}$$

where

$$L^{-1}\left(C_1\left(y,q\right)\right)=L^{-1}\left(\frac{\sqrt{\left(q+\alpha_1\right)^2-\alpha_3^2}}{q-B_4}\right)=e^{-\alpha_1\tau}\left(\alpha_3 I_1\left(\alpha_3\tau\right)+\delta\left(\tau\right)\right)$$
$$+\left(B_4+\alpha_1\right)\int_0^t e^{B_4(t-\tau)}e^{-\alpha_1\tau}\left(\alpha_3 I_1\left(\alpha_3\tau\right)+\delta(\tau)\right)d\tau$$
$$-\alpha_3^2\int_0^t e^{B_4(t-\tau)}e^{-\alpha_1\tau}I_0\left(\alpha_3\tau\right)d\tau, \tag{3.23}$$

$$L^{-1}\left(C_3\left(y,q\right)\right)=L^{-1}\left(\frac{\sqrt{\left(q+\alpha_1\right)^2-\alpha_3^2}}{q-B_5}\right)=e^{-\alpha_1\tau}\left(\alpha_3 I_1\left(\alpha_3\tau\right)+\delta\left(\tau\right)\right)$$
$$+\left(B_5+\alpha_1\right)\int_0^t e^{B_5(t-\tau)}e^{-\alpha_1\tau}\left(\alpha_3 I_1\left(\alpha_3\tau\right)+\delta\left(\tau\right)\right)d\tau$$
$$-\alpha_3^2\int_0^t e^{B_5(t-\tau)}e^{-\alpha_1\tau}I_0\left(\alpha_3\tau\right)d\tau. \tag{3.24}$$

4. Special Cases

General results are obtained time dependent velocity. Some cases are given below

4.1. **Case 1($f(t)=H(t)$).** Let $F(q)=\frac{1}{q}$ in equation (3.11) then it becomes,

$$\overline{u}\left(y,q\right)=\frac{e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}}}{q}+\left(\frac{2G_r}{\lambda A_2}-\frac{2G_r}{\lambda A_1}\right)\left(\frac{1-e^{-q}}{q}\right)\times$$
$$\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}}-e^{-y\sqrt{Pr}\sqrt{q+Q}}\right)$$
$$-\frac{4G_r}{\lambda A_3}\left(\frac{1-e^{-q}}{q^2}\right)\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}}-e^{-y\sqrt{Pr}\sqrt{q+Q}}\right)$$
$$+\frac{2G_r}{\lambda A_1}\left(\frac{1-e^{-q}}{q-A_4}\right)\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}}-e^{-y\sqrt{Pr}\sqrt{q+Q}}\right)$$
$$-\frac{2G_r}{\lambda A_2}\left(\frac{1-e^{-q}}{q-A_5}\right)\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}}-e^{-y\sqrt{Pr}\sqrt{q+Q}}\right)$$
$$+\frac{4G_m}{\lambda q B_1}\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}}-e^{-y\sqrt{Sc}\sqrt{q+R}}\right)$$
$$-\frac{G_m}{\lambda\left(q-B_4\right)B_2}\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}}-e^{-y\sqrt{Sc}\sqrt{q+R}}\right)$$
$$-\frac{G_m}{\lambda\left(q-B_5\right)B_3}\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}}-e^{-y\sqrt{Sc}\sqrt{q+R}}\right), \tag{4.1}$$

Applying laplace inverse to Eq. (4.1), we have

$$u_m\left(y,t\right)=\left(L^{-1}\left(M_3\left(y,q\right)\right)-\alpha_1 L^{-1}\left(M_1\left(y,q\right)\right)-\alpha_3^2 L^{-1}\left(M_3\left(y,q\right)\right)\right)*L^{-1}\left(P_2\left(y,q\right)\right), \tag{4.2}$$

$$
\begin{aligned}
u_T(y,t) = & \left(\frac{G_r}{\lambda} - \frac{2Gr}{\lambda A} + \frac{2Gr}{\lambda A_2}\right)\left(L^{-1}\left(E_1\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right)\right. \\
& \left. - \psi\left(y,t,0,Q,Pr\right) + u\left(t-1\right)\psi\left(y,t-1,0,Q,Pr\right)\right) \\
& - \left(\frac{4G_r}{\lambda A_3}\right)\left(L^{-1}\left(D_1\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right) - \psi\left(y,t,0,Q,Pr\right)\right) \\
& + \left(\frac{G_r}{\lambda A_1}\right)\left(L^{-1}\left(C_2\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right) - u\left(t-1\right) * L^{-1}\left(C_2\left(y,q\right)\right)\right. \\
& \left. * L^{-1}\left(P_2\left(y,t-1\right)\right) - \psi\left(y,t,A_4,Q,Pr\right) + u\left(t-1\right)\psi\left(y,t-1,A_4,Q,Pr\right)\right) \\
& + \left(\frac{2G_r}{\lambda A_2}\right)\left(L^{-1}\left(C_4\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right) - u\left(t-1\right) * L^{-1}\left(C_4\left(y,q\right)\right)\right. \\
& \left. * L^{-1}\left(P_2\left(y,t-1\right)\right) - \psi\left(y,t,A_5,Q,Pr\right) + u\left(t-1\right)\psi\left(y,t-1,A_5,Q,Pr\right)\right),
\end{aligned} \tag{4.3}
$$

$$
\begin{aligned}
u_C\left(y,t\right) = & \left(\frac{4G_m}{\lambda B_1}\right)\left(L^{-1}\left(D_1\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right) - \psi\left(y,t,0,R,Sc\right)\right) \\
& + \left(\frac{G_m}{\lambda B_2}\right)\left(L^{-1}\left(C_1\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right) - \psi\left(y,t,B_4,R,Sc\right)\right) \\
& + \left(\frac{G_m}{\lambda B_3}\right)\left(L^{-1}\left(C_3\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right) - \psi\left(y,t,B_5,R,Sc\right)\right).
\end{aligned} \tag{4.4}
$$

As $\lambda \to 0$ and $u_0 = 0$ similar results are obtain by Narahari and Debnath [35](Eq. (11a) with $a_0 = 0$) and Tokis [40](Equation (12)). This shows that our general results sport the results present in literature.

4.2. **Case 2($f(t) = H(t)e^{bt}$).** Let $F(q) = \frac{1}{q-b}$ in equation (3.11) then it becomes,

$$\begin{aligned}
\overline{u}(y,q) = & \frac{e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}}}{q-b} + \left(\frac{2G_r}{\lambda A_2} - \frac{2G_r}{\lambda A_1}\right)\left(\frac{1-e^{-q}}{q}\right)\times \\
& \left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Pr}\sqrt{q+Q}}\right) \\
& - \frac{4G_r}{\lambda A_3}\left(\frac{1-e^{-q}}{q^2}\right)\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Pr}\sqrt{q+Q}}\right) \\
& + \frac{2G_r}{\lambda A_1}\left(\frac{1-e^{-q}}{q-A_4}\right)\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Pr}\sqrt{q+Q}}\right) \\
& - \frac{2G_r}{\lambda A_2}\left(\frac{1-e^{-q}}{q-A_5}\right)\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Pr}\sqrt{q+Q}}\right) \\
& + \frac{4G_m}{\lambda q B_1}\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Sc}\sqrt{q+R}}\right) \\
& - \frac{G_m}{\lambda(q-B_4)B_2}\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Sc}\sqrt{q+R}}\right) \\
& - \frac{G_m}{\lambda(q-B_5)B_3}\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Sc}\sqrt{q+R}}\right),
\end{aligned} \tag{4.5}$$

Applying laplace inverse to Eq. (4.5), it gives

$$\begin{aligned}
u_m(y,t) = & ((1+b-\alpha_1)\int_0^t e^b(t-\tau)\, e^{-\alpha_1\tau}(\alpha_3 I_1(\alpha_3\tau) + \delta(\tau))\, d\tau \\
& - \alpha_3^2 \int_0^t H(t-\tau)\, e^{-\alpha_1\tau} I_0(\alpha_3\tau)\, d\tau) * L^{-1}(P_2(y,q)),
\end{aligned} \tag{4.6}$$

$$\begin{aligned}
u_T(y,t) = & \left(\frac{G_r}{\lambda} - \frac{2Gr}{\lambda A} + \frac{2Gr}{\lambda A_2}\right)(L^{-1}(E_1(y,q)) * L^{-1}(P_2(y,q)) \\
& - \psi(y,t,0,Q,Pr) + u(t-1)\,\psi(y,t-1,0,Q,Pr)) \\
& - \left(\frac{4G_r}{\lambda A_3}\right)\left(L^{-1}(D_1(y,q)) * L^{-1}(P_2(y,q)) - \psi(y,t,0,Q,Pr)\right) \\
& + \left(\frac{G_r}{\lambda A_1}\right)(L^{-1}(C_2(y,q)) * L^{-1}(P_2(y,q)) - u(t-1) * L^{-1}(C_2(y,q)) \\
& * L^{-1}(P_2(y,t-1)) - \psi(y,t,A_4,Q,Pr) + u(t-1)\,\psi(y,t-1,A_4,Q,Pr)) \\
& + \left(\frac{2G_r}{\lambda A_2}\right)(L^{-1}(C_4(y,q)) * L^{-1}(P_2(y,q)) - u(t-1) * L^{-1}(C_4(y,q)) \\
& * L^{-1}(P_2(y,t-1)) - \psi(y,t,A_5,Q,Pr) + u(t-1)\,\psi(y,t-1,A_5,Q,Pr)),
\end{aligned} \tag{4.7}$$

$$
\begin{aligned}
u_C(y,t) &= \left(\frac{4G_m}{\lambda B_1}\right)\left(L^{-1}(D_1(y,q)) * L^{-1}(P_2(y,q)) - \psi(y,t,0,R,Sc)\right) \\
&+ \left(\frac{G_m}{\lambda B_2}\right)\left(L^{-1}(C_1(y,q)) * L^{-1}(P_2(y,q)) - \psi(y,t,B_4,R,Sc)\right) \\
&+ \left(\frac{G_m}{\lambda B_3}\right)\left(L^{-1}(C_3(y,q)) * L^{-1}(P_2(y,q)) - \psi(y,t,B_5,R,Sc)\right).
\end{aligned} \tag{4.8}
$$

By taking $\lambda \to 0$ and $U = 0$ identicals results are exist in Pattnaik et al. [41](Equation (13) with $a = b$, $\lambda = M^2$, $\frac{1}{k_p} = 0$ and $\gamma = 0$) when magnetic field is fixed relative to the fluid. Also temperature and concentration effects are absent.

4.3. **Case 3($f(t) = \sin \omega t$).** Consider $F(q) = \frac{\omega}{q^2+\omega^2}$ in equation (3.11) and we get,

$$
\begin{aligned}
\overline{u}(y,q) &= \left(\frac{\omega}{q^2+\omega^2}\right) e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} \\
&+ \left(\frac{2G_r}{\lambda A_2} - \frac{2G_r}{\lambda A_1}\right)\left(\frac{1-e^{-q}}{q}\right)\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Pr}\sqrt{q+Q}}\right) \\
&- \frac{4G_r}{\lambda A_3}\left(\frac{1-e^{-q}}{q^2}\right)\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Pr}\sqrt{q+Q}}\right) \\
&+ \frac{2G_r}{\lambda A_1}\left(\frac{1-e^{-q}}{q-A_4}\right)\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Pr}\sqrt{q+Q}}\right) \\
&- \frac{2G_r}{\lambda A_2}\left(\frac{1-e^{-q}}{q-A_5}\right)\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Pr}\sqrt{q+Q}}\right) \\
&+ \frac{4G_m}{\lambda q B_1}\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Sc}\sqrt{q+R}}\right) \\
&- \frac{G_m}{\lambda(q-B_4)B_2}\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Sc}\sqrt{q+R}}\right) \\
&- \frac{G_m}{\lambda(q-B_5)B_3}\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Sc}\sqrt{q+R}}\right),
\end{aligned} \tag{4.9}
$$

Applying laplace inverse to Eq. (4.9), it becomes

$$
\begin{aligned}
u_m(y,t) &= \omega \int_0^t \cos(t-\tau)\, e^{-\alpha_1\tau}\left(\alpha_3 I_1(\alpha_3\tau) + \delta(\tau)\right) d\tau \\
&- \alpha_1 \int_0^t \sin(t-\tau)\, e^{-\alpha_1\tau}\left(\alpha_3 I_0(\alpha_3\tau)\right) d\tau \\
&- \alpha_3^2 \int_0^t \sin(t-\tau)\, e^{-\alpha_1\tau}\left(\alpha_3 I_1(\alpha_3\tau) + \delta(\tau)\right) d\tau,
\end{aligned} \tag{4.10}
$$

$$
\begin{aligned}
u_T(y,t) = &\left(\frac{G_r}{\lambda} - \frac{2Gr}{\lambda A} + \frac{2Gr}{\lambda A_2}\right)\left(L^{-1}\left(E_1\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right)\right. \\
&- \psi\left(y,t,0,Q,Pr\right) + u\left(t-1\right)\psi\left(y,t-1,0,Q,Pr\right)) \\
&- \left(\frac{4G_r}{\lambda A_3}\right)\big(L^{-1}\left(D_1\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right) - \psi\left(y,t,0,Q,Pr\right)\big) \\
&+ \left(\frac{G_r}{\lambda A_1}\right)(L^{-1}\left(C_2\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right) - u\left(t-1\right) * L^{-1}\left(C_2\left(y,q\right)\right) \\
&* L^{-1}\left(P_2\left(y,t-1\right)\right) - \psi\left(y,t,A_4,Q,Pr\right) + u\left(t-1\right)\psi\left(y,t-1,A_4,Q,Pr\right)) \\
&+ \left(\frac{2G_r}{\lambda A_2}\right)(L^{-1}\left(C_4\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right) - u\left(t-1\right) * L^{-1}\left(C_4\left(y,q\right)\right) \\
&* L^{-1}\left(P_2\left(y,t-1\right)\right) - \psi\left(y,t,A_5,Q,Pr\right) + u\left(t-1\right)\psi\left(y,t-1,A_5,Q,Pr\right)),
\end{aligned}
\tag{4.11}
$$

$$
\begin{aligned}
u_C\left(y,t\right) = &\left(\frac{4G_m}{\lambda B_1}\right)\big(L^{-1}\left(D_1\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right) - \psi\left(y,t,0,R,Sc\right)\big) \\
&+ \left(\frac{G_m}{\lambda B_2}\right)\big(L^{-1}\left(C_1\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right) - \psi\left(y,t,B_4,R,Sc\right)\big) \\
&+ \left(\frac{G_m}{\lambda B_3}\right)\big(L^{-1}\left(C_3\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right) - \psi\left(y,t,B_5,R,Sc\right)\big).
\end{aligned}
\tag{4.12}
$$

4.4. **Case 4($f(t)=t^{\delta}$).** Consider $F(q)=t^{\delta}$ in equation (3.11) and we have,

$$\begin{aligned}
\overline{u}(y,q) &= t^{\delta} e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} + \left(\frac{2G_r}{\lambda A_2}-\frac{2G_r}{\lambda A_1}\right)\left(\frac{1-e^{-q}}{q}\right)\times \\
&\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Pr}\sqrt{q+Q}}\right) \\
&- \frac{4G_r}{\lambda A_3}\left(\frac{1-e^{-q}}{q^2}\right)\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Pr}\sqrt{q+Q}}\right) \\
&+ \frac{2G_r}{\lambda A_1}\left(\frac{1-e^{-q}}{q-A_4}\right)\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Pr}\sqrt{q+Q}}\right) \\
&- \frac{2G_r}{\lambda A_2}\left(\frac{1-e^{-q}}{q-A_5}\right)\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Pr}\sqrt{q+Q}}\right) \\
&+ \frac{4G_m}{\lambda q B_1}\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Sc}\sqrt{q+R}}\right) \\
&- \frac{G_m}{\lambda\left(q-B_4\right)B_2}\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Sc}\sqrt{q+R}}\right) \\
&- \frac{G_m}{\lambda\left(q-B_5\right)B_3}\left(e^{-y\sqrt{\lambda}\sqrt{(q+\alpha_1)^2-\alpha_3^2}} - e^{-y\sqrt{Sc}\sqrt{q+R}}\right),
\end{aligned} \tag{4.13}$$

Applying laplace inverse to Eq. (4.13), it gives

$$\begin{aligned}
u_m\left(y,t\right) &= (\delta\int_0^t \left(t-\tau\right)^{\delta-1} e^{-\alpha_1\tau}\left(\alpha_3 I_1\left(\alpha_3\tau\right)+\delta\left(\tau\right)\right)d\tau \\
&- \alpha_1\int_0^t \left(t-1\right)^{\delta} e^{-\alpha_1\tau}\left(\alpha_3 I_1\left(\alpha_3\tau\right)+\delta\left(\tau\right)\right)d\tau \\
&- \alpha_3^2\int_0^t \left(t-1\right)^{\delta} e^{-\alpha_1\tau} I_0\left(\alpha_3\tau\right)d\tau) * L^{-1}\left(P_2\left(y,q\right)\right),
\end{aligned} \tag{4.14}$$

$$\begin{aligned}
u_T(y,t) = &\left(\frac{G_r}{\lambda} - \frac{2Gr}{\lambda A} + \frac{2Gr}{\lambda A_2}\right)\left(L^{-1}\left(E_1\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right)\right.\\
&- \psi\left(y,t,0,Q,Pr\right) + u\left(t-1\right)\psi\left(y,t-1,0,Q,Pr\right))\\
&- \left(\frac{4G_r}{\lambda A_3}\right)\left(L^{-1}\left(D_1\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right) - \psi\left(y,t,0,Q,Pr\right)\right)\\
&+ \left(\frac{G_r}{\lambda A_1}\right)\left(L^{-1}\left(C_2\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right) - u\left(t-1\right) * L^{-1}\left(C_2\left(y,q\right)\right)\right.\\
&* L^{-1}\left(P_2\left(y,t-1\right)\right) - \psi\left(y,t,A_4,Q,Pr\right) + u\left(t-1\right)\psi\left(y,t-1,A_4,Q,Pr\right))\\
&+ \left(\frac{2G_r}{\lambda A_2}\right)\left(L^{-1}\left(C_4\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right) - u\left(t-1\right) * L^{-1}\left(C_4\left(y,q\right)\right)\right.\\
&* L^{-1}\left(P_2\left(y,t-1\right)\right) - \psi\left(y,t,A_5,Q,Pr\right) + u\left(t-1\right)\psi\left(y,t-1,A_5,Q,Pr\right)),
\end{aligned} \tag{4.15}$$

$$\begin{aligned}
u_C\left(y,t\right) = &\left(\frac{4G_m}{\lambda B_1}\right)\left(L^{-1}\left(D_1\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right) - \psi\left(y,t,0,R,Sc\right)\right)\\
&+ \left(\frac{G_m}{\lambda B_2}\right)\left(L^{-1}\left(C_1\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right) - \psi\left(y,t,B_4,R,Sc\right)\right)\\
&+ \left(\frac{G_m}{\lambda B_3}\right)\left(L^{-1}\left(C_3\left(y,q\right)\right) * L^{-1}\left(P_2\left(y,q\right)\right) - \psi\left(y,t,B_5,R,Sc\right)\right).
\end{aligned} \tag{4.16}$$

5. CONCLUSION

Phenomena of heat and mass transfer is studied for MHD Maxwell fluid with ramped temperature and constant concentration over an infinite vertical plate. The generalized time dependent conditions on velocity are considered. Some special case are considered to highlight the applications of problem in the field of engineering sciences. Laplace transform is applied in order to get exact solutions for generalized velocity, their particular cases for velocity, temperature and concentration. Results from the literature can be acquired by our general results.

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