Journal of Prime Research in Mathematics Vol. **15***(2019)*, 101-114

SUM DIVISOR CORDIAL LABELING FOR PATH AND CYCLE RELATED GRAPHS

A. LOURDUSAMY[1] AND F. PATRICK[2]

ABSTRACT. A sum divisor cordial labeling of a graph G with vertex set V is a bijection f from V to $\{1, 2, \cdots, |V(G)|\}$ such that an edge uv is assigned the label 1 if 2 divides $f(u) + f(v)$ and 0 otherwise; and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a sum divisor cordial labeling is called a sum divisor cordial graph. In this paper, we prove that P_n^2, $P_n \odot mK_1$, $S(P_n \odot mK_1)$, $D_2(P_n)$, $T(P_n)$, the graph obtained by duplication of each vertex of path by an edge, $T(C_n)$, $D_2(C_n)$, the graph obtained by duplication of each vertex of cycle by an edge, $C_4^{(t)}$, book, quadrilateral snake and alternate triangular snake are sum divisor cordial graphs.

Key words : sum divisor cordial, divisor cordial.
AMS SUBJECT CLASSIFICATION 2010 : 05C78.

1. INTRODUCTION

All graphs considered here are simple, finite, connected and undirected. For all other standard terminology and notations we follow Harary [2]. A labeling of a graph is a map that carries the graph elements to the set of numbers, usually to the set of non-negative or positive integers. If the domain is the set of vertices the labeling is called vertex labeling. If the domain is the set of edges, then we speak about edge labeling. If the labels are assigned to both vertices and edges then the labeling is called total labeling. For all detailed survey of graph labeling we refer Gallian [1]. A. Lourdusamy and F. Patrick introduced the concept of sum divisor cordial labeling in [5]. In this paper, we investigate the sum divisor cordial labeling behavior of P_n^2,

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$P_n \odot mK_1$, $S(P_n \odot mK_1)$, $D_2(P_n)$, $T(P_n)$, the graph obtained by duplication of each vertex of path by an edge, $T(C_n)$, $D_2(C_n)$, the graph obtained by duplication of each vertex of cycle by an edge, $C_4^{(t)}$, book, quadrilateral snake and alternate triangular snake.

Notation 1.1. *Let $e_f(1)$ denotes the number of edges labeled with 1 and $e_f(0)$ denotes the number of edges labeled with 0.*

Definition 1.2. *Let $G = (V(G), E(G))$ be a simple graph and $f : V \rightarrow \{1, 2, \cdots, |V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if $2|(f(u) + f(v))$ and the label 0 otherwise. The function f is called a sum divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph which admits a sum divisor cordial labeling is called a sum divisor cordial graph.*

Definition 1.3. [3] *For a simple connected graph G the square of graph G is denoted by G^2 and defined as the graph with the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance 1 or 2 apart in G.*

Definition 1.4. [4] *The corona $G_1 \odot G_2$ of two graphs $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ is defined as the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2.*

Definition 1.5. [7] *The shadow graph $D_2(G)$ of a connected graph G is obtained by taking two copies of G, say G' and G''. Join each vertex u' in G' to the neighbours of corresponding vertex u'' in G''.*

Definition 1.6. [3] *For every vertex $v \in V(G)$, the open neighbourhood set $N(v)$ is the set of all vertices adjacent to v in G.*

Definition 1.7. [8] *Duplication of a vertex v_k by a new edge $e = v_k'v_k''$ in a graph G produces a new graph G' such that $N(v_k') \cap N(v_k'') = v_k$.*

Definition 1.8. [7] *The total graph $T(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent whenever they are either adjacent or incident in G.*

Definition 1.9. [6] *A one point union of regular graph G denoted by G^t is the graph obtained by taking v as a common vertex such that any two copy of G are edge disjoint and do not have any vertex in common except v.*

Definition 1.10. [3] *The subdivision graph $S(G)$ is obtained from G by subdividing each edge of G with a vertex.*

Definition 1.11. [6] *A quadrilateral snake Q_n is obtained from a path $v_1, v_2, \cdots, v_n$ by joining v_i, v_{i+1} to new vertices u_i, w_i for every $i = 1, 2, \cdots, n-1$ respectively and then joining u_i and w_i. That is every edge of the path is replaced by a cycle C_4.*

Definition 1.12. [7] *An alternate triangular snake* $A(T_n)$ *is obtained from a path* $v_1, v_2, \cdots, v_n$ *by joining* v_i *and* v_{i+1} *for every* $i = 1, 2, \cdots, n-1$ *(alternately) to a new vertex* u_i*. That is every alternate edge of the path is replaced by a cycle* C_3*.*

2. Main results

Theorem 2.1. *The graph* P_n^2 *is sum divisor cordial graph.*

Proof. Let $v_1, v_2, \cdots, v_n$ be the vertices of the path P_n. Let $V(P_n^2) = \{v_1, v_2, \cdots, v_n\}$ and $E(P_n^2) = \{v_iv_{i+1} : 1 \leq i \leq n-1\} \bigcup \{v_iv_{i+2} : 1 \leq i \leq n-2\}$. Then P_n^2 is of order n and size $2n-3$. Define $f : V(P_n^2) \to \{1, 2, \cdots, n\}$ as follows:

$$f(v_i) = i,\ 1 \leq i \leq n.$$

Then, the induced edge labels are

$$f^*(v_iv_{i+1}) = 0,\ 1 \leq i \leq n-1;$$
$$f^*(v_iv_{i+2}) = 1,\ 1 \leq i \leq n-2.$$

We observe that, $e_f(0) = n-1$ and $e_f(1) = n-2$.
Thus, $|e_f(0) - e_f(1)| \leq 1$.
Hence, P_n^2 is sum divisor cordial graph. □

Example 1. *A sum divisor cordial labeling of* P_6^2 *is shown in Figure* 2.1*.*

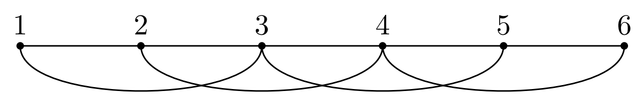

Figure 2.1

Theorem 2.2. *The graph* $P_n \odot mK_1$ *is sum divisor cordial graph.*

Proof. Let $u_1, u_2, \cdots, u_n$ be the vertices of P_n. Let u_{ij} be the vertices which is added to u_i. Then $V(P_n \odot mK_1) = \{u_i, u_{ij} : 1 \leq i \leq n, 1 \leq j \leq m\}$ and $E(P_n \odot mK_1) = \{u_iu_{ij} : 1 \leq i \leq n, 1 \leq j \leq m\} \bigcup \{u_iu_{i+1} : 1 \leq i \leq n-1\}$. Also, $P_n \odot mK_1$ is of order $mn + n$ and size $mn + n - 1$. Define $f : V(P_n \odot mK_1) \to \{1, 2, \cdots, mn + n\}$ as follows:

Case 1. m is odd.
For $1 \leq i \leq n$,

$$f(u_i) = (m+1)(i-1) + 1;$$
$$f(u_{ij}) = (m+1)(i-1) + j + 1,\ 1 \leq j \leq m.$$

Then, the induced edge labels are

$$f^*(u_iu_{i+1}) = 1,\ 1 \leq i \leq n-1;$$
$$f^*(u_iu_{ij}) = \begin{cases} 0 & \text{if } j \text{ is odd and } 1 \leq i \leq n \\ 1 & \text{if } j \text{ is even and } 1 \leq i \leq n. \end{cases}$$

Case 2. m is even.
For $1 \leq i \leq n$,

$$f(u_i) = \begin{cases} (m+1)(i-1)+1 & \text{if } i \text{ is odd} \\ (m+1)(i-1)+2 & \text{if } i \text{ is even;} \end{cases}$$

$$f(u_{ij}) = \begin{cases} (m+1)(i-1)+j+1 & \text{if } i \text{ is odd and } 1 \leq j \leq m \\ (m+1)(i-1)+j & \text{if } i \text{ is even and } j = 1 \\ (m+1)(i-1)+j+1 & \text{if } i \text{ is even and } 2 \leq j \leq m. \end{cases}$$

Then, the induced edge labels are

$$f^*(u_i u_{i+1}) = 1, \ 1 \leq i \leq n-1;$$

For $1 \leq i \leq n$,

$$f^*(u_i u_{ij}) = \begin{cases} 0 & \text{if } i \text{ is odd and } j \text{ is odd and } 1 \leq j \leq m \\ 1 & \text{if } i \text{ is odd and } j \text{ is even and } 1 \leq j \leq m; \end{cases}$$

$$f^*(u_i u_{ij}) = \begin{cases} 0 & \text{if } i \text{ is even and } j \text{ is even and } 1 \leq j \leq m \\ 0 & \text{if } i \text{ is even and } j = 1 \\ 1 & \text{if } i \text{ is even and } j \text{ is odd and } 3 \leq j \leq m. \end{cases}$$

In the above two cases, we observe that, $e_f(0) = \lceil \frac{mn+n-1}{2} \rceil$ and $e_f(1) = \lfloor \frac{mn+n-1}{2} \rfloor$. Thus, $|e_f(1) - e_f(0)| \leq 1$. Hence, $P_n \odot mK_1$ is sum divisor cordial graph. □

Example 2. *A sum divisor cordial labeling of $P_4 \odot 2K_1$ is shown in Figure 2.2.*

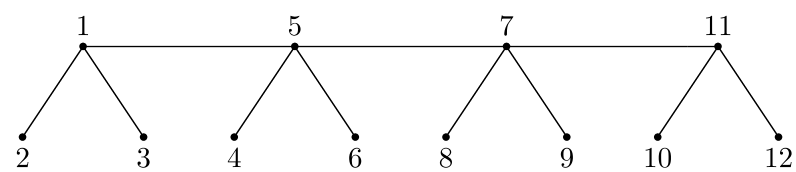

Figure 2.2

Theorem 2.3. *The graph $S(P_n \odot mK_1)$ is sum divisor cordial graph.*

Proof. Let $V(S(P_n \odot mK_1)) = \{u_i : 1 \leq i \leq n\} \bigcup \{v_j, v_j^{'} : 1 \leq j \leq m\} \bigcup \{u_i^{'} : 1 \leq i \leq n-1\}$ and $E(S(P_n \odot mK_1)) = \{u_i v_j^{'}, v_j^{'} v_j : 1 \leq i \leq n, 1 \leq j \leq m\} \bigcup \{u_i u_i^{'}, u_i^{'} u_{i+1} : 1 \leq i \leq n-1\}$. Then, $S(P_n \odot mK_1)$ is of order $2mn+2n-1$ and size $2mn+2n-2$. Define $f : V(S(P_n \odot mK_1)) \to \{1, 2, \cdots, 2mn+2n-1\}$ as follows:

For $1 \leq i \leq n$,

$$f(u_i) = \begin{cases} (m+1)2(i-1)+1 & \text{if } i \text{ is odd} \\ (m+1)2(i-1) & \text{if } i \text{ is even;} \end{cases}$$

$$f(v_j) = \begin{cases} (m+1)2(i-1)+2j & \text{if } i \text{ is odd and } 1 \leq j \leq m \\ (m+1)2(i-1)+2j+1 & \text{if } i \text{ is even and } 1 \leq j \leq m; \end{cases}$$

$$f(v_j^{'}) = \begin{cases} (m+1)2(i-1)+2j+1 & \text{if } i \text{ is odd and } 1 \leq j \leq m \\ (m+1)2(i-1)+2j & \text{if } i \text{ is even and } 1 \leq j \leq m; \end{cases}$$

For $1 \leq i \leq n-1$,

$$f(u_i^{'}) = \begin{cases} (m+1)(2i)+1 & \text{if } i \text{ is odd} \\ (m+1)2i & \text{if } i \text{ is even.} \end{cases}$$

Then, the induced edge labels are

$$f^*(u_iu_i^{'}) = 1, \ 1 \leq i \leq n-1;$$
$$f^*(u_i^{'}u_{i+1}) = 0, \ 1 \leq i \leq n-1;$$
$$f^*(u_iv_j^{'}) = 1, \ 1 \leq i \leq n, 1 \leq j \leq m;$$
$$f^*(v_j^{'}v_j) = 0, \ 1 \leq i \leq n, 1 \leq j \leq m.$$

We observe that, $e_f(0) = mn + n - 1$ and $e_f(1) = mn + n - 1$. Thus, $|e_f(1) - e_f(0)| \leq 1$. Hence, $S(P_n \odot mK_1)$ is sum divisor cordial graph. □

Example 3. *A sum divisor cordial labeling of $S(P_4 \odot 2K_1)$ is shown in Figure* 2.3.

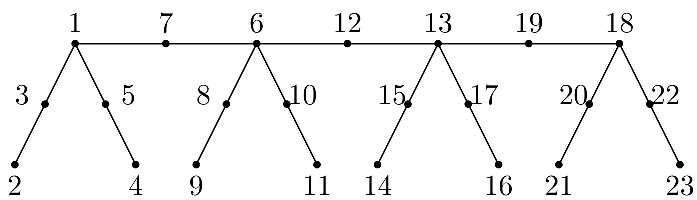

Figure 2.3

Theorem 2.4. *The graph $D_2(P_n)$ is sum divisor cordial graph.*

Proof. Let $v_1, v_2, \cdots, v_n$ be the vertices of the path P_n and $v_1^{'}, v_2^{'}, \cdots, v_n^{'}$ be the newly added vertices corresponding to the vertices $v_1, v_2, \cdots, v_n$ in order to obtain $D_2(P_n)$. Let $G = D_2(P_n)$. Then $V(G) = \{v_i, v_i^{'} : 1 \leq i \leq n\}$ and $E(G) = \{v_iv_{i+1}, v_i^{'}v_{i+1}^{'}, v_iv_{i+1}^{'}, v_i^{'}v_{i+1} : 1 \leq i \leq n-1\}$. Also, G is of order $2n$ and size $4n-4$. Define $f : V(G) \to \{1, 2, \cdots, 2n\}$ as follows:

$$f(v_i) = 2i - 1, \ 1 \leq i \leq n;$$
$$f(v_i^{'}) = 2i, \ 1 \leq i \leq n.$$

Then, the induced edge labels are

$$f^*(v_iv_{i+1}) = 1, \ 1 \leq i \leq n-1;$$
$$f^*(v_i^{'}v_{i+1}^{'}) = 1, \ 1 \leq i \leq n-1;$$
$$f^*(v_iv_{i+1}^{'}) = 0, \ 1 \leq i \leq n-1;$$
$$f^*(v_i^{'}v_{i+1}) = 0, \ 1 \leq i \leq n-1.$$

We observe that, $e_f(0) = 2n - 2$ and $e_f(1) = 2n - 2$.
Thus, $|e_f(0) - e_f(1)| \leq 1$.
Hence, $D_2(P_n)$ is sum divisor cordial graph. □

Example 4. *A sum divisor cordial labeling of $D_2(P_5)$ is shown in Figure* 2.4.

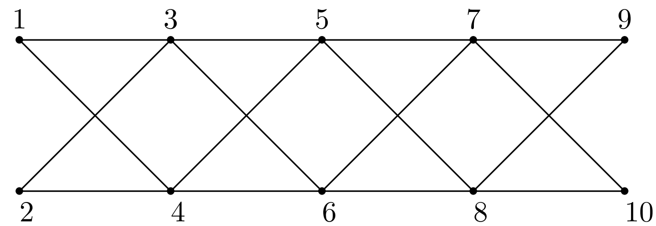

Figure 2.4

Theorem 2.5. *The graph $T(P_n)$ is sum divisor cordial graph.*

Proof. Let $G = T(P_n)$. Let $V(G) = \{v_i : 1 \leq i \leq n\} \bigcup \{u_i : 1 \leq i \leq n-1\}$ and $E(G) = \{u_iu_{i+1} : 1 \leq i \leq n-2\} \bigcup \{v_iu_{i-1} : 2 \leq i \leq n\} \bigcup \{v_iv_{i+1}, v_iu_i : 1 \leq i \leq n-1\}$. Then G is of order $2n-1$ and size $4n-5$. Define $f : V(G) \to \{1, 2, \cdots, 2n-1\}$ as follows:

$$f(v_1) = 1;$$
$$f(v_{i+1}) = 2i,\ 1 \leq i \leq n-1;$$
$$f(u_i) = 2i+1,\ 1 \leq i \leq n-1.$$

Then, the induced edge labels are

$$f^*(v_1v_2) = 0;$$
$$f^*(v_iv_{i+1}) = 1,\ 2 \leq i \leq n-1;$$
$$f^*(v_1u_1) = 1;$$
$$f^*(v_iu_i) = 0,\ 2 \leq i \leq n-1;$$
$$f^*(u_iu_{i+1}) = 1,\ 1 \leq i \leq n-2;$$
$$f^*(v_iu_{i-1}) = 0,\ 2 \leq i \leq n.$$

We observe that, $e_f(0) = 2n-2$ and $e_f(1) = 2n-3$.
Thus, $|e_f(0) - e_f(1)| \leq 1$.
Hence, $T(P_n)$ is sum divisor cordial graph. □

Example 5. *A sum divisor cordial labeling of $T(P_6)$ is shown in Figure 2.5.*

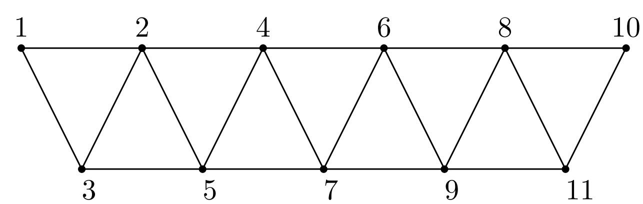

Figure 2.5

Theorem 2.6. *The graph obtained by duplication of each vertex by an edge in P_n is sum divisor cordial graph.*

Proof. Let $v_1, v_2, \cdots, v_n$ be the vertices of the path P_n and G be the graph obtained by duplication of each vertex v_i of the path P_n by an edge $v_i^{'}v_i^{''}$ for $1 \leq i \leq n$ at a time. Let $V(G) = \{v_i, v_i^{'}, v_i^{''} : 1 \leq i \leq n\}$ and $E(G) = \{v_iv_i^{'}, v_iv_i^{''}, v_i^{'}v_i^{''} : 1 \leq i \leq n\} \bigcup \{v_iv_{i+1} : 1 \leq i \leq n-1\}$. Then G is of order $3n$

and size $4n-1$. Define $f : V(G) \to \{1, 2, \cdots, 3n\}$ as follows:

$$f(v_{2i-1}) = 6i-4,\ 1 \le i \le \lceil \tfrac{n}{2} \rceil;$$
$$f(v_{2i}) = 6i-2,\ 1 \le i \le \lfloor \tfrac{n}{2} \rfloor;$$
$$f(v'_{2i-1}) = 6i-5,\ 1 \le i \le \lceil \tfrac{n}{2} \rceil;$$
$$f(v'_{2i}) = 6i-1,\ 1 \le i \le \lfloor \tfrac{n}{2} \rfloor;$$
$$f(v''_{i}) = 3i,\ 1 \le i \le n.$$

Then, the induced edge labels are

$$f^*(v_i v_{i+1}) = 1,\ 1 \le i \le n-1;$$
$$f^*(v_i v'_i) = 0,\ 1 \le i \le n;$$
$$f^*(v'_{2i-1} v''_{2i-1}) = 1,\ 1 \le i \le \lceil \tfrac{n}{2} \rceil;$$
$$f^*(v'_{2i} v''_{2i}) = 0,\ 1 \le i \le \lfloor \tfrac{n}{2} \rfloor;$$
$$f^*(v_{2i-1} v''_{2i-1}) = 0,\ 1 \le i \le \lceil \tfrac{n}{2} \rceil;$$
$$f^*(v_{2i} v''_{2i}) = 1,\ 1 \le i \le \lfloor \tfrac{n}{2} \rfloor.$$

We observe that, $e_f(0) = 2n$ and $e_f(1) = 2n-1$.
Thus, $|e_f(0) - e_f(1)| \le 1$.
Hence, G is sum divisor cordial graph. □

Example 6. *A sum divisor cordial labeling of duplicating each vertex by edge in P_4 is shown in Figure 2.6.*

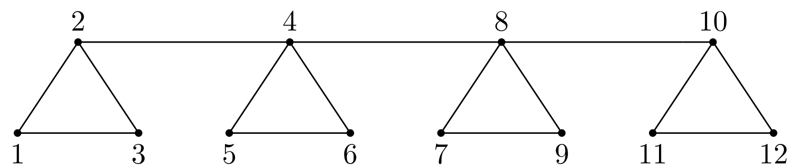

Figure 2.6

Theorem 2.7. *The graph $T(C_n)$ is sum divisor cordial graph.*

Proof. Let $v_1, v_2, \cdots, v_n$ be the vertices of the cycle C_n. Let $G = T(C_n)$. Then $V(G) = \{v_i, u_i : 1 \le i \le n\}$ and $E(G) = \{v_i v_{i+1}, u_i u_{i+1} : 1 \le i \le n-1\} \bigcup \{v_i u_i : 1 \le i \le n\} \bigcup \{v_i u_{i-1} : 2 \le i \le n\} \bigcup \{v_n v_1, u_n u_1, v_1 u_n\}$. Also, G is of order $2n$ and size $4n$. Define $f : V(G) \to \{1, 2, \cdots, 2n\}$ as follows:

$$f(v_i) = 2i-1,\ 1 \le i \le n;$$
$$f(u_i) = 2i,\ 1 \le i \le n.$$

Then, the induced edge labels are

$$f^*(v_i v_{i+1}) = 1,\ 1 \le i \le n-1;$$
$$f^*(v_n v_1) = 1;$$
$$f^*(u_i u_{i+1}) = 1,\ 1 \le i \le n-1;$$
$$f^*(u_n u_1) = 1;$$
$$f^*(v_i u_i) = 0,\ 1 \le i \le n;$$
$$f^*(v_i u_{i-1}) = 0,\ 2 \le i \le n;$$
$$f^*(v_1 u_n) = 0.$$

We observe that, $e_f(0) = 2n$ and $e_f(1) = 2n$.
Thus, $|e_f(0) - e_f(1)| \leq 1$.
Hence, $T(C_n)$ is sum divisor cordial graph. $\square$

Example 7. *A sum divisor cordial labeling of $T(C_8)$ is shown in Figure 2.7.*

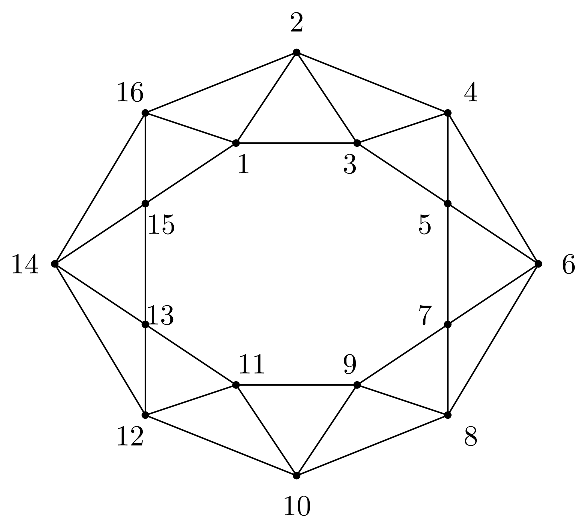

Figure 2.7

Theorem 2.8. *The graph $D_2(C_n)$ is sum divisor cordial graph.*

Proof. Let $v_1, v_2, \cdots, v_n$ be the vertices of the first copy of the cycle C_n and $u_1, u_2, \cdots, u_n$ be the vertices of the second copy of the cycle C_n. Let $G = D_2(C_n)$. Then $V(G) = \{v_i, u_i : 1 \leq i \leq n\}$ and $E(G) = \{v_iv_{i+1}, u_iu_{i+1}, v_iu_{i+1} : 1 \leq i \leq n-1\} \bigcup \{v_iu_{i-1} : 2 \leq i \leq n\} \bigcup \{v_nv_1, u_nu_1, v_nu_1, v_1u_n\}$. Also, G is of order $2n$ and size $4n$. Define $f : V(G) \to \{1, 2, \cdots, 2n\}$ as follows:

$$f(v_i) = 2i, \ 1 \leq i \leq n;$$
$$f(u_i) = 2i - 1, \ 1 \leq i \leq n.$$

Then, the induced edge labels are

$$f^*(v_iv_{i+1}) = 1, \ 1 \leq i \leq n-1;$$
$$f^*(v_nv_1) = 1;$$
$$f^*(u_iu_{i+1}) = 1, \ 1 \leq i \leq n-1;$$
$$f^*(u_nu_1) = 1;$$
$$f^*(v_iu_{i+1}) = 0, \ 1 \leq i \leq n-1;$$
$$f^*(v_nu_1) = 0;$$
$$f^*(v_iu_{i-1}) = 0, \ 2 \leq i \leq n;$$
$$f^*(u_nv_1) = 0.$$

We observe that, $e_f(0) = 2n$ and $e_f(1) = 2n$.
Thus, $|e_f(0) - e_f(1)| \leq 1$.
Hence, $D_2(C_n)$ is sum divisor cordial graph. $\square$

Example 8. *A sum divisor cordial labeling of $D_2(C_6)$ is shown in Figure 2.8.*

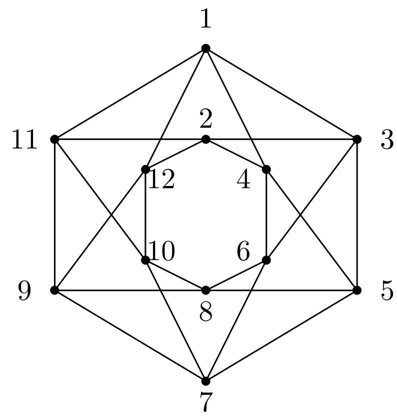

Figure 2.8

Theorem 2.9. *The graph obtained by duplication of each vertex by an edge in C_n is sum divisor cordial graph.*

Proof. Let $v_1, v_2, \cdots, v_n$ be the vertices of the cycle C_n and G be the graph obtained by duplication of each vertex v_i of the cycle C_n by an edge $v_i^{'}v_i^{''}$ for $1 \leq i \leq n$. Then $V(G) = \{v_i, v_i^{'}, v_i^{''} : 1 \leq i \leq n\}$ and $E(G) = E(C_n) \bigcup \{v_iv_i^{'}, v_iv_i^{''}, v_i^{'}v_i^{''} : 1 \leq i \leq n\}$. Also, G is of order $3n$ and size $4n$. Define $f : V(G) \to \{1, 2, \cdots, 3n\}$ as follows:

$$f(v_{2i-1}) = 6i-5,\ 1 \leq i \leq \lceil \tfrac{n}{2} \rceil;$$
$$f(v_{2i}) = 6i-1,\ 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor;$$
$$f(v_{2i-1}^{'}) = 6i-4,\ 1 \leq i \leq \lceil \tfrac{n}{2} \rceil;$$
$$f(v_{2i}^{'}) = 6i-2,\ 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor;$$
$$f(v_i^{''}) = 3i,\ 1 \leq i \leq n.$$

Then, the induced edge labels are

$$f^*(v_iv_{i+1}) = 1,\ 1 \leq i \leq n-1;$$
$$f^*(v_nv_1) = 1;$$
$$f^*(v_iv_i^{'}) = 0,\ 1 \leq i \leq n;$$
$$f^*(v_{2i-1}^{'}v_{2i-1}^{''}) = 0,\ 1 \leq i \leq \lceil \tfrac{n}{2} \rceil;$$
$$f^*(v_{2i}^{'}v_{2i}^{''}) = 1,\ 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor;$$
$$f^*(v_{2i-1}v_{2i-1}^{''}) = 1,\ 1 \leq i \leq \lceil \tfrac{n}{2} \rceil;$$
$$f^*(v_{2i}v_{2i}^{''}) = 0,\ 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor.$$

We observe that, $e_f(0) = 2n$ and $e_f(1) = 2n$.
Thus, $|e_f(0) - e_f(1)| \leq 1$.
Hence, G is sum divisor cordial graph. □

Example 9. *A sum divisor cordial labeling of duplicating each vertex by edge in C_5 is shown in Figure 2.9.*

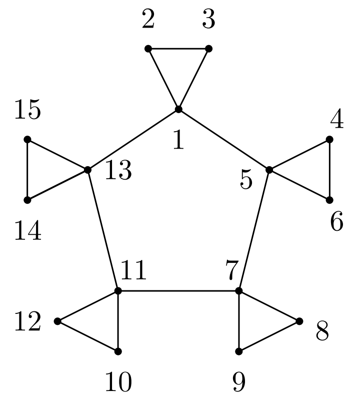

Figure 2.9

Theorem 2.10. *The graph $C_4^{(t)}$ is sum divisor cordial graph.*

Proof. Let $v_1^{(i)}, v_2^{(i)}, v_3^{(i)}, v_4^{(i)} (i = 1, 2, \cdots, t)$ be the vertices of $C_4^{(t)}$. Let $v_1^{(1)} = v_1^{(2)} = \cdots = v_1^{(t)} = v$. Let $G = C_4^{(t)}$. Then, G is of order $3t + 1$ and size $4t$. Define $f : V(G) \to \{1, 2, \cdots, 3t + 1\}$ as follows:

$$f(v) = 1;$$
$$f(v_2^{(i)}) = 3i - 1,\ 1 \le i \le t;$$
$$f(v_3^{(i)}) = 3i + 1,\ 1 \le i \le t;$$
$$f(v_4^{(i)}) = 3i,\ 1 \le i \le t.$$

Then, the induced edge labels are

$$f^*(vv_2^{(2i-1)}) = 0,\ 1 \le i \le \lceil \tfrac{t}{2} \rceil;$$
$$f^*(vv_2^{(2i)}) = 1,\ 1 \le i \le \lfloor \tfrac{t}{2} \rfloor;$$
$$f^*(v_2^{(i)} v_3^{(i)}) = 1,\ 1 \le i \le t;$$
$$f^*(v_3^{(i)} v_4^{(i)}) = 0,\ 1 \le i \le t;$$
$$f^*(vv_4^{(2i-1)}) = 1,\ 1 \le i \le \lceil \tfrac{t}{2} \rceil;$$
$$f^*(vv_4^{(2i)}) = 0,\ 1 \le i \le \lfloor \tfrac{t}{2} \rfloor.$$

We observe that, $e_f(1) = 2t$ and $e_f(0) = 2t$.
Thus, $|e_f(1) - e_f(0)| \le 1$.
Hence, $C_4^{(t)}$ is sum divisor cordial. □

Example 10. *A sum divisor cordial labeling of $C_4^{(4)}$ is shown in Figure* 2.10.

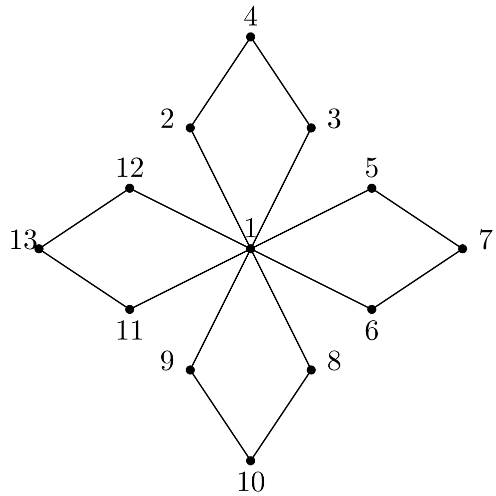

Figure 2.10

Theorem 2.11. *A book with n pentagonal pages is sum divisor cordial graph.*

Proof. Let G be a book with n pentagonal pages. Let $V(G) = \{u, v, u_i, v_i, w_i : 1 \leq i \leq n\}$ and $E(G) = \{uv\} \bigcup \{uu_i, u_i w_i, vv_i, v_i w_i : 1 \leq i \leq n\}$. Then, G is of order $3n+2$ and size $4n+1$. Define $f : V(G) \to \{1, 2, \cdots, 3n+2\}$ as follows:

$$f(u) = 1;$$
$$f(v) = 2;$$
$$f(u_{2i-1}) = 6i-3,\ 1 \leq i \leq \left\lceil \tfrac{n}{2} \right\rceil;$$
$$f(u_{2i}) = 6i+2,\ 1 \leq i \leq \left\lfloor \tfrac{n}{2} \right\rfloor;$$
$$f(w_{2i-1}) = 6i-1,\ 1 \leq i \leq \left\lceil \tfrac{n}{2} \right\rceil;$$
$$f(w_{2i}) = 6i+1,\ 1 \leq i \leq \left\lfloor \tfrac{n}{2} \right\rfloor;$$
$$f(v_{2i-1}) = 6i-2,\ 1 \leq i \leq \left\lceil \tfrac{n}{2} \right\rceil;$$
$$f(v_{2i}) = 6i,\ 1 \leq i \leq \left\lfloor \tfrac{n}{2} \right\rfloor.$$

Then, the induced edge labels are

$$f^*(uv) = 0;$$
$$f^*(uu_{2i-1}) = 1,\ 1 \leq i \leq \left\lceil \tfrac{n}{2} \right\rceil;$$
$$f^*(uu_{2i}) = 0,\ 1 \leq i \leq \left\lfloor \tfrac{n}{2} \right\rfloor;$$
$$f^*(u_{2i-1}w_{2i-1}) = 1,\ 1 \leq i \leq \left\lceil \tfrac{n}{2} \right\rceil;$$
$$f^*(u_{2i}w_{2i}) = 0,\ 1 \leq i \leq \left\lfloor \tfrac{n}{2} \right\rfloor;$$
$$f^*(vv_i) = 1,\ 1 \leq i \leq n;$$
$$f^*(v_i w_i) = 0,\ 1 \leq i \leq n.$$

We observe that,

$$e_f(0) = \begin{cases} 2n & \text{if } n \text{ is odd} \\ 2n+1 & \text{if } n \text{ is even} \end{cases}$$

$$e_f(1) = \begin{cases} 2n+1 & \text{if } n \text{ is odd} \\ 2n & \text{if } n \text{ is even} \end{cases}$$

Thus, $|e_f(1) - e_f(0)| \leq 1$.
Hence, book with n pentagonal pages is sum divisor cordial graph. □

Example 11. *A sum divisor cordial labeling of book with* 4 *pentagonal pages is shown in Figure* 2.11.

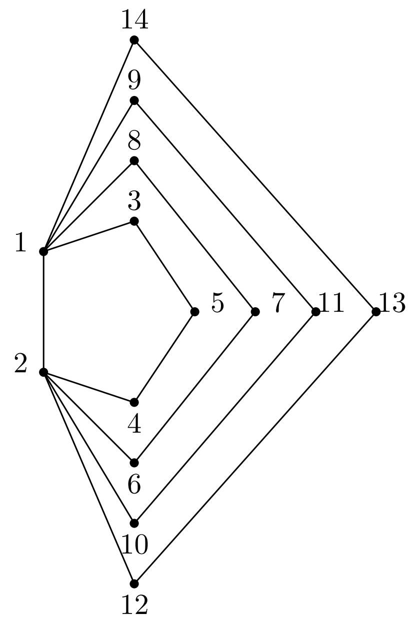

Figure 2.11

Theorem 2.12. *An alternate triangular snake $A(T_n)$ is sum divisor cordial graph.*

Proof. Let $v_1, v_2, \cdots, v_n$ be the vertices of path P_n. The graph $A(T_n)$ is obtained by joining the vertices v_iv_{i+1} (alternately) to new vertex $u_i, 1 \leq i \leq n-1$ for even n and $1 \leq i \leq n-2$ for odd n. Then, $V(G) = V(P_n) \bigcup \{u_i : 1 \leq i \leq \lfloor \frac{n}{2} \rfloor\}$ and $E(G) = E(P_n) \bigcup \{v_{2i-1}u_i : 1 \leq i \leq \lfloor \frac{n}{2} \rfloor ; u_iv_{2i} : 1 \leq i \leq \lfloor \frac{n}{2} \rfloor\}$. Also,

$$|V(G)| = \begin{cases} \frac{3n-1}{2} & \text{if } n \text{ is odd} \\ \frac{3n}{2} & \text{if } n \text{ is even} \end{cases}$$

$$|E(G)| = \begin{cases} 2n-2 & \text{if } n \text{ is odd} \\ 2n-1 & \text{if } n \text{ is even} \end{cases}$$

Case 1: n is odd
Define $f : V(G) \to \{1, 2, \cdots, \frac{3n-1}{2}\}$ as follows:

$$f(v_i) = \begin{cases} 3i-2 & \text{if } i \text{ is odd} \\ 3i-1 & \text{if } i \text{ is even}; \end{cases}$$
$$f(u_i) = 3i,\ 1 \le i \le \lfloor \tfrac{n}{2} \rfloor.$$

Then, the induced edge labels are

$$f^*(v_iv_{i+1}) = \begin{cases} 0 & \text{if } i \text{ is odd} \\ 1 & \text{if } i \text{ is even}; \end{cases}$$
$$f^*(v_{2i-1}u_i) = 1,\ 1 \le i \le \lfloor \tfrac{n}{2} \rfloor;$$
$$f^*(v_{2i}u_i) = 0,\ 1 \le i \le \lfloor \tfrac{n}{2} \rfloor.$$

We observe that, $e_f(0) = n-1$ and $e_f(1) = n-1$.

Case 2: n is even

Define $f : V(G) \to \{1, 2, \cdots, \frac{3n}{2}\}$ as follows:

$$f(v_i) = \begin{cases} 3i-1 & \text{if } i \text{ is odd} \\ 3i & \text{if } i \text{ is even}; \end{cases}$$
$$f(u_i) = 3i-2,\ 1 \le i \le \lfloor \tfrac{n}{2} \rfloor.$$

Then, the induced edge labels are

$$f^*(v_iv_{i+1}) = \begin{cases} 0 & \text{if } i \text{ is odd} \\ 1 & \text{if } i \text{ is even}; \end{cases}$$
$$f^*(v_{2i-1}u_i) = 0,\ 1 \le i \le \lfloor \tfrac{n}{2} \rfloor;$$
$$f^*(v_{2i}u_i) = 1,\ 1 \le i \le \lfloor \tfrac{n}{2} \rfloor.$$

We observe that, $e_f(0) = n$ and $e_f(1) = n-1$.

Thus, in both the cases $|e_f(0) - e_f(1)| \le 1$.

Hence, $A(T_n)$ is sum divisor cordial graph. □

Example 12. *A sum divisor cordial labeling of $A(T_7)$ is shown in Figure* 2.12.

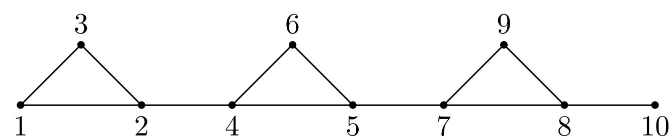

Figure 2.12

Theorem 2.13. *The quadrilateral snake Q_n is sum divisor cordial graph.*

Proof. Let $v_1, v_2, \cdots, v_n$ be the vertices of path P_n. Let $G = Q_n$. Then $V(G) = V(P_n) \bigcup \{u_i, w_i : 1 \le i \le n-1\}$ and $E(G) = E(P_n) \bigcup \{v_iu_i, u_iw_i, v_{i+1}w_i : 1 \le i \le n-1\}$. Also, G is of order $3n-2$ and size $4n-4$. Define $f : V(G) \to \{1, 2, \cdots, 3n-2\}$ as follows:

$$f(v_i) = 3i-2,\ 1 \le i \le n;$$
$$f(u_i) = 3i,\ 1 \le i \le n-1;$$
$$f(w_i) = 3i-1,\ 1 \le i \le n-1.$$

Then, the induced edge labels are

$$f^*(v_iv_{i+1}) = 0,\ 1 \le i \le n-1;$$

$$f^*(v_iu_i) = 1,\ 1 \le i \le n-1;$$
$$f^*(v_{i+1}w_i) = 1,\ 1 \le i \le n-1;$$
$$f^*(u_iw_i) = 0,\ 1 \le i \le n-1;$$

We observe that, $e_f(1) = 2n-2$ and $e_f(0) = 2n-2$.
Thus, $|e_f(1) - e_f(0)| \le 1$.
Hence, Q_n is sum divisor cordial graph. □

Example 13. *A sum divisor cordial labeling of* Q_6 *is shown in Figure* 2.13.

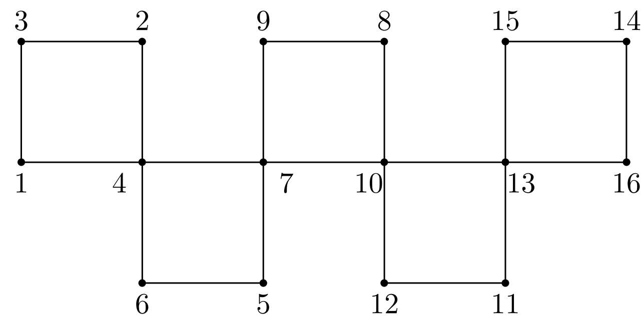

Figure 2.13

3. CONCLUSION

It is clear that $K_n, n \ge 4$, is not sum divisor cordial graph. Here we have proved P_n^2, $P_n \odot mK_1$, $S(P_n \odot mK_1)$, $D_2(P_n)$, $T(P_n)$, the graph obtained by duplication of each vertex of path by an edge, $T(C_n)$, $D_2(C_n)$, the graph obtained by duplication of each vertex of cycle by an edge, $C_4^{(t)}$, book, quadrilateral snake and alternate triangular snake are sum divisor cordial graphs.

REFERENCES

[1] J. A. Gallian, *A Dyamic Survey of Graph Labeling*, The Electronic J. Combin., 17 (2014) # DS6.
[2] F. Harary, *Graph Theory*, Addison-wesley, Reading, Mass 1972.
[3] Gary Chartrand and Ping Zhang, *A First Course in Graph Theory*, McGraw-Hill Higher Education, Boston, 2005.
[4] Gary Chartrand and Ping Zhang, *Chromatic Graph Theory*, Chapman & Hall/CRC, Taylor & Francis, 2009.
[5] A. Lourdusamy and F. Patrick, *Sum Divisor Cordial Graphs*, Proyecciones Journal of Mathematics, **35**(1), (2016), 115-132.
[6] M. Seenivasan *Some New Labeling Concepts*, PhD thesis, Manonmaniam Sundaranar University, India, 2013.
[7] S. K. Vaidya and N. J. Kothari, *Line Gracefulness of Some Path Related Graphs*, International Journal of Mathematics and Scientific Computing, **4**(1), (2014), 15-18.
[8] S. K. Vaidya and N. J. Kothari, *Line Gracefulness of Some Cycle Related Graphs*, IOSR Journal of Mathematics, **10**(6), (2014), 31-37.