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DG-domination topology in Digraph

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Abstract

Throughout this paper, a new entry from domination approaches is introduced which is called a $DG-$ dominating set and which builds its dominance by the topology that relates to digraphs called $\tau_{DG}-$ Topological space. Also, a modern definition of domination number called $DG-$domination number is created. Moreover, some properties of $DG-$ dominating set are presented. Finally, the $DG-$domination number for certain graphs are determined.

Keywords: Topology, digraphs, $DG-$dominating set, $DG-$domination number.

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1. Introduction

In the last previous years, graph theory witnessed several changes due to its relation with other fields like computer science, engineering, communication, etc. The study of domination is more interesting in graph theory, due to having many applications in different fields such as communication networks, natural science, algorithm designs, etc. The first person who defined this concept is Berge [1] and the first one who used this concept is Ore [2]. After this many new parameters of domination have emerged to address these problems.

Dominations in graphs play a wide role in different kinds of fields in graph theory as labeled graph [3], fuzzy graph [4, 5] and [6, 7, 8], soft graph [9], and others. Dominance in graph theory took a great interest of researchers, among them newly.

Additionally, recently, many new definitions of this concept have used, which depend on put some conditions put on the dominating set, pulled out of the dominating set, or on the two together as in [10, 11, 12, 13, 14, 15, 16], and [17, 18].

Topology is one of the most famous, ancient and modern topics that has taken a wide area of mathematicians thinking. With regard to the relationship between graph theory and topology, it expressed topological

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concepts by one of the graph tools such as converting a set of edges, or a set of vertices to topological space and study of other topological concepts of this space. Below some of the previous studies on the topic of the topological graph.

In [19]. Evans *et al.,* formulated the idea of topology on digraph. They found one to one correspondence between the set of all topologies with n points and the set of all transitive digraph with n points. In [20], Bhargava and Ahlborn examined the topological spaces associated with digraphs. They made the above result larger to infinite graphs. Majumdar constructed graph topology from continuous multivalued functions [21]. Subramanian in [22] connected between a dense subset of topology and a domination set of a graph. In [23, 24]. Jabor and Omran presented a new topological space that depend on the minimal dominating sets and got interesting results. Furthermore, in [25], Al'Dzhabri introduced new structures of topological spaces related from digraphs by associating new topologies with digraph which induced from certain open sets which called $DG-$topological space. Also, Al'Dzhabri introduced certain types of open sets related to graphs [26].

In this work, a new parameter of domination is introduced in $DG-$topological space where the two vertices are dominating each other's if there is a$DG-$open set contains them such that this open set is not equal to the vertex set. Some of important properties of this parameter are discussed. Also, the $DG-$domination number for certain graphs such as path, cycle, complete, complete bipartite, null, and wheel graph is computed. For all terminology and notation in graph theory, we refer to Harary [27].

Theorem 1.1. [27] *A nontrivial graph is bipartite if and only if it contains no odd cycles.*

Definition 1.2. [25]Let $G = (V, E)$be a digraph. A subset A of V is called $DG-$ set if for $u_i \in A$ and an arc$v_j u_i \in E$, then $v_j \in A$.

Theorem 1.3. [25] *Let* $G = (V, E)$*be a digraph and* $\tau_{DG} = \{\emptyset, V, A : AisDG - set\}$*, then* τ_{DG} *is a* $DG-$*topology on* V *and*(V, τ_{DG}) *is a* $DG-$*topological space.*

Note that each set in τ_{DG} is called $DG-$open.

2. Main results

Definition 2.1. Let (V, τ_{DG}) is a $DG-$ topological space associated with digraph $G = (V, E)$. A proper subset D of V is called a $DG-$dominating set of V if for every $v \in V - D$ there exists a vertex $u \in D$ such that there exists at least one $DG-$open set (does not equal to V and has at most one initial vertex to all arc incident with it) contains u and v.

Definition 2.2. The minimum cardinality of all $DG-$ dominating set is called the $DG-$domination number and denoted by γ_{DG}.

Note. From the above definition, the indiscrete$DG-$ topology was excluded because we put a condition on the $DG-$open set does not equal to V.

To address this exception, we will take the following remark:

Remark 2.3. Let (V, τ_{DG}) be the $DG-$ indiscrete topological space associated with digraph $D = (V, E)$, then$\gamma_{DG}(G) = 1$.

Observation 2.4.
1. *If there is a vertex* v *such that* v *is initial to all arcs incident with it, then* $\{v\}$ *is* $DG-$*open set.*
2. *If* v *is end vertex and there is a sequence* $v \leftarrow v_1 \leftarrow v_2 \leftarrow \ldots \leftarrow v_n$*, then each* $DG-$*open contains* v*, contains all vertices* $\{v_i, i = 1, \ldots, n\}$*.*

Proposition 2.5. *If* G *has* m *isolated vertices, then* $\gamma_{DG}(G) = m$*.*

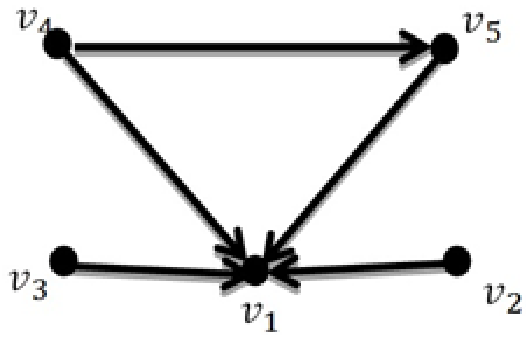

Figure 1:

Proof. If G has one isolated vertex say v, then let $D = \{v\}$. The set $\{v\}$ is $DG-$open set, and for all other $DG-$open set M the set $\{\{v\}, M\}$ is still $DG-$open set. Thus, in this case, $\gamma_{DG}(G) = 1$. Now, if G has more than one isolated vertex say m, then all singleton sets of isolated vertices are $DG-$open set. Also, each one of the isolated vertices cannot be $DG-$dominating set to other isolated vertices according to Definition 1.2. Thus, all isolated belong to each $DG-$dominating set and by part one of the proof, each one of the isolated vertices is $DG-$dominating to all vertices which are not isolated. Therefore, $\gamma_{DG}(G) = m$. □

Corollary 2.6. *Let G be a null graph of order n, then $\gamma_{DG}(G) = 1$.*

Proposition 2.7. *If v is the end vertex to all other vertices and there is an arc v_iv_j such that $v_i \neq v \neq v_j$, then $1 < \gamma_{DG}(G) < n$.*

Proof. Let D be a $DG-$dominating set of G, then $v \in D$, if not means that $v \in V - D$ so for each vertex in D the only open set contains this vertex with the vertex v is V, since v is the end vertex of all other vertices. This is a contradiction with the definition of $DG-$ dominating set. Therefore, the vertex v belongs to every $DG-$dominating set (as an example, see Figure 2). Now, if there is an edge between any two vertices (say $v_i and v_j$) different from the vertex v, the different $DG-$open set is obtained. Thus, if $D = \{v\}$, then for each other vertices, the only $DG-$open set contains v and this vertex is V, so D is not $DG-$ dominating set (as an example, see Figure 2). Thus, the set D contains more than one vertex and the required is obtained. □

Proposition 2.8. *Let G be a non-trivial digraph, then $\gamma_{DG}(G) = 1$ if one of the following statements holds.*

1. *If G has no isolated vertex of order n and there is a vertex v such that v is initial to all arcs.*
2. *If G is Hamiltonian.*

Proof. 1. There are two cases as follows:

Case 1. If $n = 2$, then τ_{DG} is$DG-$indiscrete topology, so according to Remark 2.3, the result is obtained.

Case 2. If $n \geq 3$, then let v be an initial vertex to all arc in the digraph G, and let $D = \{v\}$, then D is a $DG-$dominating set, since for all other vertices v_i there is an open set $\{v, v_i\}, \forall i$ (as an example, see Figure 4 (G_2)). Thus, $\gamma_{DG}(G) = 1$.

2. If G is a Hamiltonian digraph, then there is a cycle passing all vertices of the digraph in the same direction. Thus, for each vertex in G, the $DG-$open that contains this vertex, should be containing too all other vertices, depending on the $DG-$open set. Therefore, τ_{DG} is$DG-$indiscrete topology, so according to Remark 2.3, $\gamma_{DG}(G) = 1$.

□

Theorem 2.9. *If the underlying of a digraph G is a cycle, then $\gamma_{DG}(G) = m \leq \lfloor \frac{n}{2} \rfloor$, where the direction of arcs with different directions and m is the number of initial vertices to each arc incident with it, otherwise$\gamma_{DG}(G) = 1$.*

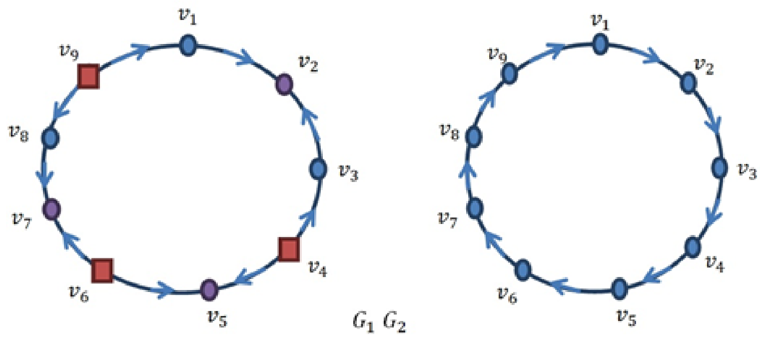

Figure 2: The direct graph of one direction (G_2) and more than one direction (G_1).

Proof. Since G contains different directions, then the number of vertices that only initial for every arc that incident with it, is equal to the number of vertices that only end arc incident with it. Let v_i be the initial vertex to all arc incident with it, so there is a sequence $v_k \leftarrow v_{k+1} \leftarrow \ldots \leftarrow v_{i-1} \leftarrow v_i \rightarrow v_{i+1} \rightarrow \ldots \rightarrow v_{j-1} \rightarrow v_j$, where v_k and v_j are the end vertices to every arc incident with them. For all vertices in this sequence that different from the vertex v_i say v_s, there is $DG-$open set $\{v_i, \ldots, v_s\}$. Thus, the vertex v_i dominates all vertices in this sequence. If v_i is the only vertex that bears the characteristics of being the initial of all the arcs incident with it, then the minimum $DG-$ dominating set$D = \{v_i\}$. Since all vertices of the cycle belong to the sequence. If not, then that means there is another vertex say v_r be initial vertex to all arcs incident with it and there is the following sequence: $v_s \leftarrow v_{s+1} \leftarrow \ldots \leftarrow v_{r-1} \leftarrow v_r \rightarrow v_{r+1} \rightarrow \ldots \rightarrow v_{t-1} \rightarrow v_t$. By following the same previous method, we prove that the vertex v_r belongs to the dominating set D and dominates all vertices belong to the sequence that contains it. Again, if there is no vertex that bears the characteristics of being the initials of all the arcs incident with it, then $D = \{v_i, v_r\}$ is the dominating set and one can be easily concluded that the vertex v_i cannot dominate the vertices $\{v_{s+1}, \ldots, v_r, \ldots, v_{t-1}\}$ and also the vertex v_i cannot dominate the vertices $\{v_{k+1}, \ldots, v_r, \ldots, v_{j-1}\}$. Thus, D is the minimum $DG-$ dominating set. Now, if there are other vertices that bear the characteristics of being the ends of all the arcs incident with them, then all these vertices belong to the dominating set. By the same technique one can be proved that the set D is the minimum $DG-$ dominating set. So, $\gamma_{DG}(G) = m$ where m is the number of initial vertices to each arc incident with it. Moreover, if G has one direction for all arcs in it, then the topology associated to this graph is the $DG-$ indiscrete topology and by Remark 2.3, $\gamma_{DG}(G) = 1$. □

Example 2.10. Let G_1 andG_2 are direct graphs as shown in the following Figure 2:

It is clear that the set $D_1 = \{v_4, v_6, v_9\}$ is the minimum cardinality of $GD-$dominating set of the direct graph G_1, so$\gamma_{DG}(G_1) = 3$. In the direct graph G_2, the topology associated with this graph is the $DG-$ indiscrete topology and by Remark 2.3, $\gamma_{DG}(G) = 1$.

Theorem 2.11. *If the underlying of a digraph G is path, then*

$$\gamma_{DG}(G) = \begin{cases} 3, & \text{if the pendant vertices are initial to the arcs incident with them} \\ & \text{and there is one vertex that end vertex to all arcs incident with it,} \\ m = \min\{m_1, m_2\} \leq \lfloor \frac{n}{2} \rfloor, & \text{where the direction of arcs more than one direction and} \\ & m_1 (m_2 \geq 2) \text{is the number of initial (end)vertices to each arc} \\ & \text{incident with it,} \\ 1, & \text{otherwise.} \end{cases}$$

Proof. There are two cases that depend on the number of directions of arcs in G as follows:

Case 1. If the pendant vertices (say $v_1 and v_2$) are initial to the arcs incident with them and there is one vertex that end vertex to all arcs incident with it say $v$as shown in the following sequence $v_1 \rightarrow v_i \rightarrow \ldots \rightarrow v \leftarrow \ldots \leftarrow v_j \leftarrow v_2$. Then in the same technique in proof of Theorem 2.9, the vertex v belongs to every $DG-$dominating set. Now, let $D = \{v, v_1, v_2\}$, one can be conclude that D is a $DG-$ dominating set and

it is minimal, since the set $\{v, v_1\}$ is not a dominating set for at least one vertex (v_2) and the set $\{v, v_2\}$ is not a dominating for at least one vertex (v_1). Thus, $\gamma_{DG}(G) = 3$.
Case 2. If the direction of the arcs is more than one direction and $m_1(m_2 \geq 2)$ is the number of initial (end) vertices to each arc incident with it, then there are three subcases as follows:
Subcase 1. If both pendant vertices are initial to the arcs incident with them, then the number of initial vertices m_1 is greater than the number of end vertices m_2 (as an example the following sequence) $v_1 \to v_2 \to v_3 \leftarrow v_4 \to v_5 \to v_6 \leftarrow v_7 \leftarrow v_8$. By the same technique in Theorem 2.9, the two sets are minimal $DG-$dominating set. Thus, $\gamma_{DG}(G) = m_2 = \min\{m_1, m_2\}$.
Subcase 2. If both pendant vertices are end to the arcs incident with them, then the number of end vertices m_2 is greater than the number of initial vertices m_1 (as an example in following sequence) $v_1 \leftarrow v_2 \to v_3 \leftarrow v_4 \to v_5 \to v_6 \leftarrow v_7 \to v_8$. Again, by the same technique in Theorem 2.9, the two sets are minimal $DG-$dominating set. Thus, $\gamma_{DG}(G) = m_1 = \min\{m_1, m_2\}$.
Subcase 3. If one of the pendant vertices is initial to the arc incident with it and the other are the end to the arc incident with it, then the number of initial vertices m_1 is equal to the number of end vertices m_2 (as an example the following sequence) $v_1 \to v_2 \to v_3 \leftarrow v_4 \to v_5 \to v_6 \leftarrow v_7 \to v_8$. Thus, $\gamma_{DG}(G) = m = m_1 = m_2$.

From all cases above, the result is obtained. □

Proposition 2.12. *If the underlying of a digraph G is a complete graph, then $\gamma_{DG}(G) = 1$.*

Proof. Let $v \in V$, then v is adjacent to all other vertices in the underlying of a digraph G. Now, let $w \in V$, then there is an arc e join v and u, so there are two cases as follows:
Case 1. If$v$is initial to the arc e, then the $DG-$ open set contains w contains v too, since w is end vertex. Thus, $vDG-$ dominates w.
Case 2. If$v$is the end to the arc e, then the $DG-$ open set contains $v$contains w too, since v is the end vertex. Thus, v $DG-$ dominates w.

From the two cases above, the result is obtained.

Since G is a complete digraph, now we can discuses the following cases:
Case1. if G has one direction for all arcs in it, then the topology associated to this graph is the $DG-$ indiscrete topology and by Remark 2.3, $\gamma_{DG}(G) = 1$.
Case 2. If the arc in the opposite direction hence then the only $DG-$ open set contains the end vertex contains the initial vertex is V and hence the topology associated to this graph is the $DG-$ indiscrete topology and by Remark 2.3, $\gamma_{DG}(G) = 1$. □

Proposition 2.13. *If the underlying of a digraph G is a complete bipartite $K_{m,n}; m \leq n$, then*

$$\gamma_{DG}(G) = \begin{cases} 1, & \text{if } m = 1 \text{ and the corresponding vertex is initial vertex to at least one edge} \\ & \text{incident with it or there is a direct cycle passing all vertices, when } n \text{ is even,} \\ 2, & \text{if } m = 1 \text{ and the corresponding vertex is end vertex to all edges incident with it,} \\ s < m, & \text{if } s \text{ is the number of disjoint direct cycles as subgraph in } G \text{ such that these cycles} \\ & \text{passing to all vertices in } G, \\ \leq m, & \text{otherwise.} \end{cases}$$

Proof. Suppose that V_1 and V_2 are the two bipartite sets of the graph G of order n and m, respectively, then there are three cases as follows:
Case 1. If $m = 1$, then

I) If the partite setV_2 contains only one vertex say v, so if v is the initial vertex to at least one edge incident with it, then there is a $DG-$open set that is not equal to V contains each vertex in the set V_1 (as an example, see Figure 4 (G_1)). Therefore, $\gamma_{DG}(G) = 1$.

II) If there is a directed cycle graph passing through all vertices in graph $K_{m,n}$, this case occurs only if n is even according to Theorem 1.1, then $\tau_{DG} = \{\emptyset, V\}$. It means that τ_{DG} is indiscrete topology, so $\gamma_{DG}(G) = 1$ according to the Remark 2.3.

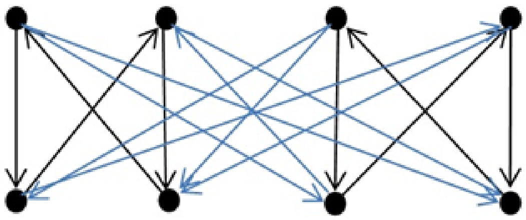

Figure 3: Complete bipartite contains two disjoint cycles, $\gamma_{DG}(G) = 2$

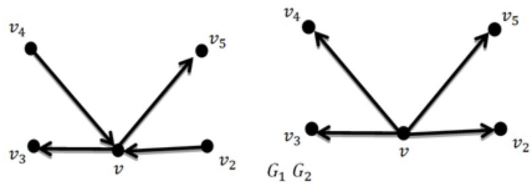

Figure 4:

III) When the partite setV_2 contains only one vertex say v. If v is the end vertex to all vertices in set V_1, then the vertex v belongs to every $DG-$dominating set. Furthermore, each vertex in the set V_2 is a $DG-$open set in the topology τ_{DG}, so the union of these vertices is still a $DG-$open set which mean there is a$DG-$open set contains all vertices in the set V_1. Let $D = \{v, v_i\}, v_i \in V_1$, it is clear that D is a $DG-$dominating set and has minimum cardinality. Thus, $\gamma_{DG}(G) = 2$

Case 3. If there are $s < r$ disjoint cycles as a subgraph in G such that each cycle has the same direction and these cycles passing through all vertices in Gthen $\gamma_{DG}(D) = s$ (as an example, see Figure 3).
Case 4. If there is a vertex in a partite that has r vertices belong to path or cycle graphs that contain more than one vertex of the other partite, then $\gamma_{DG}(G) \leq r$.
Case 5. If all edges have the same direction, then $\gamma_{DG}(G) = r$.
From all the above cases, the required is completed. □

Corollary 2.14. *If the underlying of a digraph G is a star$K_{1,n}$, and v is the center of the star, then*

$$\gamma_{DG}(G) = \begin{cases} 1, & \text{if } v \text{ is initial vertex to at least one edge incident with it,} \\ n, & \text{if } v \text{ is end vertex to all edges incident with it.} \end{cases}.$$

Proposition 2.15. *If the underlying of a digraph G is wheel $W_n \equiv K_1 + C_n$,$v \varepsilon K_1$, then*

$$\gamma_{DG}(G) = \begin{cases} 1, & \text{if } v \text{ is an initial vertex to at least one arc incident with it and the direct of} \\ & \quad \text{each arc in the cycle at the same direction,} \\ 2, & \text{if } v \text{ is an end vertex to all edges incident with it and the direct of each arc} \\ & \quad \text{in the cycle at the same direction,} \\ \lfloor \frac{n}{2} \rfloor, & \text{if } v \text{ is an initial vertex to at least one arc incident with it and the direct of} \\ & \quad \text{each arc in the cycle is more than one the direction,} \\ \lfloor \frac{n}{2} \rfloor + 1, & \text{if } v \text{ is an end vertex to all edges incident with it and the direct of each arc} \\ & \quad \text{in the cycle at the same direction.} \end{cases}$$

Proof. There are two cases that depend on the direction of arcs in the cycle as follows:
Case 1. If all arcs have the same direction, then there are two subcases as follows:
Subcase 1. If the center vertex v (the vertex of K_1) is the initial vertex to at least one arc incident with it (as an example, see Figure 5(A)), then the topology associated with this graph is the indiscrete topology and by Remark 2.3,$\gamma_{DG}(G) = 1$.

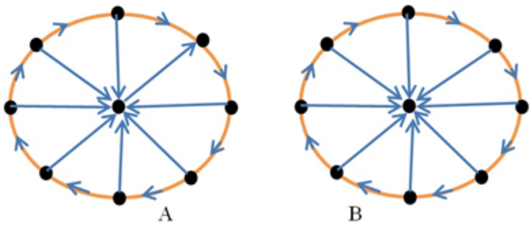

Figure 5:

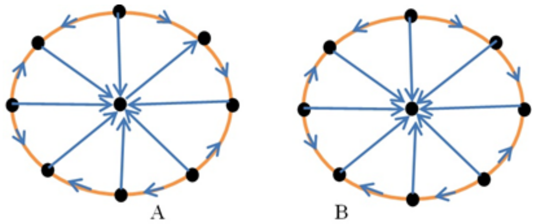

Figure 6:

Subcase 2. If the center vertex v (the vertex of K_1) is the end vertex to each arc incident with it (as an example, see Figure 5(B)), then by the same technique used in proofing Proposition 2.7, the center vertex belongs to each $DG-$dominating set. Also, since all arcs in the cycle have the same direction, then each one of them (say v_i) dominates the other by $DG-$open set that contains all vertices of the cycle. Thus, the set $\{v, v_i\}$ is the $DG-$dominating set and it is clear that it has the minimum cardinality. Therefore, $\gamma_{DG}(G) = 2$.
Case 2. If the arcs have more than one direction, then there are two subcases as follows:
Subcase 1. If the center vertex$v$ (the vertex of K_1) is the initial vertex to at least one arc incident with it (as an example, see Figure 6(A)), then the $DG-$dominating set depends on the vertices of the cycle. Since each $DG-$open set contains a vertex that is the end vertex to the arc incident with the center vertex that contains the center vertex too. Thus, according to Theorem 2.11, $\gamma_{DG}(G) = \lfloor \frac{n}{2} \rfloor$.
Subcase 2. If the center vertex$v$ (the vertex K_1) is the end vertex to each arcs incident with it (as an example, see Figure 6(B)), then by the same technique used in the proofing Proposition 2.7, the center vertex belongs to each $DG-$dominating set. Again, according to Theorem 2.11, the minimum number of vertices that $DG-$ dominates is $\lfloor \frac{n}{2} \rfloor$. Therefore, $\gamma_{DG}(G) = \lfloor \frac{n}{2} \rfloor + 1$.

From all the above cases, the required is completed.

□

3. Conclusion

Depend on the results in this paper, there are some properties about the DG-domination in DG-topological space for the digraph are been getting. Moreover, for the certain graph are mentioned above the DG-domination number is determined.

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