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Leap Zagreb indices for the Central graph of graph

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Abstract

The first, second and third leap Zagreb indices are the sum of squares of second degrees of vertices of G, the sum of products of second degrees of pairs of adjacent vertices in G and the sum of products of first and second degrees of vertices of G, respectively. In this Paper We obtained the formal of leap Zagreb Indices for the central graph of graph. Also We compute the the first, second and third leap Zagreb for the central graph of some standard graph.

Keywords: Leap Zagreb indices, Central graph of a graph.

1. Introduction

We consider only finite, connected, undirected graphs without multiple edges and loops. Let G be a graph with a vertex set $V(G)$ and an edge set $E(G)$.

Let $d(v)$ be the number of vertices adjacent to v. The distance $d(u, v)$ between any two vertices u and v of G is the number of edges in a shortest path connecting these two vertices u and v. For a positive integer k and a vertex v in G, $k-$ neighborhood of v in G is defined as $N_k(v/G) = \{u \in V(G) : d(u, v) = k\}$. The $k-$ distance degree of a vertex v in G is the number of $k-$ neighbors of v in G, and it is denoted by $d_k(v)$, see [1, 2, 3, 4].

In the interdisciplinary area where chemistry, physics and mathematics meet, molecular graph based structure descriptors, usually referred to as topological indices, are of significant importance. A topological index of a graph is a graph invariant number calculated from a graph representing a molecule. Among the most important such structure descriptors are the classical first and second Zagreb indices, which introduced

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by Gutman and Trinajestic [5], in (1972), and elaborated in [6]. They are defined as:

$$LM_1(G) = \sum_{v \in V(G)} d_2(v)^2,$$
$$LM_2(G) = \sum_{uv \in E(G)} d_2(u)d_2(v).$$

These are widely studied degree based topological indices due to their applications in chemistry. For properties of the two Zagreb indices, see [7, 8, 9] and for more details, see the survey [10] and the references cited therein. After most of the results on Zagreb indices were established, the inevitable occurred, their various modifications have been proposed, thus opening the possibility to do analogous research and publish numerous additional papers. For these modifications, see the recent survey [11].

In 2017, Naji *et al.,* [4] have introduced a new distance-degree-based topological indices conceived depending on the second degrees of vertices, and are so-called leap Zagreb indices of a graph G and are defined as:

$$LM_1(G) = \sum_{v \in V(G)} d_2(v)^2,$$
$$LM_2(G) = \sum_{uv \in E(G)} d_2(u)d_2(v),$$
$$LM_3(G) = \sum_{v \in V(G)} d(v)d_2(v).$$

The leap Zagreb indices have several chemical applications. Surprisingly, the first leap Zagreb index has very good correlation with physical properties of chemical compound, like bolling point, entropy, DHVAP, HVAP and eccentric factor [12].

Consequently, the new class of graphs, that so called leap graphs was defined and studied in [13], and was defined as: a graph G is said to be a leap graph, if and only if for every vertex $v \in V(G)$, $d(v) = d_2(v)$. For recent studying and more details on leap Zagreb indices, we refer to [12, 14, 15].

The central graph of a graph G is obtained by subdividing each edge of G exactly once and joining all the non-adjacent vertices of G, also the central graph of G is denoted by $C(G)$[16].

In this paper, we introduce the formulas of the leap Zagreb indices for Central graph of graph G, also we find the central graph of some family and standard graph.

2. Central Graph of A Graphs

In this section, we present the definition of a central graph of a graph G and we investigate some properties of degrees of vertices of central graph of a graph.

Definition 2.1. The central graph of a graph G is obtained by subdividing each edge of G exactly once and joining all the non-adjacent vertices of G, also the central graph of G is denoted by $C(G)$.

For recent results on the central graph of a graph, we refer to[16].

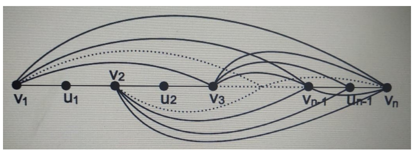

Figure 1: central graph of a P_n.

Lemma 2.2. *Let G be a connected graph of order n and size m, and let $C(G)$ be the central graph of G with vertex set $V(G)\bigcup U$, where $V(G) = \{v_1, v_2, .., v_n\}$, $U = \{u_1, u_2, ..., u_m\}$, with $i = 1, 2, ..., n$ and $j = 1, 2, ..., m$. Then*

1. $(n-1) \leq d_2(v_i/C(G)) \leq (\lceil \frac{n}{2} \rceil)^2$, *and the* $d_2(v_i/C(G)) = (\lceil \frac{n}{2} \rceil)^2$ *if and only if* $G \cong K_{r,s}$ *and* $r = s$.
2. $d_2(u_j/C(G)) \leq d_1(e_j/G) + (n-2)$, *where* $e_j \in E(G)$.
3. $d_1(v_i/C(G)) = n - 1$.
4. $d_2(u_i/C(G)) = 2$.
5. $d_2(v_i/C(G)) \leq d_1(v_i/G) + (\sum_{w \text{ not in } N(v)} d_1(w_i)$

3. Leap Zagreb indices for the Central graph of a graph

In this section, we obtain the expressions for the first, second and third leap Zagreb indices of Central of a graph G, and we also compute the exact values of these three leap zagreb indices for some standard graphs.

3.1. First Leap Zagreb indices for the Central graph of a graph

Theorem 3.1. *Let G be connected graph with n vertices and m edges. Then*

$$LM_1(C(G)) \leq n \left(\left\lceil \frac{n}{2} \right\rceil \right)^4 + F(G) + 2M_2(G) + (2n-8)M_1(G) + mn^2 - 8mn + 16m.$$

Proof. Let G be a connected graph with vertex set $V(G) = \{v_1, v_2, .., v_n\}$ and m edges, and let $C(G)$ be the central of G with vertices set $V \bigcup U$, where $U = \{u_1, u_2, ..., u_m\}$. Then

$$\begin{aligned} LM_1(C(G)) &= \sum_{v \in V(C(G))} d_2^2(v/C(G)) \\ &= \sum_{i=1}^{n} d_2^2(v_i/C(G)) + \sum_{j=1}^{m} d_2^2(u_j/C(G)). \end{aligned}$$

By Lemma 2.2, we get

$$LM_1(C(G)) \leq n \left(\left\lceil \frac{n}{2} \right\rceil \right)^4 + F(G) + 2M_2(G) + (2n-8)M_1(G) + mn^2 - 8mn + 16m.$$

□

Proposition 3.2. *Let $G \cong C(P_n)$, where $C(P_n)$ central of path P_n with $n \geq 2$ vertices. Then*

$$LM_1(C(G)) = \begin{cases} 2, & \text{if } n = 2 \text{ ;} \\ 20, & \text{if } n = 3 \text{ ;} \\ (n+2)(n-1)^2 + (n-3)n^2, & \text{if } n \geq 4. \end{cases}$$

Proposition 3.3. *Let $G \cong C(K_n)$, where $C(K_n)$ central of compleat K_n with $n \geq 3$ vertices. Then*

$$LM_1(C(G)) = \begin{cases} 24, & \text{if } n = 3 \text{ ;} \\ n^3 - 2n^2 + n + (2n-4)^2 \sum_{i=1}^{n}(n-i), & \text{if } n \geq 4. \end{cases}$$

Proposition 3.4. *Let $G \cong C(K_{r,s})$, where $C(K_{r,s})$ central of Complete bipartite graph $K_{r,s}$ with $n = s + r$ vertices. Then*

$$LM_1(C(G)) = \begin{cases} 128, & \text{if } n = 4 \text{ ;} \\ r^3(4s + s^2) + s^3(r^2 + 4r) + r^2(8s^2 - 16s) + 16rs(1-s), & \text{if } n \geq 4. \end{cases}$$

Proposition 3.5. *Let $G \cong C(C_n)$, where $C(C_n)$ central of cycle graph P_n with $n \geq 2$ vertices. Then*

$$LM_1(C(G)) = \begin{cases} 24, & \text{if } n = 2 \text{ ;} \\ 2n^3, & \text{if } n \geq 4. \end{cases}$$

Proposition 3.6. *Let $G \cong C(S_n)$, where $C(S_n)$ central star graph S_n with $n \geq 2$ vertices. Then*

$$LM_1(C(G)) = \begin{cases} 20, & \text{if } n = 3 \text{ ;} \\ 5n^3 - 22n^2 + 33n - 16, & \text{if } n \geq 4.. \end{cases}$$

3.2. **Second Leap Zagreb indices for Central graph of a graph**

Theorem 3.7. *Let G be connected graph with n vertices and m edges. Then*

$$LM_2(C(G)) \leq 2\left(\left\lceil \frac{n}{2} \right\rceil\right)^4 + 2m(n-2)\left(\left\lceil \frac{n}{2} \right\rceil\right)^2 + 2\left(\left\lceil \frac{n}{2} \right\rceil\right)^2 (M_1(G) - 2m).$$

Proof. Let G be a connected graph with vertices set $V = \{v_1, v_2, ..., v_n\}$, and m edges, and Let $C(G)$ be the central of G with vertices set $V(C(G)) = V(G) \bigcup U$, where $U = \{u_j : u_j \in V(C(G)), \text{ and } j = 1, 2, ..., s\}$ and from the definition of $C(G)$, the $E(C(G)) = \{v_i u_j : u_j \in V(G) \text{ and} i, j = 1, 2, 3, ..n i \neq j\} \bigcup \{v_i u_j : u_j \in V(C(G)) \text{ and } i = 1, 2, ...n, j = 1, 2, 3, ..s i \geq j\} \bigcup \{v_i u_j : u_j \in V(C(G)) \text{ and } i = 1, 2, ...n, j = 1, 2, 3, ..s, i < j\}$.

Hence, by Lemma 2.2, we obtain,

$$\begin{aligned} LM_2(C(G)) &= \sum_{uv \in E(C(G))} = d_2(v/C(G)) d_2(u/C(G)) \\ &= \sum_{v_i w_j \in E(C(G)) i,j=1,2,3...n, i \neq j} d_2(v_i/C(G)) d_2(w_j/C(G)) \\ &\quad + \sum_{v_i u_j \in E(C(G)) i=1,2,...n, j=1,2,3...s, i \leq j} d_2(v_i/C(G)) d_2(u_j/C(G)) \\ &\quad + \sum_{v_i u_j \in E(C(G)) i=1,2,...n, j=1,2,3...s, i < j} d_2(v_i/C(G)) d_2(u_j/C(G)). \end{aligned}$$

Therefore

$$LM_2(C(G)) \leq 2\left(\left\lceil \frac{n}{2} \right\rceil\right)^4 + 2m(n-2)\left(\left\lceil \frac{n}{2} \right\rceil\right)^2 + 2\left(\left\lceil \frac{n}{2} \right\rceil\right)^2 (M_1(G) - 2m).$$

□

Proposition 3.8. *Let $G \cong C(P_n)$, where $C(P_n)$ central of path P_n with $n \geq 2$ vertices. Then*

$$LM_2(C(G)) = \begin{cases} 20, & \text{if } n = 3 \text{ ;} \\ 2n^3 - 4n^2 - 2n + 4 + (n-1)^2 \sum_{j=2}^{n} (n-j), & \text{if } n \geq 5. \end{cases}$$

Proposition 3.9. *Let $G \cong C(K_n)$, where $C(K_n)$ central of compleat K_n with $n \geq 3$ vertices. Then*

$$LM_2(C(G)) = \begin{cases} 24, & \text{if } n = 3 \text{ ;} \\ (4n^2 - 12n + 8) \sum_{i=1}^{n} (n-i), & \text{if } n \geq 4. \end{cases}$$

Proposition 3.10. *Let $G \cong C(K_{r,s})$, where $C(K_{r,s})$ Central of Complete bipartite graph $K_{r,s}$ with $n = s + r$ vertices. Then*

$$LM_2(C(G)) = \left\{ \; 2r^2 s^2 (2r + 2s - 4) + r^2 s^2 \left(\left(\sum_{i=1}^{r} (r-i) \right) + \left(\sum_{i=1}^{s} (s-i) \right) \right), \; \text{if } n \geq 4. \right.$$

Proposition 3.11. *Let $G \cong C(C_n)$, where $C(C_n)$ Central of cycle graph P_n with $n \geq 2$ vertices. Then*

$$LM_2(C(G)) = \begin{cases} 24, & \text{if } n = 3 \ ; \\ 2n^3 + n^2 \sum_{i=2}^{n-2}, & \text{if } n \geq 4 \end{cases}$$

Proposition 3.12. *Let $G \cong C(S_n)$, where $C(S_n)$ Central star graph S_n with $n \geq 2$ vertices. Then*

$$LM_2(C(G)) = \begin{cases} 20, & \text{if } n = 3 \ ; \\ 5n^3 - 20n^2 + 25n - 10 + (n-1)^2 \sum_{i=3}^{n}(n-i), & \text{if } n \geq 4.. \end{cases}$$

3.3. Third Leap Zagreb indices for Central graph of a graph

Theorem 3.13. *Let G be connected graph with n vertices and m edges. Then*

$$LM_3(C(G)) \leq n(n-1)\left(\left\lceil \frac{n}{2} \right\rceil\right)^2 + 2M_1(G) + 2mn - 8m.$$

Proof. Let G be a connected graph with vertex set $V(G) = \{v_1, v_2, .., v_n\}$ and m edges, and let $C(G)$ be the central of G with vertices set $V \bigcup U$, where $U = \{u_1, u_2, ..., u_s\}$. Then

$$LM_3(C(G)) = \sum_{v \in V(C(G))} d_1(v/C(G) d_2(v/C(G))$$
$$= \sum_{i=1}^{n} d_1(v_i/C(G)) d_2(v_i/C(G)) + \sum_{j=1}^{s} d_1(u_j/C(G)) d_2(u_j/C(G)).$$

By Lemma 2.2,

$$ML_3(C(G)) \leq n(n-1)\left(\left\lceil \frac{n}{2} \right\rceil\right)^2 + 2M_1(G) + 2nm - 8m.$$

□

Proposition 3.14. *Let $G \cong C(P_n)$, where $C(P_n)$ Central of path P_n with $n \geq 2$ vertices. Then*

$$LM_3(C(G)) = \begin{cases} 2, & \text{if } n = 2 \ ; \\ 20, & \text{if } n = 3 \ ; \\ n^3 - n - 4, & \text{if } n \geq 4. \end{cases}$$

Proposition 3.15. *Let $G \cong C(K_n)$, where $C(K_n)$ central of compleat K_n with $n \geq 3$ vertices. Then*

$$LM_2(C(G)) = \begin{cases} 24, & \text{if } n = 3 \ ; \\ n^3 - 2n^2 + n + 2(2n-4) \sum_{i=1}^{n}(n-i), & \text{if } n \geq 4 \end{cases}$$

Proposition 3.16. *Let $G \cong C(K_{r,s})$, where $C(K_{r,s})$ central of Complete bipartite graph $K_{r,s}$ with $n = s + r$ vertices. Then*

$$LM_3(C(G)) = \begin{cases} 80, & \text{if } n = 4 \ ; \\ r^3 s + s^3 r + 2r^2 s^2 + 3r^2 s + 3s^2 r - 8rs, & \text{if } n \geq 4. \end{cases}$$

Proposition 3.17. *Let $G \cong C(C_n)$, where $C(C_n)$ Central of cycle graph P_n with $n \geq 2$ vertices. Then*

$$LM_3(C(G)) = \begin{cases} 24, & \text{if } n = 3 \ ; \\ n^3 + n^2, & \text{if } n \geq 4. \end{cases}$$

Proposition 3.18. *Let $G \cong C(S_n)$, where $C(S_n)$ Central star graph S_n with $n \geq 2$ vertices. Then*

$$LM_3(C(G)) = \begin{cases} 20, & \text{if } n = 3 \ ; \\ n^3 + 2n^2 - 11n + 8, & \text{if } n \geq 4. \end{cases}$$

4. Conclusion

We have computed the formulas of the first, second and third leap Zagreb indices for Central graph of a graph G, also we found the central graph of some family and standard graph.

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