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Reciprocal leap indices of some wheel related graphs

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Abstract

Recently, Ammar Alsinai *et al.,* [1], introduced Reciprocal leap Zagreb indices of a graph based on the inverse second degree of vertices. The first Reciprocal leap Zagreb index $RL_1(G)$ is equal to the sum of squares of the inverse second degrees of the vertices, the second Reciprocal leap Zagrab index $RL_2(G)$ is equal to the sum of the products of the inverse second degrees of pairs of adjacent vertices of G and the third Reciprocal leap Zagreb RL_3 is equal to the sum of the products of the inverse first degrees with the inverse second degrees of the vertices. In this paper, exact expression for Reciprocal leap Zagreb indices of wheel w_n, and some related graphs as gear G_n, helm H_n, flower fl_n and sunflower sf_n graphs are commuted.

Keywords: Second degree of vertex, Inverse degree, Leap Zagreb indices, Reciprocals leap indices, Wheel graphs.

1. Introduction

In last decade, graph theory has found a considerable use in the mathematical chemistry. In this area we can apply tools of graph theory to model the chemical phenomenon mathematically. This theory contributes a prominent in chemical science. A chemical structure of molecules can be represent by molecular graph, where vertices represent the atoms and edges represent the bonds between them. The graph theory based structure descriptors can be determined by considering graph vertices and edges. A simply arithmetic operators are carried out to get numerical indices. Topological indices are used in the development of Quantitative Structure Activity/Property Relations (QSAR/QSPR). A graph is a collection of points and lines connecting a subset of them. The points and lines of a graph are also called vertices and edges of the graph, respectively.

The vertex and edge sets of a graph G are denoted by $V(G)$ and $E(G)$, respectively. Let $|V(G)| = n$ and $|E(G)| = m$, if two vertices u and v of the graph G are adjacent, then the edge connecting them will

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be denoted by uv. If $u, v \in V(G)$ then the distance $d_G(u, v)$ between u and v is defined as the length of a shortest path in G connecting them. The diameter of a connected graph G is the length of any longest geodesic, denoted by $diam(G)$. In a graph G, the degree of a vertex v, denoted $d(v)$, is the number of first neighbors (the number of edges incident with v) and the second degree of v, denoted $d_2(v)$, is the number of second neighbors. The maximum and minimum degrees among the vertices of G, are denoted by $\Delta = \Delta(G)$ and $\delta = \delta(G)$, respectively. A wheel $W_{1,n}$ defined as $W_{1,n} = K_1 + C_n$. In this paper, we only conceder with a simple connected graphs. Any undefined term or notation in this paper can be found in [5, 6].

Some of the major classes of topological indices are distance based-topological indices, see [7], degree-based-topological indices, see [8, 2, 4, 3] and in [9, 10, 11] the authors introduced new topological indices based on minimal and minimum dominating sets, for more details and applications for those new indices, see [12, 13, 14].

One of the oldest and most commonly used to study the physico-chemical properties of molecules topological index are Zagreb indices introduced by Gutman and Trinajstic on based degree of vertices of G. The first and second Zagreb indices of a graph G are defined as:

$$M_1(G) = \sum_{v \in V(G)} d(v)^2,$$
$$M_2(G) = \sum_{uv \in E(G)} d(u)d(v).$$

The quantity $M_1(G)$ was first time considered in 1972 [15], whereas $M_2(G)$ in 1975 [16]. For more information on Zagreb and beyond topological indices, readers are referred to the survey [17], and the references therein.

In 2017, Naji *et al.,* [18] have introduced a new distance-degree-based topological indices conceived depending on the second degrees of vertices, and are called leap Zagreb indices of a graph G and are defined as:

$$LM_1(G) = \sum_{v \in V(G)} d_2(v)^2,$$
$$LM_2(G) = \sum_{uv \in E(G)} d_2(u)d_2(v),$$
$$LM_3(G) = \sum_{v \in V(G)} d(v)d_2(v).$$

The leap Zagreb indices have several chemical applications. Surprisingly, the first leap Zagreb index has very good correlation with physical properties of chemical compound, like bolling point, entropy, DHVAP, HVAP and eccentric factor [19]. Consequently, the new class of graphs, that so called leap graphs was defined and studied in [20], and was defined as, A graph G is said to be a leap graph, if and only if for every vertex $v \in V(G)$, $d(v) = d_2(v)$.

The inverse degree index of a graph the first was introduced in 2005 [21], and was defined by

$$ID(G) = \sum_{v \in V(G)} \frac{1}{d(v)}.$$

The inverse degree has attracted attention through a conjecture generating computer Graffiti [22]. The modified first Zagreb index ${}^mM_1(G)$ was first introduced in [23], and defined as

$${}^mM_1(G) = \sum_{v \in V(G)} \frac{1}{d(v)^2}.$$

Motivated by the inverse degree index, Alsinai *et al.,* [1], defined the inverse second degree index of a graph, as following

$$ID_2(G) = \sum_{v\in V(G)} \frac{1}{d_2(v)+1}.$$

For a connected graph G, the first, second and third reciprocal leap indices are defined in [1]:

$$RL_1 = RL_1(G) = \sum_{v\in V(G)} \frac{1}{(d_2(v)+1)^2},$$

$$RL_2 = RL_2(G) = \sum_{uv\in E(G)} \frac{1}{(d_2(u)+1)(d_2(v)+1)},$$

$$RL_3 = RL_3(G) = \sum_{v\in V(G)} \frac{1}{d(v)(d_2(v)+1)}.$$

In this paper, the explicit formulae for reciprocal leap Zagreb indices of wheel and some related graph are presented.

2. Main Result

Definition 2.1. The wheel graph W_n with $n+1$ vertices is defined to be the join of K_1 and G_n, where K_1 is the complete graph with one vertex and C_n is the cycle graph with n vertices. Clearly,$|V(W_n)| = n+1$ and $|E(W_n)| = 2n$. The vertex corresponding to K_1 s known as apex while the vertices corresponding to C_n are K_1 known as rim vertices.

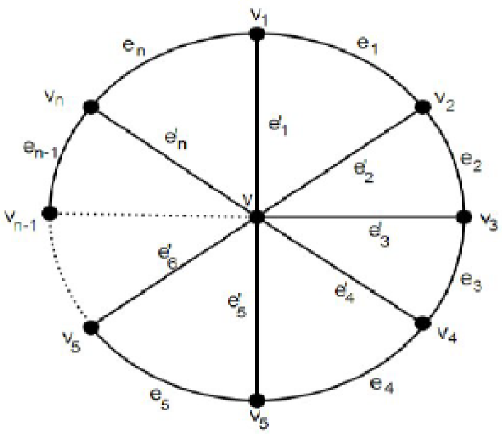

Figure 1: Wheel graph w_n

Lemma 2.2. *Let W_n be a wheel graph with $n+1$ vertices (as shown in Fig. 1). Then $d^{-1}(v_0) = \frac{1}{n}$, $d_2^{-1}(v_0) = 1$, $d^{-1}(v_i) = \frac{1}{3}$, and $d_2^{-1}(v_i) = \frac{1}{n-2}$.*

Theorem 2.3. *Let W_n , $n \geq 4$ be a wheel graph with $n+1$ vertices. Then*

1. $RL_1(W_n) = \frac{n^2-5n+9}{n^2-6n+9}$.
2. $RL_2(W_n) = \frac{2(n-1)}{n^2-4n+4}$.
3. $RL_3(W_n) = \frac{n^2+3n-6}{3n^2-6}$.

Proof. Let W_n , $n \geq 4$ be a wheel graph with $n+1$ vertices, and Let $v_0, v_1...v_n$ be a vertices set of W_n where v_0 be the apex vertex and $v_1, v_2, ...v_n$ be the rim vertices of W_n

1. By Lemma 2.2, and from definition of $RL_1(G)$, we have

$$\begin{aligned} RL_1(W_n) &= \sum_{v\in V(W_n} \frac{1}{d_2(v)+1} \\ &= \frac{1}{(d_2(v_0)+1)^2} + \sum_{i=1} \frac{1}{(d_2(v_i)+1)^2} \\ &= 1 + \frac{n}{(n-3)^2}. \end{aligned}$$

Therefore $RL_1(W_n) = \frac{n^2-5n+9}{n^2-6n+9}$.

2. From Fig. 1, we have two types of edges v_0, v_i and v_i, v_{i+1}, for indices being taken modolu n, and from definition of $RL_2(G)$, we have

$$\begin{aligned} RL_2(W_n) &= \sum_{v_0v\in V(W_n)} \frac{1}{(d_2(v_0)+1)(d_2(v)+1)} \\ &= \sum_{v_0v_i\in V(W_n)} \frac{1}{(d_2(v_0)+1)(d_2(v_i)+1)} + \sum_{v_iv_{i+1}\in V(W_n)} \frac{1}{(d_2(v_i)+1)(d_2(v_{i+1})+1)} \\ &= \frac{1}{(n-2)} + \sum_{i=1}^{n} \frac{1}{(n-2)^2} \\ &= \frac{1}{(n-2)} + \frac{n}{(n-2)^2}. \end{aligned}$$

Therefore $RL_2(W_n) = \frac{2n-2}{n^2-4n+4}$.

3. From definition of $RL_3(G)$, we have

$$\begin{aligned} RL_3(W_n) &= \sum_{v\in V(w_n)} \frac{1}{d(v)(d_2(v)+1)} \\ &= \frac{1}{d(v_0)(d_2(v)+1)} + \sum_{n}^{i=1} d(v_i) \frac{1}{(d_2(v_i+1)}. \end{aligned}$$

Therefore $RL_3(W_n) = \frac{n^2+3n-6}{3n^2-6}$.

□

Definition 2.4. The gear graph G_n , also sometimes known as a bipartite wheel graph, is a graph obtained from wheel graph W_n by added a vertex between each pair of adjacent rim vertices. It contains three type of vertices, the vertex of degree n called apex,n vertices of degree three and n vertices of degree two. The gear graph G_n has $2n+1$ vertices and $3n$ edges.

Lemma 2.5. *Let G_n be a gear graph with $2n+1$ vertices (as shown in Fig. 2). Then* $d^{-1}(v_0) = \frac{1}{n}$, $d_2^{-1}(v_0) = \frac{1}{n+1}$, $d^{-1}(v_i) = \frac{1}{3}$, $d_2^{-1}(v_i) = \frac{1}{n}$, $d^{-1}(u_i) = \frac{1}{2}$, *and* $d_2^{-1}(u_i) = \frac{1}{4}$.

Theorem 2.6. *Let G_n be a gear graph with $2n+1$ vertices. Then*

1. $RL_1(G_n) = \frac{n^2+33n+16}{6n(n+1)}$.
2. $RL_2(G_n) = \frac{(n+5)}{4(n+1)}$.
3. $RL_3(G_n) = \frac{11n^2+8n+27}{24n(n+1)}$.

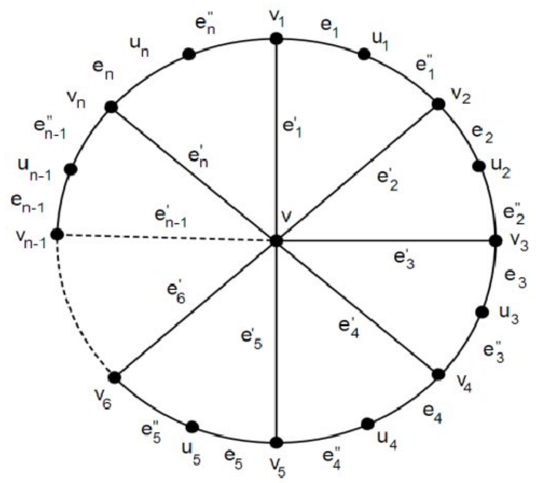

Figure 2: Gear graph G_n

Proof. Let $G_n$be a gear graph and let v_0 be the apex vertex,$v_1, v_2..., v_n$ be the vertices of G_n with three degree and $u_1, u_2..., u_n$ be the vertices of G_n with two degree

1. By Lemma 2.5, and from definition of $RL_1(G)$, we have

$$
\begin{aligned}
RL_1(G_n) &= \sum_{v\in V(G_n} \frac{1}{(d_2(v)+1)^2} \\
&= \frac{1}{(d_2(v_0)+1)^2} + \sum_{i=1} \frac{1}{(d_2(v_i)+1)^2} + \sum_{i=1} \frac{1}{(d_2(u_i)+1)^2} \\
&= \frac{1}{(n+1)} + \frac{1}{n} + \frac{n}{16}.
\end{aligned}
$$

   Therefore $RL_1(W_n) = \frac{n^2+33n+16}{6n(n+1)}$.

2. From definition of $RL_2(G)$, we have

$$
\begin{aligned}
RL_2(G_n) &= \sum_{uv\in V(G_n)} \frac{1}{(d_2(u)+1)(d_2(v)+1)} \\
&= \sum_{n}^{i=1} \frac{1}{(d_2(v_0)+1)(d_2(v_i)+1)} + \sum_{i=1}^{n} \frac{1}{(d_2(u_i)+1)(d_2(v_i)+1)} \\
&= \frac{1}{(n+1)} + \frac{1}{(4)}.
\end{aligned}
$$

   Therefore $RL_2(G_n) = \frac{n+5}{4(n+1)}$.

3. Form definition of $RL_3(G_n)$, we have

$$
\begin{aligned}
RL_3(G_n) &= \sum_{v\in V(G_n)} \frac{1}{d(v)(d_2(v)+1)} \\
&= \frac{1}{d(v_0)(d_2(v_0)+1)} + \sum_{n}^{i=1} d(v_i)\frac{1}{(d_2(v_i+1)} + \sum_{n}^{i=1} \frac{1}{d(u_i)(d_2(u_i+1)} \\
&= \frac{1}{n(n+1)} + \frac{1}{3} + \frac{n}{8}.
\end{aligned}
$$

   Therefore $RL_3(G_n) = \frac{11n^2+8n+27}{24n(n+1)}$.

□

Definition 2.7. The helm graph H_n is a graph obtained from wheel graph W_n by attaching a pendant edge to each rim vertex. The helm graph contains three types of vertices, the vertex of degree n called apex,n pendant vertices and n rim vertices of degree four. The helm graph H_n has $2n+1$ vertices and $3n$ edges.

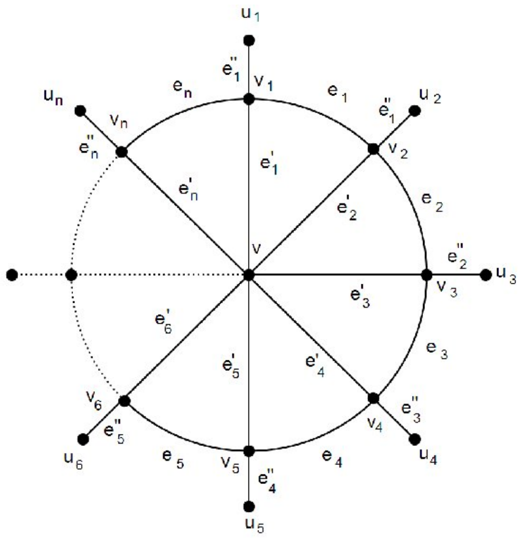

Figure 3: Helm graph H_n

Lemma 2.8. *Let H_n be a helm graph with $2n+1$ vertices (as show in Fig. 3). Then we have* $d^{-1}(v_0) = \frac{1}{n}$, $d_2^{-1}(v_0) = \frac{1}{n+1}$, $d^{-1}(v_i) = \frac{1}{4}$, $d_2^{-1}(v_i) = \frac{1}{n}$, $d^{-1}(u_i) = 1$ *and* $d_2^{-1}(u_i) = \frac{1}{4}$.

Theorem 2.9. *Let H_n be a helm graph with $2n+1$ vertices. Then*

1. $RL_1(H_n) = \frac{n^3+6n^2+13n+4}{4n(n+1)^2}$.
2. $RL_2(H_n) = \frac{(n^2+9n+4)}{4n(n+1)}$.
3. $RL_3(H_n) = \frac{n^3+2n^2+n+4}{4n(n+1)}$.

Proof. Let H_n be a helm graph with $2n+1$ vertices and let v_0 be the apex vertex, $v_1, v_2, ...v_n$ be the rim vertices with four degree and $u_1, u_2, ...u_n$ be the pendant vertices. Then

1. By Lemma 2.8, and from definition of $RL_1(G)$ we have

$$\begin{aligned} RL_1(H_n) &= \sum_{v\in V(H_n} \frac{1}{(d_2(v)+1)^2} \\ &= \frac{1}{(d_2(v_0)+1)^2} + \sum_{i=1}^{n} \frac{1}{(d_2(v_i)+1)^2} + \sum_{i=1}^{n} \frac{1}{(d_2(u_i)+1)^2} \\ &= \frac{1}{(n+1)^2} + \frac{1}{n^2} + \frac{n}{16}. \end{aligned}$$

2. From definition of $RL_2(H_n)$, we have

$$
\begin{aligned}
RL_2(H_n) &= \sum_{uv \in V(H_n)} \frac{1}{(d_2(u)+1)(d_2(v)+1)} \\
&= \sum_{v_0, v_i \in E(H_n)} \frac{1}{(d_2(v_0)+1)(d_2(v_i)+1)} + \sum_{v_i u_i} \frac{1}{(d_2(u_i)+1)(d_2(v_i)+1)} \\
&\quad + \sum_{v_i v_{i+1}} \frac{1}{(d_2(v_{i+1})+1)(d_2(v_{i+1})+1)} \\
&= \frac{1}{(n+1)} + \frac{1}{(4)} + \frac{1}{n}.
\end{aligned}
$$

3. From definition of $RL_3(H_n)$, we have

$$
\begin{aligned}
RL_3(H_n) &= \sum_{v \in V(H_n)} \frac{1}{d(v)(d_2(v)+1)} \\
&= \frac{1}{d(v_0)(d_2(v_0+1)} + \sum_{i=1}^{n} \frac{1}{d(v_i)(d_2(v_i+1)} \\
&= \frac{1}{n(n+1)} + \sum_{i=1}^{n} \frac{1}{4n} + \frac{n}{4} \\
&= \frac{1}{n(n+1)} + \frac{1}{4} + \frac{n}{4}.
\end{aligned}
$$

□

Definition 2.10. The flower graph fl_n is a graph obtained from a helm graph by joining each pendant vertex to the apex of the helm graph.There are three types of vertices, the apex of degree $2n, n$ vertices of degree four and n vertices of degree two.The flower graph fl_n has $2n+1$ vertices and $4n$ edges

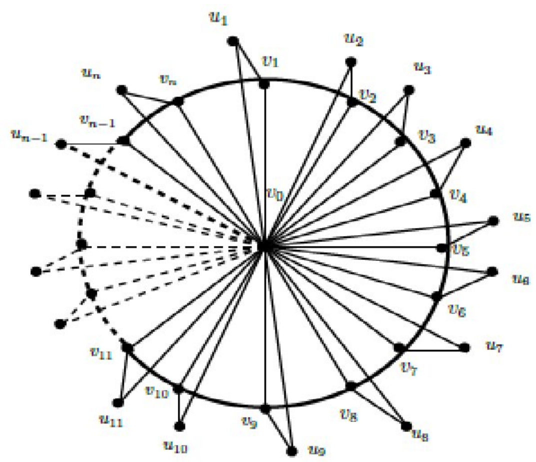

Figure 4: Flower graph fl_n

Lemma 2.11. *Let fl_n be a flower graph with $2n+1$ vertices (as shown in Fig. 4). Then* $d^{-1}(v_0) = \frac{1}{2n}$, $d_2^{-1}(v_0) = 1$, $d^{-1}(v_i) = \frac{1}{4}$, $d_2^{-1}(v_i) = \frac{1}{(n-4)}$, $d^{-1}(u_i) = 1$ *and* $d_2^{-1}(u_i) = \frac{1}{(n-1)}$.

Theorem 2.12. *Let fl_n be a flower graph with $2n+1$ vertices. Then*

1. $RL_1(fl_n) = \frac{1}{4n^2} + \frac{n}{(n-4)^2} + \frac{n}{(n-1)^2}$
2. $RL_2(fl_n) = \frac{n^3-5n^2-10n}{(n-1)(n-4)^2}$
3. $RL_3(fl_n) = \frac{3n^3-7n^2-10n+8}{4n^3-20n^2-16n}$

Proof. Let fl_n be a flower graph with $2n+1$ vertices,$4n$ edges and let$v_0$be the apex vertex,$v_1, v_2, ..., v_n$ be the rim vertices with four degrees and $u_1, u_2, u_3, ..., u_n$ be the extreme. Then

1. By Lemma 2.11, and from definition of $RL_1(G)$, we have

$$RL_1(fl_n) = \sum_{v\in V(fl_n} \frac{1}{(d_2(v)+1)^2}$$
$$= \frac{1}{(d_2(v_0)+1)^2} + \sum_{i=1} \frac{1}{(d_2(v_i)+1)^2} + \sum_{i=1} \frac{1}{(d_2(u_i)+1)^2}$$
$$= \frac{1}{(4n^2)} + \frac{n}{(n-4)^2} + \frac{n}{(n-1)^2}.$$

2. From Fig. 4, we have four type of edge v_0, v_i, v_0u_i, v_iu_i and v_iv_{i+1} for $i = 1, 2, ..n$ and definition of $RL_2(G)$, we have

$$RL_2(fl_n) = \sum_{uv\in V(fl_n)} \frac{1}{(d_2(u)+1)(d_2(v)+1)}$$
$$= \sum_{n}^{i=1} \frac{1}{(d_2(v_0)+1)(d_2(v_i)+1)}$$
$$+ \sum_{n}^{i=1} \frac{1}{(d_2v_0+1)(d_2(u_i+1)} + \sum_{i=1}^{n} \frac{1}{(d_2(u_i)+1)(d_2(v_i)+1)}$$
$$= \sum_{n}^{i=1} \frac{1}{(v_i+1)(d_2(v_{i+1}))}. = \frac{n}{(n-4)} + \frac{n}{n-1)} + \frac{n}{(n-4)(n-1)} + \frac{n}{(n-4)^2}.$$

Therefore $RL_2(fl_n) = \frac{n^3-5n^2-10n+8}{(n-1)(n-4)^4}$.

3. From definition of $RL_3(fl_n)$, we have

$$RL_3(fl_n) = \sum_{v\in V(fl_n)} \frac{1}{d(v)(d_2(v)+1)}$$
$$= \frac{1}{d(v_0)(d_2(v_0)+1)} + \sum_{n}^{i=1} d(v_i)\frac{1}{(d_2(v_i+1)} + \sum_{n}^{i=1} \frac{1}{d(u_i)(d_2(u_i+1)}$$
$$= \frac{1}{2n)} + \frac{n}{4(n-4)} + \frac{n}{2(n-1)}.$$

Therefore $RL_3(fl_n) = \frac{3n^3-7n^2-10n+8}{4n^3-20n^2-16n}$.

□

Definition 2.13. The sunflower graph $Sf_n$is the graph obtained from the flower graph fl_n by attaching n pendant edges to the apex vertex. Sfl_n has four types of vertices, the apex of $3n, n$ vertices of degree fourn vertices of degree two and n pendant vertices.

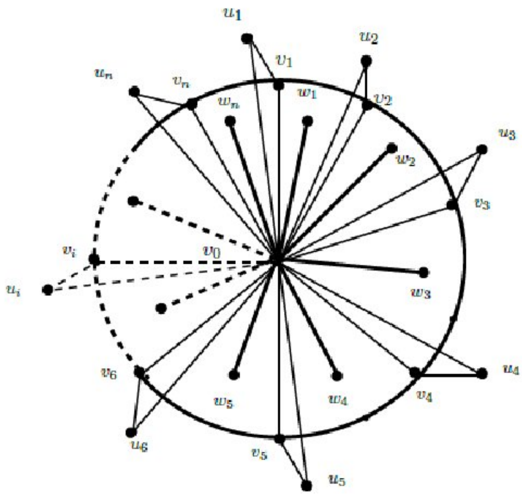

Figure 5: sunflower graph fl_n

Lemma 2.14. *Let Sfl_n be a sunflower graph with $3n+1$ vertices (as shown in Fig. 5). Then* $d^{-1}(v_0) = \frac{1}{3n}$, $d_2^{-1}(v_0) = 1$, $d^{-1}(v_i) = \frac{1}{4}$, $d_2^{-1}(v_i) = \frac{1}{3(n-1)}$, $d^{-1}(u_i) = \frac{1}{2}$, $d_2^{-1}(u_i) = \frac{1}{(3n-1)}$, $d^{-1}w_i) = 1$ *and* $d_2^{-1}(w_i) = \frac{1}{(3n)}$.

Theorem 2.15. *Let Sf_n be a sunflower graph with $3n+1$ vertices. Then*

1. $RL_1(sf_n) = n + \frac{n}{9(n-1)^2} + \frac{n}{(3n-1)^2} + \frac{1}{9n}$
2. $RL_2(sf_n) = \frac{n^3-5n^2-10n+8}{(n-1)(n-4)^4}$
3. $RL_3(sf_n) = \frac{3n^3-7n^2-10n+8}{4n^3-20n^2-16n}$

Proof. Let sf_n be a sunflower graph with $3n+1$ vertices,$5n$ edges and letv_0 be the apex vertex,$v_0, v_1, v_3, ..., v_n$be the rim vertices with four degrees and $u_1, u_2, u_3, ..., u_n$ be the extreme vertices with two degrees. since $d(v_0 = 3n$. Then

1. By Lemma 2.14, and from definition of $RL_1(G)$, we have

$$\begin{aligned} RL_1(sf_n) &= \sum_{v \in V(sf_n} \frac{1}{(d_2(v)+1)^2} \\ &= \sum_{i=1}^{n} \frac{1}{(d_2(v_0)+1)^2} + \sum_{i=1}^{n} \frac{1}{(d_2(v_i+1)^2} \\ &= \sum_{i=1}^{n} \frac{1}{(d_2(v_i+1)^2} + \sum_{i=1}^{n} \frac{1}{(d_2(w_i+1)^2} \\ &= n + \frac{n}{9(n-1)^2} + \frac{n}{(3n-1)^2} + \frac{1}{9n}. \end{aligned}$$

2. From definition of $RL_2(G)$, we have

$$\begin{aligned}
RL_2(sf_n) &= \sum_{uv\in V(sf_n)} \frac{1}{(d_2(u)+1)(d_2(v)+1)} \\
&= \sum_{n}^{i=1} \frac{1}{(d_2(v_0)+1)(d_2(v_i)+1)} + \sum_{n}^{i=1} \frac{1}{(d_2 v_0+1)(d_2(u_i+1)} + \sum_{i=1}^{n} \frac{1}{(d_2(u_i)+1)(d_2(v_i)+1)} \\
&= \sum_{n}^{i=1} \frac{1}{(v_i+1)(d_2(v_{i+1}))} \\
&= \frac{n}{(n-4)} + \frac{n}{n-1)} + \frac{n}{(n-4)(n-1)} + \frac{n}{(n-4)^2}.
\end{aligned}$$

Therefore $RL_2(fl_n) = \frac{n^3-5n^2-10n+8}{(n-1)(n-4)^4}$.

3. From definition of $RL_3(fl_n)$, we have

$$\begin{aligned}
RL_3(fl_n) &= \sum_{v\in V(fl_n)} \frac{1}{d(v)(d_2(v)+1)} \\
&= \frac{1}{d(v_0)(d_2(v_0)+1)} + \sum_{n}^{i=1} d(v_i) \frac{1}{(d_2(v_i+1)} + \sum_{n}^{i=1} \frac{1}{d(u_i)(d_2(u_i+1)} \\
&= \frac{1}{(2n)} + \frac{n}{4(n-4)} + \frac{n}{2(n-1)}.
\end{aligned}$$

Therefore $RL_3(fl_n) = \frac{3n^3-7n^2-10n+8}{4n^3-20n^2-16n}$.

□

References

[1] Ammar Alsinai, Anwar Alwardi, N.D. Soner.(2021). On Reciprocals Leap Indices of Graphs. *Int. J. Anal. Appl., 19*(1), 1-19.

[2] Ammar Alsinai, Anwar Alwardi, Mohammad Reza Farahani, Soner Nandappa D.(2021). On ψ_k polynomial of graph. *Eurasian Chemical Communcations, 3*(4), 219-226.

[3] Ammar Alsinai, Anwar Alwardi, N.D. Soner. (2021). On reciprocals leap function of graph. *Journal of Discrete Mathematical Sciences and Cryptography, 24*(2), 307-324.

[4] Ammar Alsinai, Anwar Alwardi, N.D. Soner. (2021). Topological properties of graphene using ψ_k polynomial. *Proceedings of the Jangjeon Mathematical Society, 24*(3), 375 - 388.

[5] Bondy, J. A., & Murty, U. S. R. (1976). *Graph theory with applications* (Vol. 290). London: Macmillan.

[6] Harary, F. (1969). *Graph Theory*. Addison Wesley, Reading Mass.

[7] T. Al-Fozan, T., Manuel, P., Rajasingh, I., & Rajan, R. S. (2014). Computing Szeged index of certain nanosheets using partition technique. *MATCH Commun. Math. Comput. Chem, 72*(1), 339-353.

[8] Dimitrov, D. (2016). On structural properties of trees with minimal atom-bond connectivity index II: Bounds on B 1 - and B 2 -branches. *Discrete Appl. Math. 204*, 90–116.

[9] Ahmed, A. H., Alwardi, A., & Salestina, M. R. (2020). On domination topological indices of graphs. *International Journal of Analysis and Applications, 19*(1), 47-64.

[10] Ahmed, H., Salestina, M. R., Alwardi, A., & Soner, N. D. (2021). Forgotten domination, hyper domination and modified forgotten domination indices of graphs. *Journal of Discrete Mathematical Sciences and Cryptography, 24*(2), 353-368.

[11] Ahmed, H., Alwardi, A., Salestina, M. R., & Nandappa, D. S. (2021). Domination, gamma-Domination Topological Indices and phi (P)-Polynomial of Some Chemical Structures Applied for the Treatment of COVID-19 Patients. *Biointerface Research in Applied Chemistry*, 13290-13302.

[12] Ahmed, H., Salestina, M. R., & Alwardi, A. (2021). Domination topological indices and their polynomials of a firefly graph. *Journal of Discrete Mathematical Sciences and Cryptography, 24*(2), 325-341.

[13] Ahmed, H., Farahani, M. R., Alwardi, A., & Salestina, M. R. (2021). Domination topological properties of some chemical structures using φ_P -Polynomial approach. *Eurasian Chemical Communications, 3*(4), 210-218.

[14] Javaraju, S., Ahmed, H., Alsinai, A., & Soner, N.D. (2021). Domination topological properties of carbidopa-levodopa used for treatment Parkinson's disease by using φ_P-polynomial. *Eurasian Chemical Communications, 3*(9), 614-621.

[15] Gutman, I., & Trinajstic, N. (1972). Graph theory and molecular orbitals. Total ϕ-electron energy of alternant hydrocarbons. *Chemical Physics Letters, 17*(4), 535-538.

[16] Gutman, I., Rucsc, B., Trinajstic, N., & Wilcox Jr, C. F. (1975). Graph theory and molecular orbitals. XII. Acyclic polyenes. *The Journal of Chemical Physics, 62*(9), 3399-3405.

[17] Gutman, I., Milovanovic, E., & Milovanovic, I. (2018). Beyond the Zagreb indices. *AKCE International Journal of Graphs and Combinatorics.*

[18] Naji, A. M., Soner, N. D., & Gutman, I. (2017). On leap Zagreb indices of graphs. *Commun. Comb. Optim, 2*(2), 99-117.

[19] Basavanagoud, B., & Jakkannavar, P. (2018). Computing first leap Zagreb index of some nano structures. *Int. J. Math. And Appl., 6*(2-B), 141-150.

[20] Naji, A. M., Davvaz, B., Mahde, S. S., & Soner, N. D. (2020). A study on some properties of leap graphs. *Communications in Combinatorics and Optimization, 5*(1), 9-17.

[21] Zhang, Z., Zhang, J., & Lu, X. (2005). The relation of matching with inverse degree of a graph. *Discrete mathematics, 301*(2-3), 243-246.

[22] Fajtlowicz, S. (1987). On conjectures of Graffiti-II. *Congr. Numer, 60*, 187-197.

[23] Milicevic, A., & Nikolic, S. (2004). On variable Zagreb indices. *Croatica Chemica Acta, 77*(1-2), 97-101.