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Journal of Prime Research in Mathematics, 18(1) (2022), 60-74

Journal of Prime Research in Mathematics

COMPLEX BIPOLAR FUZZY SOFT EXPERT SETS

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Abstract

The idea of representing information with its periodic nature has been extensively studied and applied in many fields. Many researchers have developed several tools to transfer uncertainty information that has the same data but with different meanings that happening in different phases/times. The novelty of combining complex numbers and uncertainty information appears in its ability to represent two values uncertainty and periodicity semantics in one mathematical tool. In this paper, we generalize existence concept of bipolar fuzzy soft expert sets (BFSES) from real number to complex numbers to be complex bipolar fuzzy soft expert sets (CBFSES). This generalization allows us to convey data that carry benefits, features, and specifications of BFSES in different phases or carrying periodic nature of the BFSE information to mathematical formula and vice versa without losing full meaning of information. The range of value becomes to be in unit disk in a complex plane for both positive and negative membership functions of BFSES. The main benefit of CBFSES that amplitude and phase terms can convey bipolar fuzzy information. Moreover, formal definition of CBFSES and illustration examples are introduced. Also, we define basic operations and their properties on CBFSES. Finally, OR and AND operations are generalized to the form of CBFSES.

Keywords: Bipolar fuzzy set, Bipolar fuzzy soft expert set, complex bipolar fuzzy soft expert set, fuzzy set, complex fuzzy set.
2010 MSC: 03E72, 94D05.

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1. Introduction

Molodtsov [44] proposed the concept of soft set to deal with uncertainties. Recognizing soft set theory as a powerful tool to describe uncertainties pushed many researchers to combine this tool to other uncertainty sets, to name but a few: Maji et al. [40, 41] introduced the notion of fuzzy soft set and, and intuitionistic fuzzy soft sets [41], Alhazaymeh et al. [24] introduced soft intuitionistic fuzzy sets, Vague soft sets were studied by Alhazaymeh and Hassan [13, 14, 15], also interval-valued vague soft set, generalized vague soft

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Received : 13 December 2019*; Accepted:* 28 April 2020*; Published Online:* 30 June 2022.

expert set, and vague soft multiset theory were introduced by Alhazaymeh and Hassan [16, 18, 17, 19, 21, 22] and others like in ref [51, 34, 35, 27, 63]. Al-Quran and Hassan [31, 32] then extended the work on vague soft expert set Hassan and Alhazaymeh [36] to neutrosophic versions. Fuzzy soft sets were also extended to Q-fuzzy soft sets Adam and Hassan [4, 5, 6, 7, 8, 9, 10, 11]. Alkhazaleh and Salleh [25] introduced the notion of a soft expert set followed by the notion of fuzzy soft expert sets [26] by combining fuzzy set with soft expert set. Also, Salleh et al. [47] introduced the definition of a multiparameterized soft set as a generalization of Molodtsovs soft set and also its basic operations like complement, union and intersection. Fuzzy sets are considered as a useful mathematical tool to represent objects whose boundary is ambiguous. In addition, fuzzy set has recognized as a very useful tool that has applications in several fields (see ref) [33, 66, 43, 61, 62]. Lee [38] introduced the concept of bipolar valued fuzzy sets which is a generalization of fuzzy sets with a membership degree range lies in [-1, 1]. Lee [39] concluded that bipolar-valued fuzzy sets can carry the satisfaction degree to counter-property, contrast to the interval-valued fuzzy set and intuitionistic fuzzy set. Several forms of bipolar fuzzy sets were then studied (see ref) [12, 42, 37, 67, 60]. A combination of bipolar fuzzy set and soft set were studied widely by many researchers like in ref [54, 45], whose introduced the notion and algebraic structure of bipolar soft sets and its basic operations, Also Abdullah et al. [3] introduced the notion of bipolar fuzzy soft set and its properties. In 2017, Al-Qudah and Hassan [28] proposed an innovative notion of the bipolar fuzzy soft expert set (BFSES) which is a combination of bipolar fuzzy sets and soft expert sets. They defined its basic operations, namely complement, union, intersection, AND, and OR, and study its properties. Also, they developed an algorithm based on BFSES and finally they illustrated an application of BFSES to a decision-making problem.

On other hand, the idea of generalizing several uncertainty set from real-valued interval [0, 1] to the realm of complex numbers (unite disk) was first introduced by Ramot et. al. [50, 49] by introducing the concept of complex fuzzy set and logic. Then many researchers discovered the power of this tool in representing both uncertainty and periodicity semantics to real life applications. To name but a few of researches who used this idea are Nguyen et. al. [46], Tamir et. al. [59, 57, 58], Alkouri & Salleh. [1, 2], Yazdanbakhsh and Dick [64], Selvachandran et al. [52], Al-Qudah and Hassan [28, 29], Singh [48]etc. The novelty of CFS acted in the additional dimension membership. The phase term of grade of membership. That is, without the phase term the CFS reduces to fuzzy set [65]. The main benefit of generalizing the fuzzy set to a complex fuzzy set is concluded in its ability to convey uncertainty and periodicity information simultaneously. Also, In ref [50] they introduced the basic theoretic operations: complements, union, intersection under the CFS. Complex fuzzy soft expert sets were introduced by Selvachandran et al. [53]. This concept combines the benefits of complex fuzzy sets and soft sets, besides having the added advantage of allowing the users to know the opinion of all the experts in only one mathematical tool without the need for any extra operations. As such, this tool successfully develops the accuracy of expressing of problem parameters that are periodic in nature, besides having a higher level of computational adeptness compared to similar models in the literature. Recently, Singh [48] suggested the bipolar complex fuzzy concept lattice with its application. He studied the problem that calculating the periodic variation in bipolar information at the given phase of time. He introduced three methods and illustrative examples for the suitable representation of bipolar complex data set. Singh represented the phase term in the complex fuzzy set as a real-valued component.

In this research, we take the benefit and properties of combining both concepts of bipolar fuzzy soft expert set and complex fuzzy set to obtain a new concept called complex bipolar fuzzy soft expert set (CBFSES). Also, the basic theoretical operation namely complement, union and intersection, and OR and AND operators are obtained with its properties. The complex bipolar fuzzy soft expert membership grade can be characterized by two bipolar fuzzy soft components with expert opinion value in the complex form $r(x).e^{i\alpha\theta(x)}$. In other words, each amplitude and phase terms have a membership degree with values lies in the interval [-1, 0] for negative pole and [0, 1] for positive pole.

2. Preliminaries

This section provides a brief outline of the relevant literature. We recall the core operative definitions, theorems, and some results of complex fuzzy set and bipolar fuzzy set. Also, the most related background and importance to this paper are briefly summarized.

Definition 2.1. [65] A fuzzy set A in a universe of discourse X is characterized by a membership function that takes values in the interval

Zadeh [65] has defined the following basic operators: let $A = \{(x, \mu_A(x)) : x \in X\}$, $B = \{(x, \mu_B(x)) : x \in X\}$. Then:

1. The complement of A is given as1) The complement of A is given as

$$A^c = \{(x, 1 - \mu_A(x)) : x \in X\}$$

2. 2) The union between two sets is given as

$$A \cup B = \{(x, \mu_A \diamond \mu_B) : x \in X\}$$

3. 3) The intersection between two sets is given as

$$A \cap B = \{(x, \mu_A * \mu_B) : x \in X\}$$

Here, $\diamond$ and * denote the s-norm and t-norm operators, respectively

Let be a set of all complex fuzzy sets in X. Then we have

Definition 2.2. [50] A complex fuzzy set A, defined on a universe of discourse X, is characterized by a membership function $\mu_A(x)$ that assigns to any element $x \in X$ a complex-valued grade of membership in A. By definition, the values of $\mu_A(x)$, may receive all lying within the unit circle in the complex plane, and are thus of the form $\mu_A(x) = r_A e^{i\omega_A(x)}$, where $i = \sqrt{-1}$ and $r_A(x)$ and $\omega_A(x)$ are both real-valued, and $r_A(x) \in [0, 1]$. The CFS A may be represented as the set of ordered pairs $A = \{(x, \mu_A(x)) : x \in X\}$

Definition 2.3. [50]. A complex fuzzy complement of A may be represented as follows:

$$\overline{A} = \{(x, \mu_{\overline{A}}(x)) : x \in X\} = \{(x, r_{\overline{A}}(x)e^{i\omega_{\overline{A}}(x)}) : x \in X\}$$

where $r_{\overline{A}}(x) = 1 - r_A(x)$ and $\omega_{\overline{A}}(x) = \omega_A(x) = 2\pi - \omega_A(x) = \pi + \omega_A(x)$.

Definition 2.4. [50] Let A and B be two complex fuzzy sets on X, $\mu_A(x) = r_A e^{i\ arg_A(x)}$ and $\mu_B(x) = r_B e^{i\ arg_B(x)}$ their membership functions, respectively. We say that A is greater than B, denoted by $A \supseteq B$ or $B \subseteq A$ if for any $x \in X$, $r_A(x) \leq r_B(x)$ and $arg_A(x) \leq arg_B(x)$.

Definition 2.5. [38]. Let X be the universe of discourse. The canonical representation of bipolar valued-fuzzy sets A in X has the following shape:

$$A = \{(x, \mu_A^P(x), \mu_A^N(x))\}$$

where $\mu_A^P : X \to [0, 1]$ and $\mu_A^N : X \to [-1, 0]$. The positive membership function $\mu_A^P(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued-fuzzy set A, and The negative membership function μ_A^N denotes the satisfaction degree of an element x to some implicit counter property to A.

Suppose A and B are two bipolar valued-fuzzy sets expressed as $A = \{(x, \mu_A^P(x), \mu_A^N(x))\}$ and $B = \{(x, \mu_B^P(x), \mu_B^N(x))\}$, the set operations of bipolar valued-fuzzy set are defined as follows:

1. $A \cup B = \{(x, \mu^P_{A\cup B}(x), \mu^N_{A\cup B}(x))\}$
   $\mu^P_{A\cup B}(x) = \max\{\mu^P_A(x), \mu^P_B(x)\}$
   $\mu^N_{A\cup B}(x) = \min\{\mu^P_A(x), \mu^P_B(x)\}$

2. $A \cap B = \{(x, \mu^P_{A\cap B}(x), \mu^N_{A\cap B}(x))\}$
   $\mu^P_{A\cap B}(x) = \min\{\mu^P_A(x), \mu^P_B(x)\}$
   $\mu^N_{A\cap B}(x) = \max\{\mu^P_A(x), \mu^P_B(x)\}$

3. $A^c = \{(x, \mu^P_{A^c}(x), \mu^N_{A^c}(x))\}$
   $\mu^P_{A^c}(x) = 1 - \mu^P_A(x)$
   $\mu^N_{A^c}(x) = 1 - \mu^N_A(x)$

Definition 2.6. [44] Let U be an initial universe, E be the set of parameters, $A \subseteq B$ and $P(U)$ be the power set of U. Then (F, A) is called a soft set, where

$$\hat{F} : A \to P(U).$$

Definition 2.7. [3] Let U be a universe, E a set of parameters and $A \subset B$. Define $F : A \to BF^U$, where BF^U is the collection of all bipolar fuzzy subsets of U. Then (F, A) is said to be the bipolar fuzzy soft set over the universe U. It is defined by

$$(F, A) = F(e_i) = \{c_i, \mu^+(c_i), \mu^-(c_i) : \forall c_i \in U,\ \forall e_i \in A\}.$$

Let U be a universe, E be the set of parameters, X the the set of experts (agents), and O a set of opinions. Let $Z = E \times X \times O$ and $A \subset Z$.

Definition 2.8. [25] A pair (F, A) is called soft expert set over U, where F is mapping given by:

$$F : A \to P(U)$$

Definition 2.9. [25] An agree-soft set $(F, A)_1$ over U is a soft expert subset of (F, A)defined as follows:

$$(F, A)_1 = \{F_1(\alpha) : \alpha \in E \times X \times \{1\}\}.$$

Definition 2.10. [25] A disagree-soft set $(F, A)_0$ over U is a soft expert subset of (F, A)defined as follows:

$$(F, A)_0 = \{F_0(\alpha) : \alpha \in E \times X \times \{0\}\}.$$

We will now give the definitions of subset and equality of two bipolar valued fuzzy sets.

Definition 2.11. Let A and B be two bipolar valued fuzzy sets over a universe U. Then,

1. $A \subset B$ if and only if $\mu^+_A(x) \leq \mu^+_B(x)$ and $\mu^-_A(x) \geq \mu^-_B(x)$ for all $x \in U$
2. $A = B$ if and only if $\mu^+_A(x) = \mu^+_B(x)$ and $\mu^-_A(x) = \mu^-_B(x)$ for all $x \in U$

Definition 2.12. Let $U = \{u_1, u_2, \cdots, u_n\}$ be a universe of elements, $E = \{e_1, e_2, \cdots, e_m\}$ be a universe of parameters, $X = \{x_1, x_2, \cdots, x_i\}$ be a set of experts (agents) and $O = \{1 = \ agree, 0 = \ disagree\}$ be set of opinions. Let $Z = \{E \times X \times O\}$ and $A \subset Z$. Let $F : A \to BF^U$ where BF^U is the collection of all bipolar fuzzy subsets of U. Suppose $F : A \to BF^U$ is a function defined by

$$F(a) = \hat{F}(a)(u_i),\ \forall u \in U.$$

Then, $F(a)$ is called a bipolar soft set expert (denoted by BFSES for simplicity) over the soft universe (U, A). For each $a_i \in A$ $F(a_i) = \hat{F}(a_i)(u_i)$ where $F(a_i)$ represents either satisfaction degree to property corresponding to a bipolar fuzzy soft expert set $F(a)$ or satisfaction degree to implicit counter- property corresponding to a bipolar fuzzy soft expert set $F(a)$ for element of U in $F(a_i)$.

$$F(a_i) = \left\{ \frac{u_i}{\hat{F}(a_i)(u_i)} \right\}$$

where $\hat{F}(a_i)(u_i) =< \mu^+_{F(a_i)}(u_i), \gamma^-_{F(a_i)}(u_i) >$ with $\mu^+_{F(a_i)}(u_i)$ denotes the satisfaction degree of each element $u_i \in U$ to the property corresponding to a bipolar fuzzy soft expert set $F(a)$. We can write F as (F, Z). If $A \subset Z$ we can also have a BFSES (F, A)

3. Complex bipolar fuzzy soft expert sets

In this section, we introduce the concept of complex bipolar fuzzy soft expert sets along with an illustrative example. We also define the subset and equality operations of this concept. We begin by proposing the definition of complex bipolar fuzzy soft expert sets.

Let U be a universe of elements, E a set of parameters, X a set of experts and $O = \{1 = \ agree, 0 = disagree\}$. Let $Z = E \times X \times O$ and $A \subset Z$.

Definition 3.1. Let $U = \{u_1, u_2, \cdots, u_n\}$ be a universe of elements, $E = \{e_1, e_2, \cdots, e_m\}$ be a universe of parameters, $X = \{x_1, x_2, \cdots, x_i\}$ be a set of experts (agents) and $O = \{1 = \ agree, 0 = \ disagree\}$ be set of opinions. Let $Z = \{E \times X \times O\}$ and $A \subset Z$. Let $CF : A \to CBF^U$ where CBF^U is the collection of all complex bipolar fuzzy subsets of U. Suppose $CF : A \to CBF^U$ is a function defined by

$$F(a) = \tilde{C}F(a)(u),\ \forall u \in U.$$

Then, $CF(a)$ is called a complex bipolar fuzzy soft expert set (denoted by CBFSES for simplicity) over the soft universe (U, A). For each $a_i \in A\, CF(a_i) = \tilde{C}F(a_i)(u) = (\mu^+_{CF(a_i)}(u) = r^+_{CF(a_i)}(u)e^{i\alpha(\omega^+_{CF(a_i)}(u))}, \mu^-_{CF(a_i)}(u) = r^-_{CF(a_i)}(u)e^{i\alpha(\omega^-_{CF(a_i)}(u))})$

where $CF(a_i)$ represents both periodicity and (uncertain either the satisfaction degree to property corresponding or satisfaction degree to implicit counter- property corresponding to a complex bipolar fuzzy soft expert set $CF(a)$ for elements of U in $CF(a_i)$.

$$CF(a_i) = \left\{ \frac{u_i}{r^+_{CF(a_i)}(u)e^{i\alpha(\omega^+_{CF(a_i)}(u))}, r^-_{CF(a_i)}(u)e^{i\alpha(\omega^-_{CF(a_i)}(u))}} \right\},\ for\ i = 1, 2, 3, \cdots.$$

where $\tilde{C}F(a_i)(u_i) =< \mu^+_{CF(a_i)}(u_i), \gamma^-_{CF(a_i)}(u_i) >$ with $\mu^+_{CF(a_i)}(u_i)$ denotes the satisfaction degree of each element $u_i \in U$ to the property corresponding to a complex bipolar fuzzy soft expert set $CF(a)$ and $\mu^-_{CF(a_i)}(u_i)$ denotes the satisfaction degree of each element $u_i \in U$ to some implicit counter-property corresponding to a complex bipolar fuzzy soft expert set $CF(a)$.

We can write CF as (CF, Z). If $A \subset Z$ we can also have a CBFSES (F, A).

In order to better understand the above definition, consider the following illustrative example:

Example 3.2. Suppose that a company wishes to produce three types of cars and wants to take the opinion of some experts about these cars. Let $U = \{u_1, u_2, u_3\}$ be a set of products, $E = \{e_1 = durability,\ e_2 = elegant,\ e_3 = full\ efficient\}$ be a set of decision parameters and $X = \{x_1, x_2\}$ be a set of experts. Suppose that

$CF(e_1, x_1, 1) =$
$\left\{ \frac{u_1}{<0.3e^{i2\pi(0,1)}, -0.4e^{i2\pi(-0,2)}>}, \frac{u_2}{<0.3e^{i2\pi(0,1)}, -0.4e^{i2\pi(-0,2)}>}, \frac{u_3}{<0.3e^{i2\pi(0,1)}, -0.4e^{i2\pi(-0,2)}>} \right\}$

$CF(e_1, x_2, 1) =$
$$\left\{\frac{u_1}{<0.1e^{i2\pi(0,2)},-0.4e^{i2\pi(-0,1)}>}, \frac{u_2}{<0.6e^{i2\pi(0,2)},-0.1e^{i2\pi(-0,3)}>}, \frac{u_3}{<0.5e^{i2\pi(0,1)},-0.1e^{i2\pi(-0,3)}>}\right\}$$
$CF(e_2, x_1, 1) =$
$$\left\{\frac{u_1}{<0.8e^{i2\pi(0,3)},-0.6e^{i2\pi(-0,2)}>}, \frac{u_2}{<0.5e^{i2\pi(0,4)},-0.5e^{i2\pi(-0,3)}>}, \frac{u_3}{<0.2e^{i2\pi(0,1)},-0.1e^{i2\pi(-0,4)}>}\right\}$$
$CF(e_1, x_2, 1) =$
$$\left\{\frac{u_1}{<0.6e^{i2\pi(0,1)},-0.5e^{i2\pi(-0,4)}>}, \frac{u_2}{<0.8e^{i2\pi(0,4)},-0.6e^{i2\pi(-0,3)}>}, \frac{u_3}{<0.1e^{i2\pi(0,5)},-0.8e^{i2\pi(-0,2)}>}\right\}$$
$CF(e_1, x_1, 0) =$
$$\left\{\frac{u_1}{<0.2e^{i2\pi(0,2)},-0.3e^{i2\pi(-0,1)}>}, \frac{u_2}{<0.5e^{i2\pi(0,1)},-0.3e^{i2\pi(-0,1}>}, \frac{u_3}{<0.2e^{i2\pi(0,2)},-0.2e^{i2\pi(-0,3)}>}\right\}$$
$CF(e_1, x_2, 0) =$
$$\left\{\frac{u_1}{<0.2e^{i2\pi(0,1)},-0.2e^{i2\pi(-0,3)}>}, \frac{u_2}{<0.5e^{i2\pi(0,2)},-0.1e^{i2\pi(-0,2)}>}, \frac{u_3}{<0.6e^{i2\pi(0,3)},-0.5e^{i2\pi(-0,3)}>}\right\}$$
$CF(e_2, x_1, 0) =$
$$\left\{\frac{u_1}{<0.2e^{i2\pi(0,2)},-0.4e^{i2\pi(-0,1)}>}, \frac{u_2}{<0.4e^{i2\pi(0,3)},-0.1e^{i2\pi(-0,2)}>}, \frac{u_3}{<0.1e^{i2\pi(0,3)},-0.2e^{i2\pi(-0,4)}>}\right\}$$
$CF(e_2, x_2, 0) =$
$$\left\{\frac{u_1}{<0.3e^{i2\pi(0,4)},-0.5e^{i2\pi(-0,5)}>}, \frac{u_2}{<0.9e^{i2\pi(0,5)},-0.2e^{i2\pi(-0,3)}>}, \frac{u_3}{<0.6e^{i2\pi(0,5)},-0.1e^{i2\pi(-0,1)}>}\right\}.$$

Then we can view the complex bipolar fuzzy soft expert set (CF, Z) as consisting of the following collection of approximations:

$(CF, Z) = \Big\{$

$(e_1, x_1, 1) =$
$$\left\{\frac{u_1}{<0.3e^{i2\pi(0,1)},-0.4e^{i2\pi(-0,2)}>}, \frac{u_2}{<0.3e^{i2\pi(0,1)},-0.4e^{i2\pi(-0,2)}>}, \frac{u_3}{<0.3e^{i2\pi(0,1)},-0.4e^{i2\pi(-0,2)}>}\right\}$$
$(e_1, x_2, 1) =$
$$\left\{\frac{u_1}{<0.1e^{i2\pi(0,2)},-0.4e^{i2\pi(-0,1)}>}, \frac{u_2}{<0.6e^{i2\pi(0,2)},-0.1e^{i2\pi(-0,3)}>}, \frac{u_3}{<0.5e^{i2\pi(0,1)},-0.1e^{i2\pi(-0,3)}>}\right\}$$
$(e_2, x_1, 1) =$
$$\left\{\frac{u_1}{<0.8e^{i2\pi(0,3)},-0.6e^{i2\pi(-0,2)}>}, \frac{u_2}{<0.5e^{i2\pi(0,4)},-0.5e^{i2\pi(-0,3)}>}, \frac{u_3}{<0.2e^{i2\pi(0,1)},-0.1e^{i2\pi(-0,4)}>}\right\}$$
$(e_1, x_2, 1) =$
$$\left\{\frac{u_1}{<0.6e^{i2\pi(0,1)},-0.5e^{i2\pi(-0,4)}>}, \frac{u_2}{<0.8e^{i2\pi(0,4)},-0.6e^{i2\pi(-0,3)}>}, \frac{u_3}{<0.1e^{i2\pi(0,5)},-0.8e^{i2\pi(-0,2)}>}\right\}$$
$(e_1, x_1, 0) =$
$$\left\{\frac{u_1}{<0.2e^{i2\pi(0,2)},-0.3e^{i2\pi(-0,1)}>}, \frac{u_2}{<0.5e^{i2\pi(0,1)},-0.3e^{i2\pi(-0,1}>}, \frac{u_3}{<0.2e^{i2\pi(0,2)},-0.2e^{i2\pi(-0,3)}>}\right\}$$
$(e_1, x_2, 0) =$
$$\left\{\frac{u_1}{<0.2e^{i2\pi(0,1)},-0.2e^{i2\pi(-0,3)}>}, \frac{u_2}{<0.5e^{i2\pi(0,2)},-0.1e^{i2\pi(-0,2)}>}, \frac{u_3}{<0.6e^{i2\pi(0,3)},-0.5e^{i2\pi(-0,3)}>}\right\}$$
$(e_2, x_1, 0) =$
$$\left\{\frac{u_1}{<0.2e^{i2\pi(0,2)},-0.4e^{i2\pi(-0,1)}>}, \frac{u_2}{<0.4e^{i2\pi(0,3)},-0.1e^{i2\pi(-0,2)}>}, \frac{u_3}{<0.1e^{i2\pi(0,3)},-0.2e^{i2\pi(-0,4)}>}\right\}$$
$(e_2, x_2, 0) =$
$$\left\{\frac{u_1}{<0.3e^{i2\pi(0,4)},-0.5e^{i2\pi(-0,5)}>}, \frac{u_2}{<0.9e^{i2\pi(0,5)},-0.2e^{i2\pi(-0,3)}>}, \frac{u_3}{<0.6e^{i2\pi(0,5)},-0.1e^{i2\pi(-0,1)}>}\right\} \Bigg\}.$$

Then (F, Z) is a complex bipolar fuzzy soft expert set over the soft universe (U, Z). Each element of the complex bipolar fuzzy soft expert sets implies the opinion of each expert based on each parameter about the cars with their own accompanying membership.

We present the concept of the subset and equality operations on two complex bipolar fuzzy soft expert sets in the following two definitions.

Definition 3.3. Let (CF, A) and (CG, B) be two complex bipolar fuzzy soft expert sets over the common universe U. (CF, A) is a complex bipolar fuzzy soft expert subset of (CG, B) if

1. $A \subseteq B$

2. $\forall e \in A$, $CF(e)$ is a complex bipolar fuzzy subset of $CG(e)$.

This relationship is denoted by $(CF, A)\tilde{\subseteq}(CG, B)$ and (CF, A) is called the complex bipolar fuzzy soft expert subset of (CG, B)

Definition 3.4. Let (CF, A) and (CG, B) be two complex bipolar fuzzy soft expert sets over the common universe U. (CF, A) and (CG, B) are complex bipolar fuzzy soft expert equal sets if (CF, A) is a complex bipolar fuzzy soft expert subset of (CG, B) and (CG, B) is a complex bipolar fuzzy soft expert subset of (CF, A).

Example 3.5. Consider Example 3.2 and suppose that the company takes the opinion of the experts once again after using the cars.
Suppose $A = \{(e_1, x_1, 1), (e_1, x_2, 0), (e_2, x_2, 0)\}$ and $B = \{(e_1, x_1, 1), (e_2, x_2, 0)\}$. Clearly $B \subset A$. Let (CF, A) and (CG, B) be defined as follows:

$(CF, A) = \Big\{$

$(e_1, x_1, 1) =$
$\left\{\frac{u_1}{<0.3e^{i2\pi(0,1)}, -0.4e^{i2\pi(-0,2)}>}, \frac{u_2}{<0.3e^{i2\pi(0,1)}, -0.4e^{i2\pi(-0,2)}>}, \frac{u_3}{<0.3e^{i2\pi(0,1)}, -0.4e^{i2\pi(-0,2)}>}\right\}$

$(e_1, x_2, 0) =$
$\left\{\frac{u_1}{<0.2e^{i2\pi(0,1)}, -0.2e^{i2\pi(-0,3)}>}, \frac{u_2}{<0.5e^{i2\pi(0,2)}, -0.1e^{i2\pi(-0,2)}>}, \frac{u_3}{<0.6e^{i2\pi(0,3)}, -0.5e^{i2\pi(-0,3)}>}\right\}$

$(e_2, x_2, 0) =$
$\left\{\frac{u_1}{<0.3e^{i2\pi(0,4)}, -0.5e^{i2\pi(-0,5)}>}, \frac{u_2}{<0.9e^{i2\pi(0,5)}, -0.2e^{i2\pi(-0,3)}>}, \frac{u_3}{<0.6e^{i2\pi(0,5)}, -0.1e^{i2\pi(-0,1)}>}\right\} \Big\}.$

$(CG, B) = \Big\{$

$(e_1, x_1, 1) =$
$\left\{\frac{u_1}{<0.3e^{i2\pi(0,1)}, -0.4e^{i2\pi(-0,2)}>}, \frac{u_2}{<0.3e^{i2\pi(0,1)}, -0.4e^{i2\pi(-0,2)}>}, \frac{u_3}{<0.3e^{i2\pi(0,1)}, -0.4e^{i2\pi(-0,2)}>}\right\}$

$(e_2, x_2, 0) =$
$\left\{\frac{u_1}{<0.3e^{i2\pi(0,4)}, -0.5e^{i2\pi(-0,5)}>}, \frac{u_2}{<0.9e^{i2\pi(0,5)}, -0.2e^{i2\pi(-0,3)}>}, \frac{u_3}{<0.6e^{i2\pi(0,5)}, -0.1e^{i2\pi(-0,1)}>}\right\} \Big\}.$

Therefore, $(CG, B)\tilde{\subset}(CF, A)$.

Now, we suggest the definitions of an agree-complex bipolar fuzzy soft expert sets and the disagree-complex bipolar fuzzy soft expert sets.

Definition 3.6. An agree-complex bipolar fuzzy soft expert set $(CF, A)_1$ over U is a complex bipolar fuzzy soft expert subset of (CF, A) defined as follows:

$$(CF, A)_1 = \{CF_1(\alpha) : \alpha \in E \times X \times \{1\}\}$$

Definition 3.7. A disagree-complex bipolar fuzzy soft expert set $(CF, A)_0$ over U is a complex bipolar fuzzy soft expert subset of (CF, A) defined as follows:

$$(CF, A)_0 = \{CF_0(\alpha) : \alpha \in E \times X \times \{0\}\}$$

Example 3.8. Consider Example 3.2. Then the agree- complex bipolar fuzzy soft expert set (CF_1, A) over U is

$(CF, A) = \Big\{$

$(e_1, x_1, 1) =$
$\left\{\frac{u_1}{<0.3e^{i2\pi(0,1)}, -0.4e^{i2\pi(-0,2)}>}, \frac{u_2}{<0.3e^{i2\pi(0,1)}, -0.4e^{i2\pi(-0,2)}>}, \frac{u_3}{<0.3e^{i2\pi(0,1)}, -0.4e^{i2\pi(-0,2)}>}\right\}$

$(e_1, x_2, 1) =$
$\left\{\frac{u_1}{<0.1e^{i2\pi(0,2)}, -0.4e^{i2\pi(-0,1)}>}, \frac{u_2}{<0.6e^{i2\pi(0,2)}, -0.1e^{i2\pi(-0,3)}>}, \frac{u_3}{<0.5e^{i2\pi(0,1)}, -0.1e^{i2\pi(-0,3)}>}\right\}$

$(e_2, x_1, 1) =$

$$\left\{\frac{u_1}{<0.8e^{i2\pi(0,3)},-0.6e^{i2\pi(-0,2)}>}, \frac{u_2}{<0.5e^{i2\pi(0,4)},-0.5e^{i2\pi(-0,3)}>}, \frac{u_3}{<0.2e^{i2\pi(0,1)},-0.1e^{i2\pi(-0,4)}>}\right\}$$

$(e_1, x_2, 1) =$

$$\left\{\frac{u_1}{<0.6e^{i2\pi(0,1)},-0.5e^{i2\pi(-0,4)}>}, \frac{u_2}{<0.8e^{i2\pi(0,4)},-0.6e^{i2\pi(-0,3)}>}, \frac{u_3}{<0.1e^{i2\pi(0,5)},-0.8e^{i2\pi(-0,2)}>}\right\}\Bigg\}.$$

and the disagree-bipolar fuzzy soft expert set $(CF, A)_0$ over U is

$(CF, A) = \Bigg\{$

$(e_1, x_1, 0) =$

$$\left\{\frac{u_1}{<0.2e^{i2\pi(0,2)},-0.3e^{i2\pi(-0,1)}>}, \frac{u_2}{<0.5e^{i2\pi(0,1)},-0.3e^{i2\pi(-0,1}>}, \frac{u_3}{<0.2e^{i2\pi(0,2)},-0.2e^{i2\pi(-0,3)}>}\right\}$$

$(e_1, x_2, 0) =$

$$\left\{\frac{u_1}{<0.2e^{i2\pi(0,1)},-0.2e^{i2\pi(-0,3)}>}, \frac{u_2}{<0.5e^{i2\pi(0,2)},-0.1e^{i2\pi(-0,2)}>}, \frac{u_3}{<0.6e^{i2\pi(0,3)},-0.5e^{i2\pi(-0,3)}>}\right\}$$

$(e_2, x_1, 0) =$

$$\left\{\frac{u_1}{<0.2e^{i2\pi(0,2)},-0.4e^{i2\pi(-0,1)}>}, \frac{u_2}{<0.4e^{i2\pi(0,3)},-0.1e^{i2\pi(-0,2)}>}, \frac{u_3}{<0.1e^{i2\pi(0,3)},-0.2e^{i2\pi(-0,4)}>}\right\}$$

$(e_2, x_2, 0) =$

$$\left\{\frac{u_1}{<0.3e^{i2\pi(0,4)},-0.5e^{i2\pi(-0,5)}>}, \frac{u_2}{<0.9e^{i2\pi(0,5)},-0.2e^{i2\pi(-0,3)}>}, \frac{u_3}{<0.6e^{i2\pi(0,5)},-0.1e^{i2\pi(-0,1)}>}\right\}\Bigg\}.$$

3.1. Basic operations on complex bipolar fuzzy soft expert sets

In this section, we introduce some basic operations on complex bipolar fuzzy soft expert sets, namely complement, union, and intersection of complex bipolar fuzzy soft expert sets, derive their properties, and give some examples. We give an illustrative example, and prove the subsequent proposition.

Definition 3.9. The complement of a complex bipolar fuzzy soft expert set (CF, A) is denoted by $(CF, A)^c$ and is defined by $(CF, A)^c = (CF^c, A)$ where $CF^c : A \to CBF^U$ is a mapping given by

$$CF^c(\alpha) = \tilde{c}(CF(\alpha)),\ \forall \alpha \in A$$

such that $\tilde{c}$ is a bipolar fuzzy complement, which is applied for both amplitude and phase terms.

Example 3.10. Consider the complex bipolar fuzzy soft expert set (CF, Z) over a soft universe (U, Z) as given in Example 3.2, by using the bipolar fuzzy complement for $CF(\alpha)$ we obtain $(CF, Z)^c$ which is defined as:

$(CF, Z)^c = \Bigg\{$

$CF^c(e_1, x_1, 1) =$

$$\left\{\frac{u_1}{<0.7e^{i2\pi(0,9)},-0.6e^{i2\pi(-0,8)}>}, \frac{u_2}{<0.7e^{i2\pi(0,9)},-0.6e^{i2\pi(-0,8)}>}, \frac{u_3}{<0.7e^{i2\pi(0,9)},-0.6e^{i2\pi(-0,8)}>}\right\}$$

$CF^c(e_1, x_2, 1) =$

$$\left\{\frac{u_1}{<0.9e^{i2\pi(0,8)},-0.6e^{i2\pi(-0,9)}>}, \frac{u_2}{<0.4e^{i2\pi(0,8)},-0.9e^{i2\pi(-0,7)}>}, \frac{u_3}{<0.5e^{i2\pi(0,9)},-0.9e^{i2\pi(-0,7)}>}\right\}$$

$CF^c(e_2, x_1, 1) =$

$$\left\{\frac{u_1}{<0.2e^{i2\pi(0,7)},-0.4e^{i2\pi(-0,8)}>}, \frac{u_2}{<0.5e^{i2\pi(0,6)},-0.5e^{i2\pi(-0,7)}>}, \frac{u_3}{<0.8e^{i2\pi(0,9)},-0.9e^{i2\pi(-0,6)}>}\right\}$$

$CF^c(e_1, x_2, 1) =$

$$\left\{\frac{u_1}{<0.4e^{i2\pi(0,9)},-0.5e^{i2\pi(-0,6)}>}, \frac{u_2}{<0.2e^{i2\pi(0,6)},-0.4e^{i2\pi(-0,7)}>}, \frac{u_3}{<0.9e^{i2\pi(0,5)},-0.8e^{i2\pi(-0,8)}>}\right\}$$

$CF^c(e_1, x_1, 0) =$

$$\left\{\frac{u_1}{<0.8e^{i2\pi(0,8)},-0.7e^{i2\pi(-0,9)}>}, \frac{u_2}{<0.5e^{i2\pi(0,9)},-0.7e^{i2\pi(-0,9)}>}, \frac{u_3}{<0.8e^{i2\pi(0,8)},-0.8e^{i2\pi(-0,7)}>}\right\}$$

$CF^c(e_1, x_2, 0) =$

$$\left\{\frac{u_1}{<0.8e^{i2\pi(0,9)},-0.8e^{i2\pi(-0,7)}>}, \frac{u_2}{<0.5e^{i2\pi(0,8)},-0.1e^{i2\pi(-0,8)}>}, \frac{u_3}{<0.4e^{i2\pi(0,7)},-0.5e^{i2\pi(-0,7)}>}\right\}$$

$CF^c(e_2, x_1, 0) =$
$\left\{ \frac{u_1}{<0.8e^{i2\pi(0,8)},-0.6e^{i2\pi(-0,9)}>}, \frac{u_2}{<0.6e^{i2\pi(0,7)},-0.9e^{i2\pi(-0,8)}>}, \frac{u_3}{<0.9e^{i2\pi(0,7)},-0.8e^{i2\pi(-0,6)}>} \right\}$
$CF^c(e_2, x_2, 0) =$
$\left\{ \frac{u_1}{<0.7e^{i2\pi(0,6)},-0.5e^{i2\pi(-0,5)}>}, \frac{u_2}{<0.1e^{i2\pi(0,5)},-0.8e^{i2\pi(-0,7)}>}, \frac{u_3}{<0.4e^{i2\pi(0,5)},-0.9e^{i2\pi(-0,9)}>} \right\} \Big\}.$

Proposition 3.11. *If (CF, A) is a complex bipolar fuzzy soft expert set over U, then $((CF, A)^c)^c = (CF, A)$.*

Proof. Suppose that (CF, A) is a complex bipolar fuzzy soft expert set over U defined as $(CF, A) = CF(e)$. Now let $(CF, A)^c = (CG, B)$. Then by Definition 3.9 $(CG, B) = CG(e)$ such that $CG(e) = \tilde{c}(CF(e))$. Thus, $(CG, B)^c = \tilde{c}(CG(e)) = \tilde{c}(\tilde{c}(CF(e))) = CF(e) = (CF, A)$.

Therefore, $((CF, A)^c)^c = (CG, B)^c = (CF, A)$. □

In the following, we introduce the definition of the union of two bipolar fuzzy soft expert sets, give an illustrative example and prove the subsequent proposition

Definition 3.12. The union of two complex bipolar fuzzy soft expert sets (CF, A) and (CG, B) over U is denoted by $(CF, A)\tilde{\cup}(CG, B)$, and it is a complex bipolar fuzzy soft expert set (CH, C) where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$(CH, C) = \begin{cases} CF(\varepsilon), & if\ \varepsilon \in A - B \\ CG(\varepsilon), & if\ \varepsilon \in B - A \\ CF(\varepsilon) \cup CG(\varepsilon), & if\ \varepsilon \in B \cap A \end{cases}$$

such that $\cup$ denotes the bipolar fuzzy union, which is applied for both amplitude and phase terms.

Example 3.13. Suppose that a company produces new types of children's games and wants to take the opinion of the experts twice again over a period of time after using those games. Let $U = \{u_1, u_2, u_3\}$ be the set of games under consideration, $E = \{e_1 = education, e_2 = fun, e_3 = entertaining\}$ be the set of decision parameters, and $X = \{x_1, x_2\}$ be the set of experts such that $A = \{e_1, e_2\}$, $B = \{e_1, e_3\}$. Suppose (CF, A) and (CG, B) are two complex bipolar fuzzy soft expert sets over U such that:

$(CF, A) = \Big\{$
$(e_1, x_1, 1) =$
$\left\{ \frac{u_1}{<0.5e^{i2\pi(0,2)},-0.1e^{i2\pi(-0,3)}>}, \frac{u_2}{<0.4e^{i2\pi(0,8)},-0.3e^{i2\pi(-0,6}>}, \frac{u_3}{<0.5e^{i2\pi(0,4)},-6.2e^{i2\pi(-0,2)}>} \right\}$
$(e_1, x_1, 0) =$
$\left\{ \frac{u_1}{<0.7e^{i2\pi(0,6)},-0.1e^{i2\pi(-0,7)}>}, \frac{u_2}{<0.2e^{i2\pi(0,2)},-0.3e^{i2\pi(-0,5)}>}, \frac{u_3}{<0.1e^{i2\pi(0,4)},-0.5e^{i2\pi(-0,7)}>} \right\}$
$(e_2, x_2, 1) =$
$\left\{ \frac{u_1}{<0.1e^{i2\pi(0,5)},-0.3e^{i2\pi(-0,4)}>}, \frac{u_2}{<0.3e^{i2\pi(0,4)},-0.2e^{i2\pi(-0,1)}>}, \frac{u_3}{<0.7e^{i2\pi(0,1)},-0.1e^{i2\pi(-0,8)}>} \right\} \Big\}.$

$(CG, B) = \Big\{$
$(e_1, x_1, 1) =$
$\left\{ \frac{u_1}{<0.5e^{i2\pi(0,1)},-0.8e^{i2\pi(-0,5)}>}, \frac{u_2}{<0.4e^{i2\pi(0,1)},-0.2e^{i2\pi(-0,8)}>}, \frac{u_3}{<0.1e^{i2\pi(0,2)},-0.2e^{i2\pi(-0,3)}>} \right\}$
$(e_1, x_1, 0) =$
$\left\{ \frac{u_1}{<0.5e^{i2\pi(0,2)},-0.7e^{i2\pi(-0,6)}>}, \frac{u_2}{<0.1e^{i2\pi(0,7)},-0.6e^{i2\pi(-0,4)}>}, \frac{u_3}{<0.1e^{i2\pi(0,2)},-0.8e^{i2\pi(-0,1)}>} \right\}$
$(e_3, x_2, 1) =$
$\left\{ \frac{u_1}{<0.6e^{i2\pi(0,2)},-0.8e^{i2\pi(-0,5)}>}, \frac{u_2}{<0.3e^{i2\pi(0,4)},-0.3e^{i2\pi(-0,5)}>}, \frac{u_3}{<0.1e^{i2\pi(0,2)},-0.6e^{i2\pi(-0,4)}>} \right\} \Big\}.$

By using basic bipolar fuzzy union, we have $(CF, A)\tilde{\cup}(CG, B) = (CH, C)$ where

$(CH, C) = \Bigg\{$

$(e_1, x_1, 1) =$
$\left\{\frac{u_1}{<0.5e^{i2\pi(0,2)}, -0.8e^{i2\pi(-0,5)}>}, \frac{u_2}{<0.4e^{i2\pi(0,8)}, -0.3e^{i2\pi(-0,8)}>}, \frac{u_3}{<0.5e^{i2\pi(0,4)}, -0.6e^{i2\pi(-0,3)}>}\right\}$

$(e_1, x_1, 0) =$
$\left\{\frac{u_1}{<0.7e^{i2\pi(0,6)}, -0.7e^{i2\pi(-0,7)}>}, \frac{u_2}{<0.2e^{i2\pi(0,7)}, -0.6e^{i2\pi(-0,5)}>}, \frac{u_3}{<0.1e^{i2\pi(0,4)}, -0.8e^{i2\pi(-0,7)}>}\right\}$

$(e_2, x_2, 1) =$
$\left\{\frac{u_1}{<0.1e^{i2\pi(0,5)}, -0.3e^{i2\pi(-0,4)}>}, \frac{u_2}{<0.3e^{i2\pi(0,4)}, -0.6e^{i2\pi(-0,5)}>}, \frac{u_3}{<0.7e^{i2\pi(0,1)}, -0.1e^{i2\pi(-0,8)}>}\right\}$

$(e_3, x_2, 1) =$
$\left\{\frac{u_1}{<0.6e^{i2\pi(0,2)}, -0.8e^{i2\pi(-0,5)}>}, \frac{u_2}{<0.3e^{i2\pi(0,4)}, -0.3e^{i2\pi(-0,5)}>}, \frac{u_3}{<0.1e^{i2\pi(0,2)}, -0.6e^{i2\pi(-0,4)}>}\right\} \Bigg\}.$

Proposition 3.14. *Let (CF, A), (CG, B) and (CH, C) be any three complex bipolar fuzzy soft expert sets over a soft universe (U, Z). Then the following properties hold:*

1. $(CF, A)\tilde{\cup}(CF, A) = (CF, A)$
2. $(CF, A)\tilde{\cup}(CG, B) = (CG, B)\tilde{\cup}(CF, A)$
3. $(CF, A)\tilde{\cup}\Phi = (CF, A)$ *where Φ is a null complex bipolar fuzzy soft expert set*
4. $(CF, A)\tilde{\cup}((CG, B)\tilde{\cup}(CH, C)) = ((CF, A)\tilde{\cup}(CG, B))\tilde{\cup}(CH, C)$

Proof. We will provide the proof of assertion (4) since the proof of assertions (1),(2) and (3) are straightforward from Definition 3.12. Suppose that $(CG, B)\tilde{\cup}(CH, C) = (CK, D)$ where $D = B \cup C$. By Definition 3.12 we have $(CG, B)\tilde{\cup}(CH, C)$ is a complex bipolar fuzzy soft expert set (CK, D) where $D = B \cup C$ and $\varepsilon \in D$

$$(CK, D) = \begin{cases} CF(\varepsilon), & if\ \varepsilon \in B - C \\ CG(\varepsilon), & if\ \varepsilon \in C - B \\ CF(\varepsilon) \cup CG(\varepsilon), & if\ \varepsilon \in B \cap C. \end{cases}$$

We consider the case when $\varepsilon \in B \cap C$ as the other cases are trivial, then we have $(CK, D) = CG(\varepsilon) \cup CH(\varepsilon)$.

Now let $(CF, A)\tilde{\cup}((CG, B)\tilde{\cup}(CH, C)) = (CF, A)\tilde{\cup}(CK, D)$. By Definition 3.12 we consider the case when $\varepsilon \in A \cap D$ the other cases are trivial. Then we have

$$\begin{aligned} (CF, A)\tilde{\cup}(CK, D) &= CF(\varepsilon) \cup (CG(\varepsilon) \cup CH(\varepsilon)) \\ &= (CF(\varepsilon) \cup CG(\varepsilon)) \cup CH(\varepsilon) \\ &= ((CF, A)\tilde{\cup}(CG, B))\tilde{\cup}(CH, C) \end{aligned}$$

Therefore, we have $(CF, A)\tilde{\cup}((CG, B)\tilde{\cup}(CH, C)) = ((CF, A)\tilde{\cup}(CG, B))\tilde{\cup}(CH, C)$ □

Now, we introduce the definition of the intersection of two complex bipolar fuzzy soft expert sets, give an illustrative example, and prove the subsequent proposition.

Definition 3.15. The intersection of two complex bipolar fuzzy soft expert sets (CF, A) and (CG, B) over U is denoted by $(CF, A)\tilde{\cap}(CG, B)$, and it is a complex bipolar fuzzy soft expert set (CH, C) where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$(CH, C) = \begin{cases} CF(\varepsilon), & if\ \varepsilon \in A - B \\ CG(\varepsilon), & if\ \varepsilon \in B - A \\ CF(\varepsilon) \cap CG(\varepsilon), & if\ \varepsilon \in B \cap A \end{cases}$$

such that $\cap$ denotes the bipolar fuzzy intersection, which is applied for both amplitude and phase terms.

Example 3.16. Consider Example 3.13, by using basic bipolar fuzzy intersection, we have $(CF, A)\tilde{\cap}(CG, B) = (CH, C)$ where

$(CH, C) = \Big\{$

$(e_1, x_1, 1) =$
$\left\{\frac{u_1}{<0.5e^{i2\pi(0,1)}, -0.1e^{i2\pi(-0,3)}>}, \frac{u_2}{<0.4e^{i2\pi(0,1)}, -0.2e^{i2\pi(-0,6)}>}, \frac{u_3}{<0.1e^{i2\pi(0,2)}, -0.2e^{i2\pi(-0,2)}>}\right\}$

$(e_1, x_1, 0) =$
$\left\{\frac{u_1}{<0.5e^{i2\pi(0,2)}, -0.1e^{i2\pi(-0,6)}>}, \frac{u_2}{<0.1e^{i2\pi(0,2)}, -0.3e^{i2\pi(-0,4)}>}, \frac{u_3}{<0.1e^{i2\pi(0,2)}, -0.5e^{i2\pi(-0,1)}>}\right\}$

$(e_2, x_2, 1) =$
$\left\{\frac{u_1}{<0.1e^{i2\pi(0,5)}, -0.3e^{i2\pi(-0,4)}>}, \frac{u_2}{<0.3e^{i2\pi(0,4)}, -0.2e^{i2\pi(-0,1)}>}, \frac{u_3}{<0.7e^{i2\pi(0,1)}, -0.1e^{i2\pi(-0,8)}>}\right\}$

$(e_3, x_2, 1) =$
$\left\{\frac{u_1}{<0.6e^{i2\pi(0,2)}, -0.8e^{i2\pi(-0,5)}>}, \frac{u_2}{<0.3e^{i2\pi(0,4)}, -0.3e^{i2\pi(-0,5)}>}, \frac{u_3}{<0.1e^{i2\pi(0,2)}, -0.6e^{i2\pi(-0,4)}>}\right\} \Big\}.$

Proposition 3.17. *Let (CF, A), (CG, B) and (CH, C) be any three complex bipolar fuzzy soft expert sets over a soft universe (U, Z). Then the following properties hold:*

1. $(CF, A)\tilde{\cap}(CF, A) = (CF, A)$
2. $(CF, A)\tilde{\cap}(CG, B) = (CG, B)\tilde{\cap}(CF, A)$
3. $(CF, A)\tilde{\cap}\Phi = (CF, A)$ *where Φ is a null complex bipolar fuzzy soft expert set*
4. $(CF, A)\tilde{\cap}((CG, B)\tilde{\cap}(CH, C)) = ((CF, A)\tilde{\cap}(CG, B))\tilde{\cap}(CH, C)$

Proof. The proofs are straightforward by Definition 3.15 and therefore it omitted. □

4. OR and AND operations on bipolar fuzzy soft expert sets

In this section, we introduce the definitions of AND and OR operations for complex bipolar fuzzy soft expert sets and derive their properties.

Definition 4.1. Let (CF, A) and (CG, B) be any two complex bipolar fuzzy soft expert sets over a soft universe (U, Z). Then '(CF, A) AND (CG, B)' denoted by $(CF, A)\tilde{\wedge}(CG, B)$ is $(CF, A)\tilde{\wedge}(CG, B) = (CH, A \times B)$ where $(CH, A \times B) = CH(a, b)$ such that $CH(a, b) = CF(a) \cap CG(b)$ for all $(a, b) \in A \times B$, and $\cap$ is the intersection operation of bipolar fuzzy sets.

Definition 4.2. Let (CF, A) and (CG, B) be any two complex bipolar fuzzy soft expert sets over a soft universe (U, Z). Then, (CF, A) OR (CG, B) denoted by $(CF, A)\tilde{\vee}(CG, B)$ is $(CF, A)\tilde{\vee}(CG, B) = (CH, A \times B)$ where $(CH, A \times B) = CH(a, b)$ such that $CH(a, b) = CF(a) \cup CG(b)$ for all $(a, b) \in A \times B$, and $\cup$ is the union operation of bipolar fuzzy sets.

Proposition 4.3. *If (CF, A) and (CG, B) be any two complex bipolar fuzzy soft expert sets over a soft universe (U, Z), then*

1. $((CF, A)\tilde{\wedge}(CG, B))^c = (CF, A)^c\tilde{\vee}(CG, B)^c$
2. $((CF, A)\tilde{\vee}(CG, B))^c = (CF, A)^c\tilde{\wedge}(CG, B)^c$

Proof. (1) Suppose that (CF, A) and (CG, B) are two complex bipolar fuzzy soft expert sets over a soft universe (U, Z) defined as $(CF, A) = F(\alpha)$ for all $\alpha \in A \subseteq Z$ and $(CG, B) = G(\beta)$ for all $\beta \in B \subseteq Z$.

By Definitions 4.1 and 4.2 it follows that:

$$\begin{aligned}((CF, A)\tilde{\wedge}(CG, B))^c &= (F(\alpha)\tilde{\wedge}G(\beta))^c \\ &= (F(\alpha) \cap G(\beta))^c \\ &= \hat{c}(F(\alpha) \cap G(\beta)) \\ &= \hat{c}(F(\alpha)) \cup \hat{c}(G(\beta)) \\ &= (F(\alpha))^c\tilde{\vee}(G(\beta))^c \\ &= (CF, A)^c\tilde{\vee}(CG, B)^c\end{aligned}$$

(2) The proof is similar to that in part (1) and therefore is omitted. □

Example 4.4. Let (CF, A) and (CG, B) be two complex bipolar fuzzy soft expert sets. If $A = \{(e_1, x, 1), (e_2, x, 0)\}$ and $B = \{(s_1, x, 1), (s_2, x, 0)\}$

$(CF, A) = \Big\{$

$(e_1, x, 1) =$
$\left\{\frac{u_1}{<0.2e^{i2\pi(0,3)},-0.6e^{i2\pi(-0,5)}>}, \frac{u_2}{<0.1e^{i2\pi(0,2)},-0.2e^{i2\pi(-0,2)}>}, \frac{u_3}{<0.4e^{i2\pi(0,2)},-0.8e^{i2\pi(-0,9)}>}\right\}$

$(e_1, x, 0) =$
$\left\{\frac{u_1}{<0.1e^{i2\pi(0,2)},-0.1e^{i2\pi(-0,5)}>}, \frac{u_2}{<0.5e^{i2\pi(0,2)},-0.3e^{i2\pi(-0,5)}>}, \frac{u_3}{<0.4e^{i2\pi(0,2)},-0.6e^{i2\pi(-0,7)}>}\right\}\Big\}$

and

$(CG, B) = \Big\{$

$(e_1, s, 1) =$
$\left\{\frac{u_1}{<0.5e^{i2\pi(0,8)},-0.2e^{i2\pi(-0,2)}>}, \frac{u_2}{<0.2e^{i2\pi(0,5)},-0.3e^{i2\pi(-0,4)}>}, \frac{u_3}{<0.6e^{i2\pi(0,2)},-0.4e^{i2\pi(-0,3)}>}\right\}$

$(e_2, s, 0) =$
$\left\{\frac{u_1}{<0.6e^{i2\pi(0,2)},-0.2e^{i2\pi(-0,3)}>}, \frac{u_2}{<0.9e^{i2\pi(0,4)},-0.3e^{i2\pi(-0,2)}>}, \frac{u_3}{<0.7e^{i2\pi(0,5)},-0.9e^{i2\pi(-0,6)}>}\right\}\Big\}.$

Thus,

$(CF, A)\ AND\ (CG, B) = (CH, A \times B) = \Big\{$

$((e_1, x, 1), (e_1, s, 1)) =$
$\left\{\frac{u_1}{<0.2e^{i2\pi(0,8)},-0.2e^{i2\pi(-0,2)}>}, \frac{u_2}{<0.1e^{i2\pi(0,2)},-0.2e^{i2\pi(-0,2)}>}, \frac{u_3}{<0.4e^{i2\pi(0,2)},-0.4e^{i2\pi(-0,8)}>}\right\}$

$((e_1, x, 1), (e_1, s, 0)) =$
$\left\{\frac{u_1}{<0.5e^{i2\pi(0,2)},-0.2e^{i2\pi(-0,5)}>}, \frac{u_2}{<0.1e^{i2\pi(0,2)},-0.2e^{i2\pi(-0,2)}>}, \frac{u_3}{<0.4e^{i2\pi(0,2)},-0.8e^{i2\pi(-0,6)}>}\right\}$

$((e_1, x, 0), (e_1, s, 1)) =$
$\left\{\frac{u_1}{<0.1e^{i2\pi(0,2)},-0.1e^{i2\pi(-0,2)}>}, \frac{u_2}{<0.2e^{i2\pi(0,2)},-0.3e^{i2\pi(-0,4)}>}, \frac{u_3}{<0.4e^{i2\pi(0,2)},-0.4e^{i2\pi(-0,7)}>}\right\}$

$((e_1, x, 0), (e_1, s, 0)) =$
$\left\{\frac{u_1}{<0.1e^{i2\pi(0,2)},-0.1e^{i2\pi(-0,5)}>}, \frac{u_2}{<0.5e^{i2\pi(0,2)},-0.3e^{i2\pi(-0,2)}>}, \frac{u_3}{<0.4e^{i2\pi(0,2)},-0.6e^{i2\pi(-0,6)}>}\right\}\Big\}$

Also, for (CF, A) OR $(\mathrm{CG, B}) = (\mathrm{CH, A \times B}) = \Big\{$

$((e_1, x, 1), (e_1, s, 1)) =$
$\left\{\frac{u_1}{<0.5e^{i2\pi(0,8)},-0.6e^{i2\pi(-0,5)}>}, \frac{u_2}{<0.2e^{i2\pi(0,5)},-0.8e^{i2\pi(-0,4)}>}, \frac{u_3}{<0.6e^{i2\pi(0,2)},-0.8e^{i2\pi(-0,9)}>}\right\}$

$((e_1, x, 1), (e_1, s, 0)) =$
$\left\{\frac{u_1}{<0.6e^{i2\pi(0,8)},-0.6e^{i2\pi(-0,8)}>}, \frac{u_2}{<0.9e^{i2\pi(0,4)},-0.3e^{i2\pi(-0,2)}>}, \frac{u_3}{<0.7e^{i2\pi(0,5)},-0.9e^{i2\pi(-0,9)}>}\right\}$

$((e_1, x, 0), (e_1, s, 1)) =$
$\left\{\frac{u_1}{<0.5e^{i2\pi(0,8)},-0.2e^{i2\pi(-0,5)}>}, \frac{u_2}{<0.5e^{i2\pi(0,5)},-0.8e^{i2\pi(-0,5)}>}, \frac{u_3}{<0.6e^{i2\pi(0,2)},-0.6e^{i2\pi(-0,8)}>}\right\}$

$((e_1, x, 0), (e_1, s, 0)) =$
$\left\{\frac{u_1}{<0.6e^{i2\pi(0,2)},-0.2e^{i2\pi(-0,5)}>}, \frac{u_2}{<0.9e^{i2\pi(0,4)},-0.3e^{i2\pi(-0,5)}>}, \frac{u_3}{<0.7e^{i2\pi(0,5)},-0.9e^{i2\pi(-0,7)}>}\right\}\Big\}$

5. Conclusion

In this paper, a new mathematical tool called complex bipolar fuzzy soft expert sets (CBFSES) was obtained to combine the benefit of complex fuzzy soft expert sets and bipolar fuzzy sets, besides having the added advantage of allowing the users to know the opinion of all the experts in only one mathematical tool without the need for any additional operations in periodic nature. Also, the basic theoretical operations were studied with their properties. In next stage, as a future work, we may modify current tool by combining it with neutrosophic, hesitant fuzzy, and intuitionistic fuzzy sets to get the ability to deal with different complex and extraordinary information. Also, it will be useful and applicable to choose and define, as examples, the best site to opening a new restaurant, factory, or tourist office, and the best time to produce, distribute, or make an advertisement for a specific product.

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