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# Comparing Zagreb Indices of Rhombus Networks

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**Abstract**

The study of networks by using topological indices (TIs) have been significantly become a useful attention in the physicochemical properties of compounds, pharmacology and drug delivery in the field of experimental sciences. Thus, TIs help us to study the new networks and they also play an essential role in the study of the quantitative structure property and activity relationships. In this paper, we compute the connection number (CN) based Zagreb indices in the form of first general-Zagreb connection index (ZCI), generalized first, second, third and fourth ZCIs of two rhombus type networks such as rhombus oxide and rhombus silicate. In particularly, we also find the first, second, modified first, second, third and fourth ZCIs by using main results of the abovementioned general & generalized connection based Zagreb indices. In addition, a comparison between degree and CN based Zagreb indices is done with the help of their numerical values and graphical demonstration.

*Keywords:* Degree, connection number, Zagreb indices, general and generalized Zagreb indices.
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## 1. Introduction

Topological index (TI) is a numerical parameter of networks which deal its topology, and also known as network invariants. The chemical applicability of their molecular structures can be characterized with the help of these TIs. In molecular networks, atoms have correspondence with vertices and covalent bounds between atoms have correspondence with edges. Therefore, TIs check the physical and chemical properties of isomers as well as compounds liked density, octanol-water partition coefficient, boiling point, molar volume, total surface area, enthalpy of formation and vaporization, see [1, 2, 3]. The fusion of chemistry, information science, and mathematics provides to a new horizon called by cheminformatics. In mathematical chemistry these molecular invariants are very used for the characteristics of chemical compounds. They are also very good tools in the behavior of quantitative-structure property and activity relationships. In modern chemistry as well as mathematics, TIs are attached by a link that two isomorphic networks have to derive the

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same TI. For more information, see [4, 5, 6]. Generally, a TI can be classified into three different ways: degree, distance and polynomial based TIs.

In graph theory history, the first TI was defined by American chemist Wiener [7] (1947) when he was working on the chemical compound which was boiling point of paraffin. The well known class of TIs is degree based which depend on degree of the vertex. This class is further classified into two subclasses degree as well connection number (CN). Gutman and Trinajstić [8] defined these both subclasses in 1972 to find the entire energy relation of a chemical compound between their atoms and bounds. Later on, Gutman and Rucic [9] defined another TI named as second Zagreb index (ZI) in 1975. Furtula and Gutman [10] investigated another index of the Zagreb series after a long gap in 2015 named as third ZI. For the late discovery of this index and due to that instance, every researcher called it forgotten index. This Zagreb series based on degree of a vertex was generalized by Azari and Iranmanesh [11] (2011) in the name of generalized ZI. Azari [12] also used this generalized ZI to compute the exact formulas of product graphs such as sum, union, disjunction, Cartesian product, symmetric difference, lexicographic product, corona product, direct product and strong product.

Javaid et al. [13] (2017) computed degree based generalized versions ZIs such as first general ZI, generalized ZI, multiplicative ZI and some other TIs of the rhombus silicate and oxide networks. Kulli [14] computed reverse Zagreb and hyper ZIs of rhombus silicate network. Mondal et al. [15] developed neighborhood versions ZIs to find the exact solutions of silicate and oxide networks. Sarkar et al. [16] (2019) called this generalized ZI as (a,b) Zagreb index to find some derived-networks liked butterfly, Benes and Sierpinski. Kulli [17] computed exact formulae for $(a, b)$-Kulli-Basava index of some special graphs. In 2020, Awais et al. [18] used this generalized version to find some particular indices such as first, second & third ZIs, symmetric division deg index, redefined index, general first ZI and general Randić index of the metal organic networks. Liu et al. [19] computed some well known Zagreb and other TIs of certain networks such as silicate, hexagonal, chain silicate, oxide, cellular and Sierpinski. Zhao et al. [20] computed some reverse Zagreb-type indices for planar-metal organic networks. Recently, a topological index Remdesivir compound has been used in the treatment of COVID-19 [21].

The another subclass of degree based TI is connection number (CN). CN represents second degree of the vertex $\lambda$. Gutman and Trinajstić defined another TI in this paper [8]. But after that no one researcher checked its suitability for any connected networks. In 2019, Tang et al. [22] computed different exact solutions of CN based ZIs in the form of first ZCI, second ZCI, and modified first ZCI for sub-division related operations on graphs. Ali et al. [23] enhanced this concept to find modified second ZCI and modified third ZCI for $T$-sum graphs. Asif et al. [24] computed exact values for ZCIs of newly developed chemical structures $\theta^{\phi}\Omega$. Recently, Hussain et al. [25] compared zinc based metal organic networks for ZCIs and modified ZCIs. At this time, many papers on the topic of CN based ZIs have been revolutionized, see [26, 27, 28].

In this paper, we compute the CN based ZIs in the form of the first general ZCI, generalized first, second, third and fourth ZCIs of the rhombus oxide and silicate networks. Moreover, a recent developed ZCIs are computed with the help of above mentioned generalized ZCIs. In addition, a comparison between degree and CN based ZIs is included by using numerical values and graphical demonstrations. This article is framed as Section II presents the preliminaries, definitions & definitions related Tables and some important results which help the derivations of main results, Section III covers partitions of rhombus oxide and silicate networks, Section IV and V gives the main results of aforesaid networks and Section VI gives comparison and conclusions.

## 2. Notations and Preliminaries

Any simple, finite and connected network $Q$, there is a path for each pair of possible vertices $\kappa$ and $\lambda$. If the path between two vertices $\kappa$ and $\lambda$ shall be shortest then its becomes distance as $d(\kappa, \lambda)$. This shortest path is also known as $\kappa - \lambda$ geodesic otherwise $\kappa - \lambda$ detour. $V(Q)$ and $E(Q)$ are the vertex and edge sets having order $u$ and size $v$, respectively. Let $d(\lambda)$ and $\tau(\lambda)$ be the degree and CN of vertex $\lambda$ in (molecular) network $Q$. A network $Q$ becomes molecular if vertex and edge of $Q$ are equal to atom and bond, respectively.

**Definition 2.1** (See [29]). Let $Q$ ba a (molecular) network. Then $\forall m \in R - \{0\}$, the first general Zagreb index ($Z_m(Q)$) is given as

$$Z_m(Q) = \sum_{\lambda \in V(Q)} [d(\lambda)^m].$$

By using Definition 2.1, we make modified first ZI in the Table 1 as gives:

Table 1: Degree based modified first ZI.

| Name/Symbol | Formula |
|---|---|
| Modified first ZI (See [30]) $^mM_1(Q) = Z_{-2}(Q)$ | $\sum_{\lambda \in V(Q)} \frac{1}{[\tau(\lambda)]^2}$ |

Corresponding to this degree based ZI, CN based the first general ZI is as gives:

**Definition 2.2** (See [31]). Let $Q$ ba a (molecular) network. Then $\forall m, n \in R \wedge n \neq 0$, the first general Zagreb connection index ($G_{m,n}(Q)$) is given as

$$G_{m,n}(Q) = \sum_{\lambda \in V(Q)} [d(\lambda)^m \times \tau(\lambda)^n].$$

By using Definition 2.2, we make the first Zagreb connection index in the Table 2 as gives:

Table 2: Connection number based first ZI.

| Name/Symbol | Formula |
|---|---|
| First ZCI (See [22]) $ZC_1(Q) = G_{0,2}(Q)$ | $\sum_{\lambda \in V(Q)} \tau(\lambda)^2$ |

**Definition 2.3** (See [32]). Let $Q$ ba a (molecular) network. Then $\forall m, n \in R$ and both are not zero at the same time. The $k$-distance generalized Zagreb index ($Z^k_{m,n}(Q)$) for $k \geq 1$, is given as

$$Z^k_{m,n}(Q) = \sum_{\kappa\lambda \in E(Q)} [d_k(\kappa)^m d_k(\lambda)^n + d_k(\lambda)^m d_k(\kappa)^n].$$

If $k = 1$, then $Z_{m,n}(Q)$ becomes degree of vertices based the generalized Zagreb index as

$$Z_{m,n}(Q) = \sum_{\kappa\lambda \in E(Q)} [d(\kappa)^m d(\lambda)^n + d(\lambda)^m d(\kappa)^n].$$

The generalized Zagreb index [11] (2011) was introduced by Azari and Iranmanesh to find some networks of nanotori and nanotubes. By using Definition 2.3, we make some important degree based ZIs in the Table 3 as gives:

In Definition 2.3, if we put $k = 2$, we can obtain connection number ($d_2 = \tau$) based Zagreb indices in the form of generalized Zagreb connection indices [32] as gives:

Table 3: Some more Degree based ZIs.

| Name/Symbol | Formula |
|---|---|
| First ZI (See [8]) $M_1(Q) = Z_{1,0}(Q)$ | $\sum_{\kappa\lambda \in E(Q)}[d(\kappa) + d(\lambda)]$ |
| Second ZI (See [9]) $M_2(Q) = \frac{1}{2}Z_{1,1}(Q)$ | $\sum_{\kappa\lambda \in E(Q)}[d(\kappa) \times d(\lambda)]$ |
| Third ZI (See [10]) $M_3(Q) = Z_{2,0}(Q)$ | $\sum_{\kappa\lambda \in E(Q)}[d(\kappa)^2 + d(\lambda)^2]$ |
| Modified second ZI (See [33]) ${}^mM_2(Q) = \frac{1}{2}Z_{-1,-1}(Q)$ | $\sum_{\kappa\lambda \in E(Q)} \frac{1}{[d(\kappa)\times d(\lambda)]}$ |
| Redefined third ZI (See [34]) $ReZ_3(Q) = Z_{2,1}(Q)$ | $\sum_{\kappa\lambda \in E(Q)} d(\kappa)d(\lambda)[d(\kappa) + d(\lambda)]$ |

**Definition 2.4**(See [32]). Let $Q$ ba a (molecular) network. Then $\forall m, n \in R$ and both are not zero at the same time, the generalized first Zagreb connection index ($C_{m,n}(Q)$) is given as

$$C_{m,n}(Q) = \sum_{\kappa\lambda \in E(Q)} [\tau(\kappa)^m \tau(\lambda)^n + \tau(\lambda)^m \tau(\kappa)^n].$$

By using Definition 2.4, we make some important CN based Zagreb indices in the Table 4 as gives:

Table 4: Connection number based ZIs.

| Name/Symbol | Formula |
|---|---|
| Second ZCI (See [22]) $ZC_2(Q) = \frac{1}{2}C_{1,1}(Q)$ | $\sum_{\kappa\lambda \in E(Q)}[\tau(\kappa) \times \tau(\lambda)]$ |
| Modified first ZCI (See [22]) $ZC_1^*(Q) = C_{1,0}(Q)$ | $\sum_{\kappa\lambda \in E(Q)}[\tau(\kappa) + \tau(\lambda)]$ |

**Definition 2.5** See [32]). Let $Q$ ba a (molecular) network. Then $\forall m, n \in R - \{0\}$, the generalized second Zagreb connection index ($C^2_{m,n}(Q)$) is given as

$$C^2_{m,n}(Q) = \sum_{\kappa\lambda \in E(Q)} [d(\kappa)^m \tau(\lambda)^n + d(\lambda)^m \tau(\kappa)^n].$$

By using Definition 2.5, we make the modified second Zagreb connection index in the Table 5 as gives:

Table 5: Connection number based modified second ZI.

| Name/Symbol | Formula |
|---|---|
| Modified second ZCI (See [23]) $ZC_2^*(Q) = C^2_{1,1}(Q)$ | $\sum_{\kappa\lambda \in E(Q)}[d(\kappa)\tau(\lambda) + d(\lambda)\tau(\kappa)]$ |

**Definition 2.6** (See [32]). Let $Q$ ba a (molecular) network. Then $\forall m, n \in R - \{0\}$, the generalized third Zagreb connection index ($C^3_{m,n}(Q)$) is given as

$$C^3_{m,n}(Q) = \sum_{\kappa\lambda \in E(Q)} [d(\kappa)^m \tau(\kappa)^n + d(\lambda)^m \tau(\lambda)^n].$$

By using Definition 2.6, we make the modified third Zagreb connection index in the Table 6 as gives:

Table 6: Connection number based modified third ZI.

| Name/Symbol | Formula |
|---|---|
| Modified third ZCI (See [23]) $ZC_3^*(Q) = C_{1,1}^3(Q)$ | $\sum_{\kappa\lambda\in E(Q)}[d(\kappa)\tau(\kappa) + d(\lambda)\tau(\lambda)]$ |

**Definition 2.7** (See [32]). Let $Q$ ba a (molecular) network. Then $\forall m, n \in R - \{0\}$, the generalized fourth Zagreb connection index ($C_{m,n}^4(Q)$) is given as

$$C_{m,n}^4(Q) = \sum_{\kappa\lambda\in E(Q)} [d(\kappa)^m\tau(\kappa)^n \times d(\lambda)^m\tau(\lambda)^n].$$

By using Definition 2.7, we make the modified fourth Zagreb connection index in the Table 7 as gives:

Table 7: Connection number based modified fourth ZI.

| Name/Symbol | Formula |
|---|---|
| Modified fourth ZCI (See [25]) $ZC_4^*(Q) = C_{1,1}^4(Q)$ | $\sum_{\kappa\lambda\in E(Q)}[d(\kappa)\tau(\kappa) \times d(\lambda)\tau(\lambda)]$ |

Now, we discuss some valuable results which can be utilized in the main outcomes.

**Lemma 2.1** (See [35]). Let $Q$ be a connected network of size $v$. Then,
$\sum\limits_{\lambda\in V(Q)} d(\lambda) = 2v$.

**Lemma 2.2** (See [22]). Let $Q$ be a connected network of order $u$ and size $v$. Then,
$\sum\limits_{\lambda\in V(Q)} \tau(\lambda) = M_1(Q) - 2v$, equality holds if $Q$ is $\{C_3, C_4\}$-free network.

**Corollary 2.1** (See [13]). Let $A \cong RHOX(p)$ be a rhombus oxide network on dimension $p \geq 3$. Then, the first general Zagreb index of A is

$Z_m(A) = 2^m \times 4p + 4^m(3p^2 - 2p)$.

By using Corollary 2.1, we derive the modified first ZI as follows:
${}^mM_1(A) = Z_{-2}(A) = \frac{3}{16}p^2 + \frac{15}{8}p$.

**Corollary 2.2** (See [13]). Let $A \cong RHOX(p)$ be a rhombus oxide network on dimension $p \geq 3$. Then, the generalized Zagreb index of A is

$Z_{m,n}(A) = 2^{m+n+2}[3p^2(2^{m+n}) + 2p(2^m + 2^n - 2^{m+n+1}) + (1 - 2^m - 2^n + 2^{m+n+1})]$.

By using Corollary 2.2, we derive the degree of vertices based ZIs as follows:
**i**. $M_1(A) = Z_{1,0}(A) = 48p^2 - 16p + 16$,
**ii**. $M_2(A) = \frac{1}{2}Z_{1,1}(A) = 96p^2 - 64p + 40$,
**iii**. $M_3(A) = Z_{2,0}(A) = 192p^2 - 96p + 64$,
**iv**. ${}^mM_2(A) = \frac{1}{2}Z_{-1,-1}(A) = \frac{3}{8}p^2 + \frac{1}{2}p + \frac{1}{4}$,
**v**. $ReZ_3(A) = Z_{2,1}(A) = 768p^2 - 640p + 352$.

**Corollary 2.3** (See [13]). Let $B \cong RHSL(p)$ be a rhombus silicate network on dimension $p \geq 3$. Then, the first general Zagreb index of B is

$Z_m(B) = p^2(2\times 3^m + 3\times 6^m) + 2p(2\times 3^m - 6^m)$.

By using Corollary 2.3, we derive the modified first ZI as follows:
${}^{m}M_1(B) = Z_{-2}(B) = \frac{11}{36}p^2 + \frac{7}{18}p.$

**Corollary 2.4** (See [13]). Let $B \cong RHSL(p)$ be a rhombus silicate network on dimension $p \geq 3$. Then, the generalized Zagreb index of B is

$$Z_{m,n}(B) = (2\times 3^{m+n})[3p^2(2^m+2^n+2^{m+n+1})+2p(2+2^m+2^n-2^{m+n+2})+2(1-2^m-2^n+2^{m+n})].$$

By using Corollary 2.4, we derive the degree of vertices based ZIs as follows:
**i**. $M_1(B) = Z_{1,0}(B) = 126p^2 - 36p$,
**ii**. $M_2(B) = \frac{1}{2}Z_{1,1}(B) = 324p^2 - 180p + 18$,
**iii**. $M_3(B) = Z_{2,0}(B) = 702p^2 - 324p$,
**iv**. ${}^{m}M_2(B) = \frac{1}{2}Z_{-1,-1}(B) = \frac{1}{2}p^2 + \frac{4}{9}p + \frac{1}{18}$,
**v**. $ReZ_3(B) = Z_{2,1}(B) = 3564p^2 - 2592p + 324$.

## 3. Under Study Molecular-Networks

In this section, we study molecular networks which are rhombus oxide and rhombus silicate. The silicate is the most interesting, most wonderful, most complicated and the largest class of minerals. $SiO_4$ tetrahedron is used as the basic chemical unit of silicate. So, $SiO_4$ is the mixture of sand and metal oxide. In graph theory (or chemistry), we represent centre vertices and corner vertices of silicate with silicon nodes (or silicon ions) and oxygen nodes (or oxygen ions), respectively. A silicate sheet is a ring of tetrahedron which is attached with sharing oxygen nodes to other rings in a two dimensional plane. As such silicate sheets are recognized sheet-like networks. Some more well known networks of silicate are pyrosilicate, orthosilicate, chain silicate, cyclic silicate and sheet silicate. We can easily develop rhombus oxide networks by the deletion of all silicon ions from rhombus silicate networks. The set of orders and sizes of rhombus oxide and rhombus silicate networks are $\{3p^2+2p, 6p^2\}$ and $\{5p^2+2p, 12p^2\}$ respectively. For more discussion, see Figures 1 and 2.

### 3.1. Partitions of Rhombus Oxide Network ($RHOX(p)$):

Let $A \cong RHOX(p)$ be the rhombus oxide network of dimensions $p$, see Figure 1.

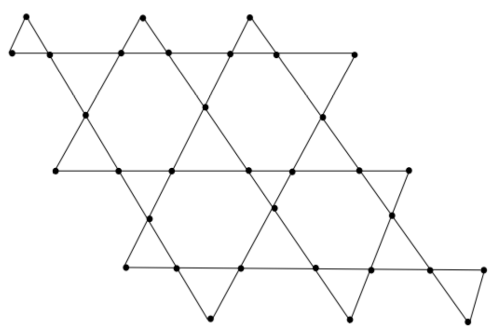

**Figure 1.** Rhombus oxide network ($RHOX(p) \cong A$) on dimension $p = 3$.

The partition of $A$ according to vertex set $V(A)$ and edge set $E(A)$. We see that each vertex of degree and connection number sets are $\{2,3\}$ and $\{2,4,6,8\}$ respectively. We have $V_1 = \{\lambda \in V(A)|d(\lambda) = 2\}$ and $V_2 = \{\lambda \in V(A)|d(\lambda) = 4\}$, where $|V_1| = 4p$ and $|V_2| = 3p^2 - 2p$. Therefore, $|V(A)| = |V_1| + |V_2| = 3p^2 + 2p$. The partition of vertices according to connection numbers are $V_1^c = \{\lambda \in V(A)|\tau(\lambda) = 2\}$, $V_2^c = \{\lambda \in$

$V(A)|\tau(\lambda) = 4\}$, $V_3^c = \{\lambda \in V(A)|\tau(\lambda) = 6\}$ and $V_4^c = \{\lambda \in V(A)|\tau(\lambda) = 8\}$, where $|V_1^c| = 4$, $|V_2^c| = 4p$, $|V_3^c| = 8p-12$ and $|V_4^c| = 3p^2-10p+8$. Therefore, $|V(A)| = |V_1^c|+|V_2^c|+|V_3^c|+|V_4^c| = 3p^2+2p$. The partition of vertices according to both degree and connection numbers are $V_{d,\tau} = V_{2,2} = \{\lambda \in V(A)|d(\lambda) = 2, \tau(\lambda) = 2\}$, $V_{2,4} = \{\lambda \in V(A)|d(\lambda) = 2, \tau(\lambda) = 4\}$, $V_{4,4} = \{\lambda \in V(A)|d(\lambda) = 4, \tau(\lambda) = 4\}$, $V_{4,6} = \{\lambda \in V(A)|d(\lambda) = 4, \tau(\lambda) = 6\}$ and $V_{4,8} = \{\lambda \in V(A)|d(\lambda) = 4, \tau(\lambda) = 8\}$, where $|V_{2,2}| = 4$, $|V_{2,4}| = 8$, $|V_{4,4}| = 4$, $|V_{4,6}| = 8p-12$ and $|V_{4,8}| = 3p^2 - 10p + 8$. Therefore, $|V(A)| = |V_{2,2}| + |V_{2,4}| + |V_{4,4}| + |V_{4,6}| + |V_{4,8}| = 3p^2 - 2p + 12$. The partitions of network $A$ vertices are presented in the following Tables 8, 9 and 10.

Table 8: The partitions of network $A$ vertices with respect to degree.

| $\boldsymbol{V_d}$ | 2 | 4 |
|---|---|---|
| $\lvert V_d\rvert$ | 4p | $3p^2 - 2p$ |

Table 9: The partitions of network $A$ vertices with respect to connection number.

| $\boldsymbol{V_\tau}$ | 2 | 4 | 6 | 8 |
|---|---|---|---|---|
| $\lvert V_\tau\rvert$ | 4 | 4p | $8p - 12$ | $3p^2 - 10p + 8$ |

Table 10: The partitions of network $A$ vertices with respect to degree and connection number.

| $\boldsymbol{V_{d,\tau}}$ | 2,2 | 2,4 | 4,4 | 4,6 | 4,8 |
|---|---|---|---|---|---|
| $\lvert V_\tau\rvert$ | 4 | 8 | 4 | $8p - 12$ | $3p^2 - 10p + 8$ |

Now, there are eight types partitions of edge set $E(A)$ with respect to connection number as $E_{2,2}^c = \{\kappa\lambda \in E(A)|\tau(\kappa) = 2, \tau(\lambda) = 2\}$, $E_{2,4}^c = \{\kappa\lambda \in E(A)|\tau(\kappa) = 2, \tau(\lambda) = 4\}$, $E_{4,4}^c = \{\kappa\lambda \in E(A)|\tau(e) = 4, \tau(\lambda) = 4\}$, $E_{4,6}^{c1} = \{\kappa\lambda \in E(A)|\tau(\kappa) = 4, \tau(j) = 6\}$, $E_{4,6}^{c2} = \{\kappa\lambda \in E(A)|\tau(\kappa) = 4, \tau(\lambda) = 6\}$, $E_{6,6}^c = \{\kappa\lambda \in E(A)|\tau(\kappa) = 6, \tau(\lambda) = 6\}$, $E_{6,8}^c = \{\kappa\lambda \in E(A)|\tau(\kappa) = 6, \tau(\lambda) = 8\}$ and $E_{8,8}^c = \{\kappa\lambda \in E(A)|\tau(\kappa) = 8, \tau(\lambda) = 8\}$, where $|E_{2,2}^c| = 2$, $|E_{2,4}^c| = 4$, $|E_{4,4}^c| = 4$, $|E_{4,6}^{c1}| = 8p - 12$, $|E_{4,6}^{c2}| = 8$, $|E_{6,6}^c| = 8p - 14$, $|E_{6,8}^c| = 8p - 16$ and $|E_{8,8}^c| = 6p^2-24p+24$. Therefore, $|E(A)| = |E_{2,2}^c|+|E_{2,4}^c|+|E_{4,4}^c|+|E_{4,6}^{c1}|+|E_{4,6}^{c2}|+|E_{6,6}^c|+|E_{6,8}^c|+|E_{8,8}^c| = 6p^2$. This edge partitions with respect to connection number is shown in Table 11.

**3.2. Partitions of Rhombus Silicate Network ($RHSL(p)$):**

Let $B \cong RHSL(p)$ be the rhombus silicate network of dimensions $p$, see Figure 2.

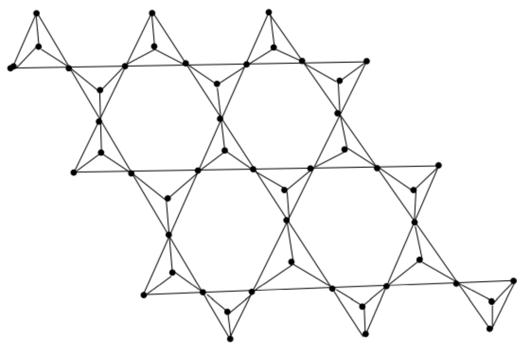

**Figure 2.** Rhombus silicate network ($RHSL(p) \cong B$) on dimension $p = 3$.

The partition of $B$ according to vertex set $V(B)$ and edge set $E(B)$. We see that each vertex of degree and connection number sets are $\{3, 6\}$ and $\{3, 6, 9, 12\}$ respectively. We have $V_1 = \{\lambda \in V(B)|d(\lambda) = 3\}$ and $V_2 = \{\lambda \in V(B)|d(\lambda) = 6\}$, where $|V_1| = 2p^2 + 4p$ and $|V_2| = 3p^2 - 2p$. Therefore, $|V(B)| = |V_1| + |V_2| =$

Table 11: The partitions of network $A$ edges with respect to connection numbers.

| $\boldsymbol{E^c_{\tau(\kappa),\tau(\lambda)}}$ | $\boldsymbol{E^c_{2,2}}$ | $\boldsymbol{E^c_{2,4}}$ | $\boldsymbol{E^c_{4,4}}$ | $\boldsymbol{E^{c1}_{4,6}}$ |
|---|---|---|---|---|
| $\lvert E^c_{\tau(\kappa),\tau(\lambda)}\rvert$ | 2 | 4 | 4 | 8p-12 |
| $\boldsymbol{E^c_{\tau(\kappa),\tau(\lambda)}}$ | $\boldsymbol{E^{c2}_{4,6}}$ | $\boldsymbol{E^c_{6,6}}$ | $\boldsymbol{E^c_{6,8}}$ | $\boldsymbol{E^c_{8,8}}$ |
| $\lvert E^c_{\tau(\kappa),\tau(\lambda)}\rvert$ | 8 | $8p-14$ | $8p-16$ | $6p^2-24p+24$ |

$5p^2+2p$. The partition of vertices according to connection numbers are $V^c_1=\{\lambda\in V(B)|\tau(\lambda)=3\}$, $V^c_2=\{\lambda\in V(B)|\tau(\lambda)=6\}$, $V^c_3=\{\lambda\in V(B)|\tau(\lambda)=9\}$ and $V^c_4=\{\lambda\in V(B)|\tau(\lambda)=12\}$, where $|V^c_1|=6$, $|V^c_2|=8p-4$, $|V^c_3|=2p^2+4p-10$ and $|V^c_4|=3p^2-10p+8$. Therefore, $|V(B)|=|V^c_1|+|V^c_2|+|V^c_3|+|V^c_4|=5p^2+2p$. The partition of vertices according to both degree and connection numbers are $V_{d,\tau}=V_{3,3}=\{\lambda\in V(B)|d(\lambda)=3,\tau(\lambda)=3\}$, $V_{3,6}=\{\lambda\in V(B)|d(\lambda)=3,\tau(\lambda)=6\}$, $V_{3,9}=\{\lambda\in V(B)|d(\lambda)=3,\tau(\lambda)=9\}$, $V_{6,6}=\{\lambda\in V(B)|d(\lambda)=6,\tau(\lambda)=6\}$, $V_{6,9}=\{\lambda\in V(B)|d(\lambda)=6,\tau(\lambda)=9\}$ and $V_{6,12}=\{\lambda\in V(B)|d(\lambda)=6,\tau(\lambda)=12\}$, where $|V_{3,3}|=6$, $|V_{3,6}|=8p-8$, $|V_{3,9}|=8p-16$, $|V_{6,6}|=4$, $|V_{6,9}|=8p-12$ and $|V_{6,12}|=3p^2-10p+8$. Therefore, $|V(B)|=|V_{3,3}|+|V_{3,6}|+|V_{3,9}|+|V_{6,6}|+|V_{6,9}|+|V_{6,12}|=3p^2+14p-18$. The partitions of network B verticesThese vertex partitions are presented in the following Tables 12, 13 and 14.

Table 12: The partitions of network $B$ vertices with respect to degree.

| $\boldsymbol{V_d}$ | 3 | 6 |
|---|---|---|
| $\lvert V_d\rvert$ | $2p^2+4p$ | $3p^2-2p$ |

Table 13: The partitions of network $B$ vertices according to connection number.

| $\boldsymbol{V_\tau}$ | 3 | 6 | 9 | 12 |
|---|---|---|---|---|
| $\lvert V_\tau\rvert$ | 6 | $8p-4$ | $2p^2+4p-10$ | $3p^2-10p+8$ |

Now, there are eleven types partitions of edge set $E(B)$ according to connection number as $E^c_{3,3}=\{\kappa\lambda\in E(B)|\tau(\kappa)=3,\tau(\lambda)=3\}$, $E^c_{3,6}=\{\kappa\lambda\in E(B)|\tau(\kappa)=3,\tau(\lambda)=6\}$, $E^{c1}_{6,6}=\{\kappa\lambda\in E(B)|\tau(\kappa)=6,\tau(\lambda)=6\}$, $E^{c2}_{6,6}=\{\kappa\lambda\in E(B)|\tau(\kappa)=6,\tau(\lambda)=6\}$, $E^{c1}_{6,9}=\{\kappa\lambda\in E(B)|\tau(\kappa)=6,\tau(\lambda)=9\}$, $E^{c2}_{6,9}=\{\kappa\lambda\in E(B)|\tau(\kappa)=6,\tau(\lambda)=9\}$, $E^{c1}_{9,9}=\{\kappa\lambda\in E(B)|\tau(\kappa)=9,\tau(\lambda)=9\}$, $E^{c2}_{9,9}=\{\kappa\lambda\in E(B)|\tau(\kappa)=9,\tau(\lambda)=9\}$, $E^{c1}_{9,12}=\{\kappa\lambda\in E(B)|\tau(\kappa)=9,\tau(\lambda)=12\}$, $E^{c2}_{9,12}=\{\kappa\lambda\in E(B)|\tau(\kappa)=9,\tau(\lambda)=12\}$ and $E^c_{12,12}=\{\kappa\lambda\in E(B)|\tau(\kappa)=12,\tau(\lambda)=12\}$, where $|E^c_{3,3}|=6$, $|E^c_{3,6}|=6$, $|E^{c1}_{6,6}|=8$, $|E^{c2}_{6,6}|=4p-4$, $|E^{c1}_{6,9}|=8$, $|E^{c2}_{6,9}|=16p-22$, $|E^{c1}_{9,9}|=8p-12$, $|E^{c2}_{9,9}|=8p-14$, $|E^{c1}_{9,12}|=8p-16$, $|E^{c2}_{9,12}|=6p^2-20p+16$ and $|E^c_{12,12}|=6p^2-24p+24$. Therefore, $|E(B)|=|E^c_{3,3}|+|E^c_{3,6}|+|E^{c1}_{6,6}|+|E^{c2}_{6,6}|+|E^{c1}_{6,9}|+|E^{c2}_{6,9}|+|E^{c1}_{9,9}|+|E^{c2}_{9,9}|+|E^{c1}_{9,12}|+|E^{c2}_{9,12}|+|E^c_{12,12}|=12p^2$. This edge partitions according to connection number is shown in Table 15.

## 4. Main Results under Rhombus Oxide Network ($\boldsymbol{RHOX(p)}$)

This section computes the main results for the first general Zagreb connection index, first, second, third and fourth generalized Zagreb connection indices of rhombus oxide network. We also compute first, second & modified first, second, third and fourth Zagreb connection indices of rhombus oxide network $RHOX(p)$ on dimension $p\geq 3$.

**Theorem 4.1.**
Let $A\cong RHOX(p)$ be a rhombus oxide network on dimension $p\geq 3$. Then, the first general Zagreb connection index of network $A$ is

$$G_{m,n}(A)=4(2^m\times 2^n)+8(2^m\times 4^n)+4(4^m\times 4^n)+(8p-12)(4^m\times 6^n)+(3p^2-10p+8)(4^m\times 8^n).$$

Table 14: The partitions of network $B$ vertices with respect to degree and connection number.

| $V_{d,\tau}$ | 3,3 | 3,6 | 3,9 | 6,6 | 6,9 | 6,12 |
|---|---|---|---|---|---|---|
| $\|V_\tau\|$ | 6 | $8p-8$ | $8p-16$ | 4 | $8p-12$ | $3p^2-10p+8$ |

Table 15: The partitions of network $B$ edges with respect to connection numbers.

| $E^c_{\tau(\kappa),\tau(\lambda)}$ | $\|E^c_{\tau(\kappa),\tau(\lambda)}\|$ | $E^c_{\tau(\kappa),\tau(\lambda)}$ | $\|E^c_{\tau(\kappa),\tau(\lambda)}\|$ |
|---|---|---|---|
| $E^c_{3,3}$ | 6 | $E^{c1}_{9,9}$ | $8p-12$ |
| $E^c_{3,6}$ | 6 | $E^{c2}_{9,9}$ | $8p-14$ |
| $E^{c1}_{6,6}$ | 8 | $E^{c1}_{9,12}$ | $8p-16$ |
| $E^{c2}_{6,6}$ | 4p-4 | $E^{c2}_{9,12}$ | $6p^2-20p+16$ |
| $E^{c1}_{6,9}$ | 8 | $E^c_{12,12}$ | $6p^2-24p+24$ |
| $E^{c2}_{6,9}$ | $16p-22$ | - | - |

**Proof.**By definition,

$$G_{m,n}(Q)=\sum_{\lambda\in V(Q)}[d(\lambda)^m\times\tau(\lambda)^n]$$

$$G_{m,n}(A)=\sum_{\lambda\in V_{2,2}}[d(\lambda)^m\times\tau(\lambda)^n]+\sum_{\lambda\in V_{2,4}}[d(\lambda)^m\times\tau(\lambda)^n]+\sum_{\lambda\in V_{4,4}}[d(\lambda)^m\times\tau(\lambda)^n]$$

$$+\sum_{\lambda\in V_{4,6}}[d(\lambda)^m\times\tau(\lambda)^n]+\sum_{\lambda\in V_{4,8}}[d(\lambda)^m\times\tau(\lambda)^n]$$

By using Table 10, we have

$$=4(2^m\times2^n)+8(2^m\times4^n)+4(4^m\times4^n)+(8p-12)(4^m\times6^n)+(3p^2-10p+8)(4^m\times8^n).$$

**Corollary 4.1.** By putting $m=0$ and $n=2$ in the Theorem 4.1, we get the first Zagreb connection index as

$$ZC_1(A)=G_{0,2}(A)=192p^2-352p+288.$$

**Theorem 4.2.**
Let $A\cong RHOX(p)$ be a rhombus oxide network on dimension $p\geq3$. Then, the generalized first Zagreb connection index of network $A$ is

$$C_{m,n}(A)=2(2^m2^n+2^m2^n)+4(2^m4^n+4^m2^n)+4(4^m4^n+4^m4^n)+(8p-12)(4^m6^n+6^m4^n)+8(4^m6^n+6^m4^n)$$

$$+(8p-14)(6^m6^n+6^m6^n)+(8p-16)(6^m8^n+8^m6^n)+(6p^2-24p+24)(8^m8^n+8^m8^n).$$

**Proof.**By definition,

$$C_{m,n}(Q)=\sum_{\kappa\lambda\in E(Q)}[\tau(\kappa)^m\tau(\lambda)^n+\tau(\lambda)^m\tau(\kappa)^n]$$

$$C_{m,n}(A)=\sum_{\kappa\lambda\in E^c_{2,2}}[\tau(\kappa)^m\tau(\lambda)^n+\tau(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda\in E^c_{2,4}}[\tau(\kappa)^m\tau(\lambda)^n+\tau(\lambda)^m\tau(\kappa)^n]$$

$$+\sum_{\kappa\lambda\in E^c_{4,4}}[\tau(\kappa)^m\tau(\lambda)^n+\tau(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda\in E^{c1}_{4,6}}[\tau(\kappa)^m\tau(\lambda)^n+\tau(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda\in E^{c2}_{4,6}}[\tau(\kappa)^m\tau(\lambda)^n+\tau(\lambda)^m\tau(\kappa)^n]$$

$$+\sum_{\kappa\lambda\in E^c_{6,6}}[\tau(\kappa)^m\tau(\lambda)^n+\tau(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda\in E^c_{6,8}}[\tau(\kappa)^m\tau(\lambda)^n+\tau(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda\in E^c_{8,8}}[\tau(\kappa)^m\tau(\lambda)^n+\tau(\lambda)^m\tau(\kappa)^n]$$

By using Table 11, we have

$$=2(2^m2^n+2^m2^n)+4(2^m4^n+4^m2^n)+4(4^m4^n+4^m4^n)+(8p-12)(4^m6^n+6^m4^n)+8(4^m6^n+6^m4^n)$$
$$+(8p-14)(6^m6^n+6^m6^n)+(8p-16)(6^m8^n+8^m6^n)+(6p^2-24p+24)(8^m8^n+8^m8^n).$$

**Corollary 4.2.** By putting $m=1$ and $n=1$ in the Theorem 4.2, we get the second Zagreb connection index as

$$ZC_2(A)=\frac{1}{2}C_{1,1}(A)=384p^2-672p+272.$$

**Corollary 4.3.** By putting $m=1$ and $n=0$ in the Theorem 4.2, we get the modified first Zagreb connection index as

$$ZC^*_1(A)=C_{1,0}(A)=96p^2-96p+16.$$

**Theorem 4.3.**
Let $A\cong RHOX(p)$ be a rhombus oxide network on dimension $p\geq 3$. Then, the generalized second Zagreb connection index of network $A$ is

$$C^2_{m,n}(A)=2(2^m2^n+2^m2^n)+4(2^m4^n+4^m2^n)+4(2^m4^n+4^m4^n)+(8p-12)(2^m6^n+4^m4^n)+8(4^m6^n+4^m4^n)$$
$$+(8p-14)(4^m6^n+4^m6^n)+(8p-16)(4^m8^n+4^m6^n)+(6p^2-24p+24)(4^m8^n+4^m8^n).$$

**Proof.** By definition,

$$C^2_{m,n}(Q)=\sum_{\kappa\lambda\in E(Q)}[d(\kappa)^m\tau(\lambda)^n+d(\lambda)^m\tau(\kappa)^n]$$

$$C^2_{m,n}(A)=\sum_{\kappa\lambda\in E^c_{2,2}}[d(\kappa)^m\tau(\lambda)^n+d(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda\in E^c_{2,4}}[d(\kappa)^m\tau(\lambda)^n+d(\lambda)^m\tau(\kappa)^n]$$

$$+\sum_{\kappa\lambda\in E^c_{4,4}}[d(\kappa)^m\tau(\lambda)^n+d(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda\in E^{c1}_{4,6}}[d(\kappa)^m\tau(\lambda)^n+d(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda\in E^{c2}_{4,6}}[d(\kappa)^m\tau(\lambda)^n+d(\lambda)^m\tau(\kappa)^n]$$

$$+\sum_{\kappa\lambda\in E^c_{6,6}}[d(\kappa)^m\tau(\lambda)^n+d(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda\in E^c_{6,8}}[d(\kappa)^m\tau(\lambda)^n+d(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda\in E^c_{8,8}}[d(\kappa)^m\tau(\lambda)^n+d(\lambda)^m\tau(\kappa)^n]$$

By using Table 11, we have

$$=2(2^m2^n+2^m2^n)+4(2^m4^n+4^m2^n)+4(2^m4^n+4^m4^n)+(8p-12)(2^m6^n+4^m4^n)+8(4^m6^n+4^m4^n)$$
$$+(8p-14)(4^m6^n+4^m6^n)+(8p-16)(4^m8^n+4^m6^n)+(6p^2-24p+24)(4^m8^n+4^m8^n).$$

**Corollary 4.4.** By putting $m=1$ and $n=1$ in the Theorem 4.3, we get the modified second Zagreb connection index as

$$ZC^*_2(A)=C^2_{1,1}(A)=384p^2-480p+128.$$

**Theorem 4.4.**
Let $A\cong RHOX(p)$ be a rhombus oxide network on dimension $p\geq 3$. Then, the generalized third Zagreb connection index of network $A$ is

$$C^3_{m,n}(A)=2(2^m2^n+2^m2^n)+4(2^m2^n+4^m4^n)+4(2^m4^n+4^m4^n)+(8p-12)(2^m4^n+4^m6^n)+8(4^m4^n+4^m6^n)$$

$+(8p-14)(4^m6^n+4^m6^n)+(8p-16)(4^m6^n+4^m8^n)+(6p^2-24p+24)(4^m8^n+4^m8^n)$.

**Proof.** By definition,

$$C^3_{m,n}(Q)=\sum_{\kappa\lambda\in E(Q)}[d(\kappa)^m\tau(\kappa)^n+d(\lambda)^m\tau(\lambda)^n]$$

$$C^3_{m,n}(A)=\sum_{\kappa\lambda\in E^c_{2,2}}[d(\kappa)^m\tau(\kappa)^n+d(\lambda)^m\tau(\lambda)^n]+\sum_{\kappa\lambda\in E^c_{2,4}}[d(\kappa)^m\tau(\kappa)^n+d(\lambda)^m\tau(\lambda)^n]$$

$$+\sum_{\kappa\lambda\in E^c_{4,4}}[d(\kappa)^m\tau(\kappa)^n+d(\lambda)^m\tau(\lambda)^n]+\sum_{\kappa\lambda\in E^{c1}_{4,6}}[d(\kappa)^m\tau(\kappa)^n+d(\lambda)^m\tau(\lambda)^n]+\sum_{\kappa\lambda\in E^{c2}_{4,6}}[d(\kappa)^m\tau(\kappa)^n+d(\lambda)^m\tau(\lambda)^n]$$

$$+\sum_{\kappa\lambda\in E^c_{6,6}}[d(\kappa)^m\tau(\kappa)^n+d(\lambda)^m\tau(\lambda)^n]+\sum_{\kappa\lambda\in E^c_{6,8}}[d(\kappa)^m\tau(\kappa)^n+d(\lambda)^m\tau(\lambda)^n]+\sum_{\kappa\lambda\in E^c_{8,8}}[d(\kappa)^m\tau(\kappa)^n+d(\lambda)^m\tau(\lambda)^n]$$

By using Table 11, we have

$$=2(2^m2^n+2^m2^n)+4(2^m2^n+4^m4^n)+4(2^m4^n+4^m4^n)+(8p-12)(2^m4^n+4^m6^n)+8(4^m4^n+4^m6^n)$$
$$+(8p-14)(4^m6^n+4^m6^n)+(8p-16)(4^m6^n+4^m8^n)+(6p^2-24p+24)(4^m8^n+4^m8^n).$$

**Corollary 4.5.** By putting $m=1$ and $n=1$ in the Theorem 4.4, we get the modified third Zagreb connection index as

$$ZC^*_3(A)=C^3_{1,1}(A)=384p^2-448p+96.$$

**Theorem 4.5.**
Let $A\cong RHOX(p)$ be a rhombus oxide network on dimension $p\geq 3$. Then, the generalized fourth Zagreb connection index of network $A$ is

$$C^4_{m,n}(A)=2(2^m2^n\times2^m2^n)+4(2^m2^n\times4^m4^n)+4(2^m4^n\times4^m4^n)+(8p-12)(2^m4^n\times4^m6^n)+8(4^m4^n\times4^m6^n)$$
$$+(8p-14)(4^m6^n\times4^m6^n)+(8p-16)(4^m6^n\times4^m8^n)+(6p^2-24p+24)(4^m8^n\times4^m8^n).$$

**Proof.** By definition,

$$C^4_{m,n}(Q)=\sum_{\kappa\lambda\in E(Q)}[d(\kappa)^m\tau(\kappa)^n\times d(\lambda)^m\tau(\lambda)^n]$$

$$C^4_{m,n}(A)=\sum_{\kappa\lambda\in E^c_{2,2}}[d(\kappa)^m\tau(\kappa)^n\times d(\lambda)^m\tau(\lambda)^n]+\sum_{\kappa\lambda\in E^c_{2,4}}[d(\kappa)^m\tau(\kappa)^n\times d(\lambda)^m\tau(\lambda)^n]$$

$$+\sum_{\kappa\lambda\in E^c_{4,4}}[d(\kappa)^m\tau(\kappa)^n\times d(\lambda)^m\tau(\lambda)^n]+\sum_{\kappa\lambda\in E^{c1}_{4,6}}[d(\kappa)^m\tau(\kappa)^n\times d(\lambda)^m\tau(\lambda)^n]+\sum_{\kappa\lambda\in E^{c2}_{4,6}}[d(\kappa)^m\tau(\kappa)^n\times d(\lambda)^m\tau(\lambda)^n]$$

$$+\sum_{\kappa\lambda\in E^c_{6,6}}[d(\kappa)^m\tau(\kappa)^n\times d(\lambda)^m\tau(\lambda)^n]+\sum_{\kappa\lambda\in E^c_{6,8}}[d(\kappa)^m\tau(\kappa)^n\times d(\lambda)^m\tau(\lambda)^n]+\sum_{\kappa\lambda\in E^c_{8,8}}[d(\kappa)^m\tau(\kappa)^n\times d(\lambda)^m\tau(\lambda)^n]$$

By using Table 11, we have

$$=2(2^m2^n\times2^m2^n)+4(2^m2^n\times4^m4^n)+4(2^m4^n\times4^m4^n)+(8p-12)(2^m4^n\times4^m6^n)+8(4^m4^n\times4^m6^n)$$
$$+(8p-14)(4^m6^n\times4^m6^n)+(8p-16)(4^m6^n\times4^m8^n)+(6p^2-24p+24)(4^m8^n\times4^m8^n).$$

**Corollary 4.6.** By putting $m=1$ and $n=1$ in the Theorem 4.5, we get the modified fourth Zagreb connection index as

$$ZC^*_4(A)=C^4_{1,1}(A)=6144p^2-12288p+5792.$$

## 5. Main Results under Rhombus Silicate Network ($RHSL(p)$)

This section computes the main results for the first general Zagreb connection index (ZCI), first, second, third and fourth generalized Zagreb connection indices of rhombus silicate network. We also compute first, second & modified first, second, third and fourth Zagreb connection indices of rhombus silicate network $RHSL(p)$ on dimension $p \geq 3$.

**Theorem 5.1.**
Let $B \cong RHSL(p)$ be a rhombus silicate network on dimension $p \geq 3$. Then, the first general Zagreb connection index of network $B$ is

$$G_{m,n}(B) = 6(3^m{\times}3^n){+}(8p{-}8)(3^m{\times}6^n){+}(8p{-}16)(3^m{\times}9^n){+}4(6^m{\times}6^n){+}(8p{-}12)(6^m{\times}9^n)$$

$$+(3p^2{-}10p{+}8)(6^m{\times}12^n).$$

**Proof.** By definition,

$$G_{m,n}(Q) = \sum_{\lambda \in V(Q)} [d(\lambda)^m{\times}\tau(\lambda)^n]$$

$$G_{m,n}(B) = \sum_{\lambda \in V_{3,3}} [d(\lambda)^m{\times}\tau(\lambda)^n]+\sum_{\lambda \in V_{3,6}} [d(\lambda)^m{\times}\tau(\lambda)^n]+\sum_{\lambda \in V_{3,9}} [d(\lambda)^m{\times}\tau(\lambda)^n]+\sum_{\lambda \in V_{6,6}} [d(\lambda)^m{\times}\tau(\lambda)^n]$$

$$+\sum_{\lambda \in V_{6,9}} [d(\lambda)^m{\times}\tau(\lambda)^n]+\sum_{\lambda \in V_{6,12}} [d(\lambda)^m{\times}\tau(\lambda)^n]$$

By using Table 14, we have

$$= 6(3^m{\times}3^n){+}(8p{-}8)(3^m{\times}6^n){+}(8p{-}16)(3^m{\times}9^n){+}4(6^m{\times}6^n){+}(8p{-}12)(6^m{\times}9^n){+}(3p^2{-}10p{+}8)(6^m{\times}12^n).$$

**Corollary 5.1.** By putting $m = 0$ and $n = 2$ in the Theorem 5.1, we get the first Zagreb connection index as

$$ZC_1(B) = G_{0,2}(B) = 432p^2{+}144p{-}1206.$$

**Theorem 5.2.**
Let $B \cong RHSL(p)$ be a rhombus silicate network on dimension $p \geq 3$. Then, the generalized first Zagreb connection index of network $B$ is

$$C_{m,n}(B) = 6(3^m3^n{+}3^m3^n){+}6(3^m6^n{+}6^m3^n){+}8(6^m6^n{+}6^m6^n){+}(8p{-}4)(6^m6^n{+}6^m6^n){+}8(6^m9^n{+}9^m6^n)$$

$$+(16p{-}22)(6^m9^n{+}9^m6^n){+}(8p{-}12)(9^m9^n{+}9^m9^n){+}(8p{-}14)(9^m9^n{+}9^m9^n){+}(8p{-}16)(9^m12^n{+}12^m9^n)$$

$$+(6p^2{-}20p{+}16)(9^m12^n{+}12^m9^n){+}(6p^2{-}24p{+}24)(12^m12^n{+}12^m12^n).$$

**Proof.** By definition,

$$C_{m,n}(Q) = \sum_{\kappa\lambda \in E(Q)} [\tau(\kappa)^m\tau(\lambda)^n{+}\tau(\lambda)^m\tau(\kappa)^n]$$

$$C_{m,n}(B) = \sum_{\kappa\lambda \in E^c_{3,3}} [\tau(\kappa)^m\tau(\lambda)^n{+}\tau(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda \in E^c_{3,6}} [\tau(\kappa)^m\tau(\lambda)^n{+}\tau(\lambda)^m\tau(\kappa)^n]$$

$$+\sum_{\kappa\lambda \in E^{c1}_{6,6}} [\tau(\kappa)^m\tau(\lambda)^n{+}\tau(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda \in E^{c2}_{6,6}} [\tau(\kappa)^m\tau(\lambda)^n{+}\tau(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda \in E^{c1}_{6,9}} [\tau(\kappa)^m\tau(\lambda)^n{+}\tau(\lambda)^m\tau(\kappa)^n]$$

$$+\sum_{\kappa\lambda\in E^{c2}_{6,9}}[\tau(\kappa)^m\tau(\lambda)^n+\tau(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda\in E^{c1}_{9,9}}[\tau(\kappa)^m\tau(\lambda)^n+\tau(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda\in E^{c2}_{9,9}}[\tau(\kappa)^m\tau(\lambda)^n+\tau(\lambda)^m\tau(\kappa)^n]$$

$$+\sum_{\kappa\lambda\in E^{c1}_{9,12}}[\tau(\kappa)^m\tau(\lambda)^n+\tau(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda\in E^{c2}_{9,12}}[\tau(\kappa)^m\tau(\lambda)^n+\tau(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda\in E^{c}_{12,12}}[\tau(\kappa)^m\tau(\lambda)^n+\tau(\lambda)^m\tau(\kappa)^n]$$

By using Table 15, we have

$$=6(3^m3^n+3^m3^n)+6(3^m6^n+6^m3^n)+8(6^m6^n+6^m6^n)+(4p-4)(6^m6^n+6^m6^n)+8(6^m9^n+9^m6^n)+(16p-22)$$
$$(6^m9^n+9^m6^n)+(8p-12)(9^m9^n+9^m9^n)+(8p-14)(9^m9^n+9^m9^n)+(8p-16)(9^m12^n+12^m9^n)+(6p^2-20p+16)$$
$$(9^m12^n+12^m9^n)+(6p^2-24p+24)(12^m12^n+12^m12^n).$$

**Corollary 5.2.** By putting $m=1$ and $n=1$ in the Theorem 5.2, we get the second Zagreb connection index as

$$ZC_2(B)=\frac{1}{2}C_{1,1}(A)=1512p^2-2448p+900.$$

**Corollary 5.3.** By putting $m=1$ and $n=0$ in the Theorem 5.2, we get the modified first Zagreb connection index as

$$ZC^*_1(B)=C_{1,0}(B)=270p^2-252p+36.$$

**Theorem 5.3.**
Let $B\cong RHSL(p)$ be a rhombus silicate network on dimension $p\geq 3$. Then, the generalized second Zagreb connection index of network $B$ is

$$C^2_{m,n}(B)=6(3^m3^n+3^m3^n)+6(3^m6^n+6^m3^n)+8(3^m6^n+6^m6^n)+(4p-4)(3^m6^n+3^m6^n)+8(6^m9^n+6^m6^n)$$
$$+(16p-22)(3^m9^n+6^m6^n)+(8p-12)(6^m9^n+3^m9^n)+(8p-14)(6^m9^n+6^m9^n)+(8p-16)(6^m12^n+6^m9^n)$$
$$+(6p^2-20p+16)(3^m12^n+6^m9^n)+(6p^2-24p+24)(6^m12^n+6^m12^n).$$

**Proof.** By definition,

$$C^2_{m,n}(Q)=\sum_{\kappa\lambda\in E(Q)}[d(\kappa)^m\tau(\lambda)^n+d(\lambda)^m\tau(\kappa)^n]$$

$$C^2_{m,n}(B)=\sum_{\kappa\lambda\in E^{c}_{3,3}}[d(\kappa)^m\tau(\lambda)^n+d(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda\in E^{c}_{3,6}}[d(\kappa)^m\tau(\lambda)^n+d(\lambda)^m\tau(\kappa)^n]$$

$$+\sum_{\kappa\lambda\in E^{c1}_{6,6}}[d(\kappa)^m\tau(\lambda)^n+d(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda\in E^{c2}_{6,6}}[d(\kappa)^m\tau(\lambda)^n+d(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda\in E^{c1}_{6,9}}[d(\kappa)^m\tau(\lambda)^n+d(\lambda)^m\tau(\kappa)^n]$$

$$+\sum_{\kappa\lambda\in E^{c2}_{6,9}}[d(\kappa)^m\tau(\lambda)^n+d(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda\in E^{c1}_{9,9}}[d(\kappa)^m\tau(\lambda)^n+d(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda\in E^{c2}_{9,9}}[d(\kappa)^m\tau(\lambda)^n+d(\lambda)^m\tau(\kappa)^n]$$

$$+\sum_{\kappa\lambda\in E^{c1}_{9,12}}[d(\kappa)^m\tau(\lambda)^n+d(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda\in E^{c2}_{9,12}}[d(\kappa)^m\tau(\lambda)^n+d(\lambda)^m\tau(\kappa)^n]+\sum_{\kappa\lambda\in E^{c}_{12,12}}[d(\kappa)^m\tau(\lambda)^n+d(\lambda)^m\tau(\kappa)^n]$$

By using Table 15, we have

$$=6(3^m3^n+3^m3^n)+6(3^m6^n+6^m3^n)+8(3^m6^n+6^m6^n)+(4p-4)(3^m6^n+3^m6^n)+8(6^m9^n+6^m6^n)+(16p-22)$$
$$(3^m9^n+6^m6^n)+(8p-12)(6^m9^n+3^m9^n)+(8p-14)(6^m9^n+6^m9^n)+(8p-16)(6^m12^n+6^m9^n)$$

$+(6p^2-20p+16)(3^m12^n+6^m9^n)+(6p^2-24p+24)(6^m12^n+6^m12^n)$.

**Corollary 5.4.** By putting $m=1$ and $n=1$ in the Theorem 5.3, we get the modified second Zagreb connection index as

$$ZC_2^*(B)=C_{1,1}^2(B)=1404p^2-1584p+342.$$

**Theorem 5.4.**
Let $B\cong RHSL(p)$ be a rhombus silicate network on dimension $p\geq 3$. Then, the generalized third Zagreb connection index of network $B$ is

$$C_{m,n}^3(B)=6(3^m3^n+3^m3^n)+6(3^m3^n+6^m6^n)+8(3^m6^n+6^m6^n)+(4p-4)(3^m6^n+3^m6^n)+8(6^m6^n+6^m9^n)$$
$$+(16p-22)(3^m6^n+6^m9^n)+(8p-12)(3^m9^n+6^m9^n)+(8p-14)(6^m9^n+6^m9^n)+(8p-16)(6^m9^n+6^m12^n)$$
$$+(6p^2-20p+16)(3^m9^n+6^m12^n)+(6p^2-24p+24)(6^m12^n+6^m12^n).$$

**Proof.** By definition,

$$C_{m,n}^3(Q)=\sum_{\kappa\lambda\in E(Q)}[d(\kappa)^m\tau(\kappa)^n+d(\lambda)^m\tau(\lambda)^n]$$

$$C_{m,n}^3(B)=\sum_{\kappa\lambda\in E_{3,3}^c}[d(\kappa)^m\tau(\kappa)^n+d(\lambda)^m\tau(\lambda)^n]+\sum_{\kappa\lambda\in E_{3,6}^c}[d(\kappa)^m\tau(\kappa)^n+d(\lambda)^m\tau(\lambda)^n]$$
$$+\sum_{\kappa\lambda\in E_{6,6}^{c1}}[d(\kappa)^m\tau(\kappa)^n+d(\lambda)^m\tau(\lambda)^n]+\sum_{\kappa\lambda\in E_{6,6}^{c2}}[d(\kappa)^m\tau(\kappa)^n+d(\lambda)^m\tau(\lambda)^n]+\sum_{\kappa\lambda\in E_{6,9}^{c1}}[d(\kappa)^m\tau(\kappa)^n+d(\lambda)^m\tau(\lambda)^n]$$
$$+\sum_{\kappa\lambda\in E_{6,9}^{c2}}[d(\kappa)^m\tau(\kappa)^n+d(\lambda)^m\tau(\lambda)^n]+\sum_{\kappa\lambda\in E_{9,9}^{c1}}[d(\kappa)^m\tau(\kappa)^n+d(\lambda)^m\tau(\lambda)^n]+\sum_{\kappa\lambda\in E_{9,9}^{c2}}[d(\kappa)^m\tau(\kappa)^n+d(\lambda)^m\tau(\lambda)^n]$$
$$+\sum_{\kappa\lambda\in E_{9,12}^{c1}}[d(\kappa)^m\tau(\kappa)^n+d(\lambda)^m\tau(\lambda)^n]+\sum_{\kappa\lambda\in E_{9,12}^{c2}}[d(\kappa)^m\tau(\kappa)^n+d(\lambda)^m\tau(\lambda)^n]+\sum_{\kappa\lambda\in E_{12,12}^{c}}[d(\kappa)^m\tau(\kappa)^n+d(\lambda)^m\tau(\lambda)^n]$$

By using Table 15, we have

$$=6(3^m3^n+3^m3^n)+6(3^m3^n+6^m6^n)+8(3^m6^n+6^m6^n)+(4p-4)(3^m6^n+3^m6^n)+8(6^m6^n+6^m9^n)+(16p-22)$$
$$(3^m6^n+6^m9^n)+(8p-12)(3^m9^n+6^m9^n)+(8p-14)(6^m9^n+6^m9^n)+(8p-16)(6^m9^n+6^m12^n)+(6p^2-20p+16)$$
$$(3^m9^n+6^m12^n)+(6p^2-24p+24)(6^m12^n+6^m12^n).$$

**Corollary 5.5.** By putting $m=1$ and $n=1$ in the Theorem 5.4, we get the modified third Zagreb connection index as

$$ZC_3^*(B)=C_{1,1}^3(B)=1458p^2-1620p+342.$$

**Theorem 5.5.**
Let $B\cong RHSL(p)$ be a rhombus silicate network on dimension $p\geq 3$. Then, the generalized fourth Zagreb connection index of network $B$ is

$$C_{m,n}^4(B)=6(3^m3^n\times 3^m3^n)+6(3^m3^n\times 6^m6^n)+8(3^m6^n\times 6^m6^n)+(4p-4)(3^m6^n\times 3^m6^n)+8(6^m6^\times 6^m9^n)$$
$$+(16p-22)(3^m6^n\times 6^m9^n)+(8p-12)(3^m9^n\times 6^m9^n)+(8p-14)(6^m9^n\times 6^m9^n)+(8p-16)(6^m9^n\times 6^m12^n)$$
$$+(6p^2-20p+16)(3^m9^n\times 6^m12^n)+(6p^2-24p+24)(6^m12^n\times 6^m12^n).$$

**Proof.** By definition,

$$C^4_{m,n}(Q) = \sum_{\kappa\lambda \in E(Q)} [d(\kappa)^m \tau(\kappa)^n \times d(\lambda)^m \tau(\lambda)^n]$$

$$C^4_{m,n}(B) = \sum_{\kappa\lambda \in E^c_{3,3}} [d(\kappa)^m \tau(\kappa)^n \times d(\lambda)^m \tau(\lambda)^n] + \sum_{\kappa\lambda \in E^c_{3,6}} [d(\kappa)^m \tau(\kappa)^n \times d(\lambda)^m \tau(\lambda)^n]$$

$$+ \sum_{\kappa\lambda \in E^{c1}_{6,6}} [d(\kappa)^m \tau(\kappa)^n \times d(\lambda)^m \tau(\lambda)^n] + \sum_{\kappa\lambda \in E^{c2}_{6,6}} [d(\kappa)^m \tau(\kappa)^n \times d(\lambda)^m \tau(\lambda)^n] + \sum_{\kappa\lambda \in E^{c1}_{6,9}} [d(\kappa)^m \tau(\kappa)^n \times d(\lambda)^m \tau(\lambda)^n]$$

$$+ \sum_{\kappa\lambda \in E^{c2}_{6,9}} [d(\kappa)^m \tau(\kappa)^n \times d(\lambda)^m \tau(\lambda)^n] + \sum_{\kappa\lambda \in E^{c1}_{9,9}} [d(\kappa)^m \tau(\kappa)^n \times d(\lambda)^m \tau(\lambda)^n] + \sum_{\kappa\lambda \in E^{c2}_{9,9}} [d(\kappa)^m \tau(\kappa)^n \times d(\lambda)^m \tau(\lambda)^n]$$

$$+ \sum_{\kappa\lambda \in E^{c1}_{9,12}} [d(\kappa)^m \tau(\kappa)^n \times d(\lambda)^m \tau(\lambda)^n] + \sum_{\kappa\lambda \in E^{c2}_{9,12}} [d(\kappa)^m \tau(\kappa)^n \times d(\lambda)^m \tau(\lambda)^n] + \sum_{\kappa\lambda \in E^{c}_{12,12}} [d(\kappa)^m \tau(\kappa)^n \times d(\lambda)^m \tau(\lambda)^n]$$

By using Table 15, we have

$$= 6(3^m 3^n \times 3^m 3^n) + 6(3^m 3^n \times 6^m 6^n) + 8(3^m 6^n \times 6^m 6^n) + (4p-4)(3^m 6^n \times 3^m 6^n) + 8(6^m 6^n \times 6^m 9^n) + (16p-22)$$
$$(3^m 6^n \times 6^m 9^n) + (8p-12)(3^m 9^n \times 6^m 9^n) + (8p-14)(6^m 9^n \times 6^m 9^n) + (8p-16)(6^m 9^n \times 6^m 12^n) + (6p^2-20p+16)$$
$$(3^m 9^n \times 6^m 12^n) + (6p^2-24p+24)(6^m 12^n \times 6^m 12^n).$$

**Corollary 5.6.** By putting $m = 1$ and $n = 1$ in the Theorem 5.5, we get the modified fourth Zagreb connection index as

$$ZC^*_4(B) = C^4_{1,1}(B) = 42768p^2 - 80352p + 35478.$$

## 6. Comparison and Conclusions

In this section, we compare degree based ZIs ($M_1, M_2, M_3, {}^m M_1, {}^m M_2$ & $ReZ_3$) and connection number based ZIs ($ZC_1, ZC_2, ZC^*_1, ZC^*_2, ZC^*_3$ & $ZC^*_4$) for both rhombus oxide network ($RHOX(p) \cong A$) and rhombus silicate network ($RHSL(p) \cong B$). For this comparison, we make Tables 16 and 17 by using Corollary 2.1-2.2 and Corollary 4.1-4.6 respectively, for network $A$. We also make Table 18 and 19 by using Corollary 2.3-2.4 and Corollary 5.1-5.6 respectively, for network $B$. These Tables 16-19 are demonstrated with the help of numerical values for indicated ZIs. The graphical depictions of these indicated ZIs for both networks $A$ and $B$ are shown in Figures 3-6.

By observing Tables 16-21 and Figures 3-6, they help us to make the conclusions of the comparative study for ZIs and ZCIs of two rhombus oxide network ($RHOX \cong A$) and rhombus silicate network ($RHSL \cong B$) on dimensions $3 \leq p \leq 10$.

$(i)$ From Tables 16-19 and Figures 3-6, we observe that the level of degree and connection number based ZIs for networks $A$ and $B$ have the following orders:
*a.* $ReZ_3(A) \geq M_3(A) \geq M_2(A) \geq M_1(A) \geq^m M_1(A) \geq^m M_2(A)$,
*b.* $ZC^*_4(A) \geq ReZ_3(A) \geq ZC^*_3(A) \geq ZC^*_2(A) \geq ZC_2(A) \geq ZC_1(A) \geq ZC^*_1(A)$,
*c.* $ReZ_3(B) \geq M_3(B) \geq M_2(B) \geq M_1(B) \geq^m M_1(B) \geq^m M_2(B)$,
*d.* $ZC^*_4(B) \geq ReZ_3(B) \geq ZC^*_3(B) \geq ZC^*_2(B) \geq ZC_2(B) \geq ZC_1(B) \geq ZC^*_1(B)$.

$(ii)$ From Tables 16-19 and Figures 3-6, we observe that modified fourth ZCI ($ZC^*_4$) gives more values and most upper lines than other ZIs and ZCIs. These relations show that $ZC^*_4$ gives better results for correlation coefficient values of both networks $A$ and $B$ than $ZC_1^*$.

$(iii)$ Tables 20 and 21 show that rhombus silicate network B of dimension $p$ for the above mentioned indices has gained supreme position than rhombus oxide network $A$.

$(iv)$ Moreover, these general relations (Tables 20-21) conclude that the physicochemical properties of rhombus silicate network $B$ is more predict than rhombus oxide network $A$ on dimension $p$.

Table 16: Numerical values for ZIs of rhombus oxide network $A$.

| p | $M_1(A)$ | $M_2(A)$ | $M_3(A)$ | $^mM_1(A)$ | $^mM_2(A)$ | $ReZ_3(A)$ |
|---|---|---|---|---|---|---|
| 3 | 400 | 712 | 1504 | 7.3125 | 5.125 | 5344 |
| 4 | 720 | 1320 | 2752 | 10.5 | 8.25 | 10080 |
| 5 | 1136 | 2120 | 4384 | 14.0625 | 12.125 | 16352 |
| 6 | 1648 | 3112 | 6400 | 18 | 16.75 | 24160 |
| 7 | 2256 | 4296 | 8800 | 22.3125 | 22.125 | 33505 |
| 8 | 2960 | 5672 | 11584 | 27 | 28.25 | 44384 |
| 9 | 3760 | 7240 | 14752 | 32.0625 | 35.125 | 56800 |
| 10 | 4656 | 9000 | 18304 | 37.5 | 42.75 | 70752 |

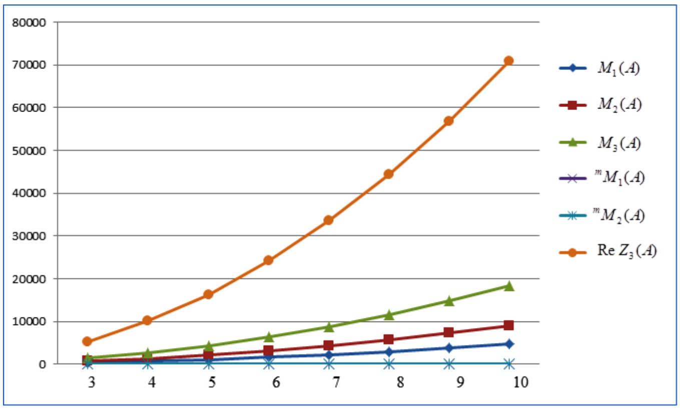


**Figure 3.** Comparisons for ZIs of network A on dimensions $3 \leq p \leq 10$.

Table 17: Numerical values for ZCIs of rhombus oxide network $A$.

| p | $ZC_1(A)$ | $ZC_2(A)$ | $ZC_1^*(A)$ | $ZC_2^*(A)$ | $ZC_3^*(A)$ | $ZC_4^*(A)$ |
|---|---|---|---|---|---|---|
| 3 | 960 | 1712 | 592 | 2144 | 2208 | 24224 |
| 4 | 1952 | 3728 | 1168 | 4352 | 4448 | 54944 |
| 5 | 3328 | 6512 | 1936 | 7328 | 7456 | 97952 |
| 6 | 5088 | 10064 | 2896 | 11072 | 11232 | 153248 |
| 7 | 7232 | 14384 | 4048 | 15584 | 15776 | 220832 |
| 8 | 9760 | 19472 | 5392 | 20864 | 21088 | 300704 |
| 9 | 12672 | 25328 | 6928 | 26912 | 27168 | 392864 |
| 10 | 15968 | 31952 | 8656 | 33728 | 34016 | 497312 |

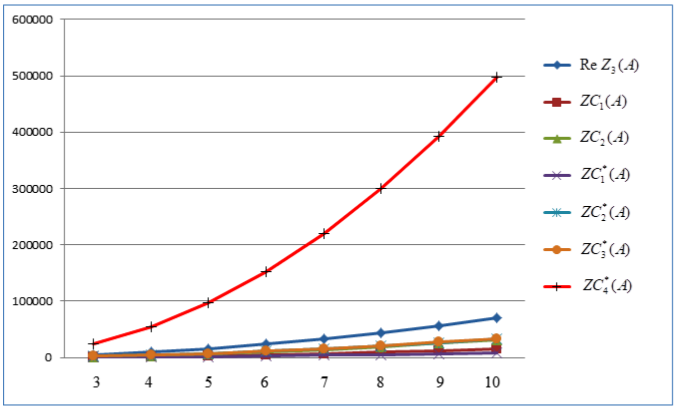


**Figure 4.** Comparisons for ZCIs of network A on dimensions $3 \leq p \leq 10$.

Table 18: Numerical values for ZIs of rhombus silicate network $B$.

| p | $\boldsymbol{M_1(B)}$ | $\boldsymbol{M_2(B)}$ | $\boldsymbol{M_3(B)}$ | $\boldsymbol{^mM_1(B)}$ | $\boldsymbol{^mM_2(B)}$ | $\boldsymbol{ReZ_3(B)}$ |
|---|---|---|---|---|---|---|
| 3 | 1026 | 2394 | 5346 | 3.9167 | 5.8889 | 24624 |
| 4 | 1872 | 4482 | 9936 | 6.4444 | 9.8333 | 46980 |
| 5 | 2970 | 7218 | 15930 | 9.5833 | 14.7778 | 76464 |
| 6 | 4320 | 10602 | 23328 | 13.3333 | 20.7222 | 113076 |
| 7 | 5922 | 14634 | 32130 | 17.6944 | 27.6667 | 156816 |
| 8 | 7776 | 19314 | 42336 | 22.6667 | 35.6111 | 207684 |
| 9 | 9882 | 24642 | 53946 | 28.25 | 44.5556 | 265680 |
| 10 | 12240 | 30618 | 66960 | 34.4444 | 54.5 | 330804 |

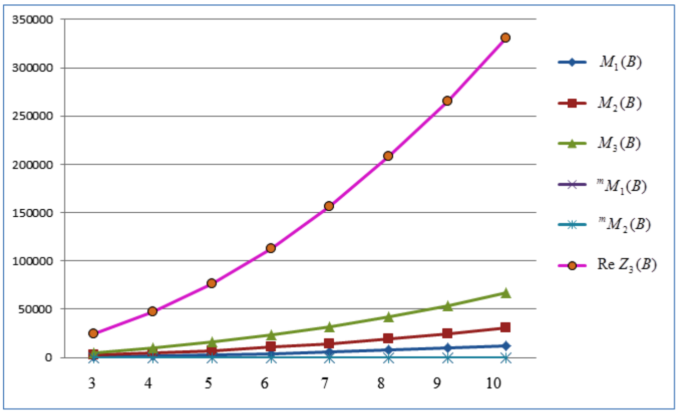


**Figure 5.** Comparisons for ZIs of network B on dimensions $3 \leq p \leq 10$.

Table 19: Numerical values for ZCIs of rhombus silicate network $B$.

| p | $ZC_1(B)$ | $ZC_2(B)$ | $ZC_1^*(B)$ | $ZC_2^*(B)$ | $ZC_3^*(B)$ | $ZC_4^*(B)$ |
|---|---|---|---|---|---|---|
| 3 | 3114 | 7164 | 1710 | 8226 | 8604 | 179334 |
| 4 | 6282 | 15300 | 3348 | 16470 | 17190 | 398358 |
| 5 | 10314 | 26460 | 5526 | 27522 | 28692 | 702918 |
| 6 | 15210 | 40644 | 8244 | 41382 | 43110 | 1093014 |
| 7 | 20970 | 57852 | 11502 | 58050 | 60444 | 1568646 |
| 8 | 27594 | 78084 | 15300 | 77526 | 80694 | 2129814 |
| 9 | 35082 | 101340 | 19638 | 99810 | 103860 | 2776518 |
| 10 | 43434 | 127620 | 24516 | 124902 | 129942 | 3508758 |

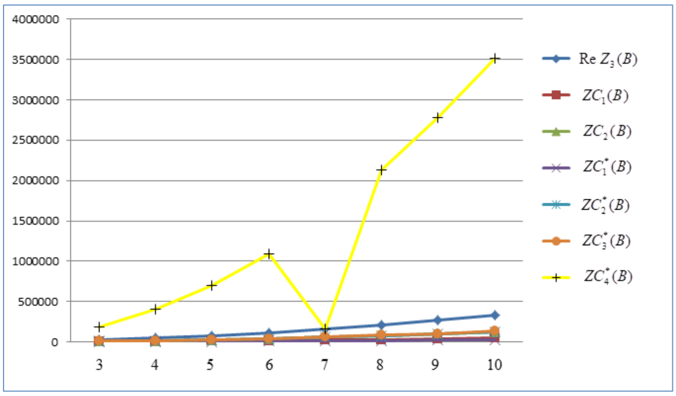


**Figure 6.** Comparisons for ZCIs of network B on dimension $3 \leq p \leq 10$.

Table 20: Comparisons of ZIs for all $p$.

| ZIs | $B - A = RHSL(p) - RHOX(p)$ | Results |
|---|---|---|
| $M_1$ | $78p^2 - 20p - 16$ | $B > A$ |
| $M_2$ | $96p^2 + 64p - 40$ | $B > A$ |
| $M_3$ | $510p^2 - 228p - 64$ | $B > A$ |
| ${}^mM_1$ | $\frac{17}{144}p^2 - \frac{107}{72}p$ | $B > A$ |
| ${}^mM_2$ | $\frac{1}{8}p^2 - \frac{1}{18}p - \frac{7}{36}$ | $B > A$ |
| $ReZ_3$ | $2796p^2 - 1952p - 28$ | $B > A$ |

Table 21: Comparisons of ZCIs for all $p$.

| ZCIs | $B - A = RHSL(p) - RHOX(p)$ | Results |
|---|---|---|
| $ZC_1$ | $240p^2 + 496p - 1494$ | $B > A$ |
| $ZC_2$ | $1128p^2 - 1776p + 628$ | $B > A$ |
| $ZC_1^*$ | $174p^2 - 156p + 20$ | $B > A$ |
| $ZC_2^*$ | $1020p^2 - 1104p + 214$ | $B > A$ |
| $ZC_3^*$ | $1074p^2 - 1172p + 246$ | $B > A$ |
| $ZC_4^*$ | $36624p^2 - 68064p + 29686$ | $B > A$ |

These general versions Zagreb indices based on connection number can be computed for the other molecular networks.

**Conflicts of Interest:** No any conflict of interest from our side.

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