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Two-sided and modified two-sided group chain sampling plan for Pareto distribution of the 2nd kind

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Abstract

In this article two-sided and modified two-sided group chain sampling plans for Pareto distribution of the 2nd kind is proposed. The minimum group size and mean ratios are obtained by satisfying the consumer's risk at the pre-specified quality level. Several tables are presented for easy selection of comparative study of the proposed plan with established plans.

Keywords: Two-sided chain sampling, truncated lifetime, group sampling, operating characteristic values, consumer's risk.

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1. Introduction

Acceptance sampling plan is an essential tool in Statistical Quality Control. In industries acceptance sampling plans are commonly used to examine the quality of final or partially final products. An appropriate plan protects the producer's product at acceptable quality level and increase the chances the selection of submitted product. At the same time, this plan protects the consumer risk that is, accepting a bad quality lot. These plans are very useful when testing are destructive, process not in control and 100 prcent inspection is inefficient. During World War II, the US Military has adopted acceptance sampling plan for testing the bullets. In most plans for a truncated lifetime test, the main concern is to decide the sample size from the lot under consideration. In single acceptance sampling plan, only a one product is put in tester but practically it is possible to examine more than one product. For instance, if a manufacturer has the capacity to examine the average lifetime of 100 products at a time and wants to inspect 1000 products. Then 100 products are placed into 10 groups. The life testing based on this assumption is known as group acceptance sampling and it save the cost, time, energy and labor. Several ordinary and group acceptance sampling plans for different lifetime distributions have been suggested by [1, 2, 3, 4, 5, 6].

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A single acceptance sampling plan having acceptance number zero with smaller sample size is used when testing is costly or destructive. The OC curves of such plans have exclusively a poor shape, in that the probability of lot acceptance starts to decrease even for a small value of proportion defective. Dodge [7] introduced a chain sampling plan known as ChSP-1, making use of cumulative recorded results of various samples which is very useful to overcome the deficiency of single acceptance sampling plan when, c=0. The authors [9, 10, 11, 12, 13, 14] have introduced group chain, resubmitted and economic reliability sampling plans that might be substituted for the established single acceptance sampling plans. The group chain sampling plan developed by Mughal et al. [9] based only partial (preceding lot) chaining but the consumers are not satisfied due to only consider the preceding sample information. Deva et al. [15] introduced two-sided (preceding and succeeding lots) chain sampling plan to give more protection to the producer as well as consumer and it reduces to the plan developed by Govindaraju and Lai [16]. An effort has been investigated to develop two-Sided chain sampling plans and best to our knowledge none has proposed this approach.

In this research, two-sided and modified two-sided group chain acceptance sampling plan are introduced by making the cumulative information of immediately preceding, i, current, as well as immediately succeeding, j lots, if the current sample does not lead to the correct decision.In the new development of manufacturing process, past, current and future lots can be examined in many fabrication industries. The purpose of this present research is to find the minimum group size and probability of lot acceptance for two-sided chain sampling plan assuming the life test is truncated at pre-assigned testing time. The proposed plans are able to provide higher probability of lot acceptance with minimum sample size based on several values of mean ratios and proportion defective than existing plans.

2. Operating procedurefor two sided group chain sampling plan

The methodology of two sided group chain sampling plans based on the following steps:

(I) Find the minimum number of groups g and allocate r products to each group.

(II) Select a sample of size$n(n = r * g)$and observe the number of defective products, d.

(III) Accept the current lot if the observed number of defectives is zero.

(IV) Also accept the lot if only one defective product is observed in current lot but preceding ι, and succeeding j lots free from defectives. Reject the lot if $d > 1$.

The probability of lot acceptance for two-sided group chain sampling plans can be written as respectively, when $i = j$,

$$P_{(a(TS))} = (P_{(0,(r*g))}) + (P_{(0,(r*g))})^i (P_{(1,(r*g))})(P_{(0,(r*g))})^j, \qquad i = j, \tag{2.1}$$

$$P_{(a(TS))} = P_{(0,(r*g))}[1 + P^{(2i-1)}_{(0,(r*g)}P_{(1,(r*g))}], \tag{2.2}$$

where P_0 and P_1 denote the probability of getting zero and one defective product respectively. Under the assumptions of Binomial distribution, $(P_{(0,(r*g))}) = (1-p)^{(r*g)}$ and $(P_{(1,(r*g))} = (r*g*p)(1-p)^{((r*g)-1)}$, the probability of lot acceptance can be obtained after solving Equation (2.2)

$$P_{(a(TS))}(p) = (1-p)^{((r*g))}[1 + (r*g*p)(1-p)^{(2(r*g*i)-1)}] \tag{2.3}$$

3. Operating procedure for modified two sided group chain sampling plan

The methodology of modified two sided group chain sampling plan functions in the following steps:

(I) For each lot, find the optimal number of groups g and allocate r products to each group.

(II) Randomly select a sample(n=r*g)and calculate the number of defective products.

(III) Accept the current lot if the sample from the current lot contains no defectives and previous i and j successive samples (lots) contain the maximum of only one defective (either in i previous lots or j successive lots).

The probability of lot acceptance for two-sided group chain sampling plans can be written as respectively, when i=j,

$$P_a(MTS) \tag{3.1}$$
$$= (P_{(0,(r*g))})^i(P_{(0,(r*g))})(P_{(0,(r*g))})^j + (P_{(1,(r*g))})^i(P_{(0,(r*g))})(P_{(0,(r*g))})^j + (P_{(0,(r*g))})^i(P_{(0,(r*g))})(P_{(1,(r*g))})^j$$
$$P_{a(MTS)} = (P_{(0,(r*g))})^{((i+j+1))} + (i+j)(P_{(1,(r*g))})(P_{(0,(r*g))})^{((i+j))} i = j, \tag{3.2}$$
$$P_{(a(MTS))} = (P_{(0,(r*g))})^{((2i+1))}[1 + (2i)(P_{(1,(r*g))})/(P_{(0,(r*g))})] \tag{3.3}$$

whereP_0 and P_1 denote the probability of zero and one defective product respectively. Under the assumptions of Binomial distribution, $(P_{(0,(r*g))}) = (1-p)^{((r*g))}$ and $(P_{(1,(r*g))}) = (r*g*p)(1-p)^{((r*g)-1)}$ the probability of lot acceptance can be obtained after solving Equation (3.3);

$$P_{(a(MTS))}(p) = (1-p)^{(r*g)(2i+1)} 1 + 2i(r*g)(p)/(1-p). \tag{3.4}$$

According to Pareto (1897) [17], the cumulative distribution function (CDF) and the mean of a Pareto distribution of the 2nd kind are respectively,

$$F(t,\sigma,\lambda) = 1 - (1 + \frac{t}{\sigma})^{-\lambda} \qquad t > 0, \sigma > 0, \lambda > 0 \tag{3.5}$$
$$\mu = \frac{\sigma}{\lambda - 1}\lambda > 1. \tag{3.6}$$

Equations (2.3) and (3.3) are used to construct Tables 1 and 2. Furthermore, p which represents the probability of failure of a submitted product during test termination time when $t_0 = a\mu_0$, and can be written in this form,

$$P = F(t,\sigma,\lambda) = 1 - (1 + \frac{a}{(\lambda-1)(\mu/\mu_0)})^{-\lambda}. \tag{3.7}$$

As discussed earlier, when the main objective is to obtain minimum sample size and more accurate probability of lot acceptance, consumer's risk, β is considered. The consumer's risk (probability of accepting the bad lot) also defines the poorest quality level that the consumer can tolerate. In Tables 1 and 2, the minimum values of group size g are presented based on iterative procedure for various values of consumer's risk β, pre-assumed testing time a and allowable acceptance number (i,j) when satisfy the following inequalities respectively,

$$P_{(a(TS))}(p) \leq \beta, \tag{3.8}$$
$$P_{(a(MTS))}(p) \leq \beta. \tag{3.9}$$

For a pre-fixed value of a, r, i, j and shape parameter of Pareto distribution of the 2^{nd} kind, the operating characteristic values as a function of μ/μ_0 are found and presented in Tables 3 and 4. The Operating characteristic values and required sample size of proposed plan having $i = j = 1, r = 3, g = 1 and \lambda = 2$, placed in Tables 6 and 7, for comparison purpose.

4. Comparisons

In this section, real lifetime data discussed by Rao and Ramesh [20] are also measured for validation of the proposed plansfor industrial uses. The data are the number of million revolutions before failure for each of the 23 ball bearings in life test: 17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.80, 51.84, 51.96, 54.12, 55.56, 67.80, 68.44, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04, and 173.40. The maximum likelihood estimates of the Pareto distribution of the 2^{nd} kind for these data are $\lambda = 1.6293$ and $\sigma = 133.97$, denoting the shape and scale parameters respectively. The Kolmogorov-Smirnov distance between the empirical distribution functions and the fitted distribution functions is 0.2358.

Table 1: Number of minimum groups required for the two sided group chain sampling plan for Pareto distribution of the 2nd kind with $\lambda = 2$

β	r	$i=j$	0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.10	2	1	2	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.05	2	1	2	2	2	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.01	2	1	2	2	2	2	2	2
	3	2	2	2	2	1	1	1
	4	3	2	1	1	1	1	1
	5	4	1	1	1	1	1	1

Table 2: Number of minimum groups required for the modified two sided group chain sampling plan for Pareto distribution of the 2nd kind with $\lambda = 2$

β	r	$i=j$	0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.10	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.05	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.01	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1

Table 3: Operating characteristic values having $i = j = 1$, $r = 3, g = 1$ for the two sided group chain sampling plan for Pareto distribution of the 2nd kind with $\lambda = 2$

β	α	2	4	6	8	10	12
0.25	0.7	0.1763	0.4426	0.6174	0.7256	0.7950	0.8415
0.10	0.8	0.1396	0.3845	0.5616	0.6778	0.7551	0.8083
0.05	1.0	0.0903	0.2925	0.4641	0.5889	0.6778	0.7419
0.01	1.2	0.0609	0.2256	0.3845	0.5105	0.6059	0.6778
	1.5	0.0351	0.1566	0.2925	0.4124	0.5105	0.5889
	2.0	0.0157	0.0903	0.1911	0.2925	0.3845	0.4641

Table 4: Operating characteristic values having $i = j = 1, r = 3, g = 1$ for the modified two sided group chain sampling plan for Pareto distribution of the 2nd kind with $\lambda = 2$

β	α	2	4	6	8	10	12
0.25	0.7	0.0378	0.2428	0.4421	0.5841	0.6817	0.7498
0.10	0.8	0.0225	0.1863	0.3741	0.5197	0.6250	0.7010
0.05	1.0	0.0082	0.1092	0.2650	0.4069	0.5197	0.6066
0.01	1.2	0.0031	0.0641	0.1863	0.3153	0.4277	0.5197
	1.5	0.0008	0.0292	0.1092	0.2127	0.3153	0.4069
	2.0	0.0001	0.0082	0.0451	0.1092	0.1863	0.2650

Table 5: Goodness of Fit-Summary

Lifetime Distributions	**Kolmogorov- Smirnov Statistic**	**Lifetime Distributions**	**Kolmogorov- Smirnov Statistic**
Pareto 2nd kind	0.23587	Normal	0.46872
Inv. Gaussian (3Parameter)	0.24914	Logistic	0.47529
Inv. Gaussian	0.26892	Hypersecant	0.48084
Gen. Gamma (4 Parameter)	0.27032	Exponential	0.48267
Weibull (3 Parameter)	0.27129	Reciprocal	0.49366
Pareto	0.2988	Error	0.49949
Levy (2 Parameter)	0.29947	Laplace	0.49949
Gamma (3 Parameter)	0.33437	Exponential (2 Parameter)	0.50574
Chi-Squared (2 Parameter)	0.35376	Error Function	0.51099
Kumaraswamy	0.36506	Johnson SB	0.52291
Fatigue Life (3 Parameter)	0.3663	Rayleigh (2 Parameter)	0.53063
Dagum	0.38245	Gamma	0.53827
Levy	0.38539	Gumbel Min	0.53887
Fatigue Life	0.40547	Beta	0.67274
Gumbel Max	0.4093	Rayleigh	0.69759
Burr (4 Parameter)	0.42473	Pert	0.70408
Gen. Gamma	0.4326	Triangular	0.81569
Power Function	0.4521	Rice	0.8578
Uniform	0.45534	Chi-Squared	0.91996

Table 6: Comparisons of mean ratios when $\alpha = 0.7$, $r = 3$, $g = 1$, $\lambda = 2$

β	(μ/μ_o)	Proposed Plan(TS) $i = j = 1$	Proposed Plan (MTS) $i = j = 1$	[8] $i = 2$	[10] $i = j = 1$
0.25	2	0.1763	0.0378	0.1763	0.0156
0.10	4	0.4426	0.2428	0.4426	0.1175
0.05	6	0.6174	0.4421	0.6174	0.2389
0.01	8	0.7256	0.5841	0.7256	0.3420
	10	0.7950	0.6817	0.7950	0.4245
	12	0.8415	0.7498	0.8415	0.4902

Table 7: Comparisons of sample size

β	α	Proposed Plan (TS) $i = j = 1$	Proposed Plan (MTS) $i = j = 1$	[18] $i = 2$	[19] $i = 2$
0.10	0.7	4	2	24	44
	1.0	2	2	12	8
	1.2	2	2	6	4
	1.5	2	2	4	2
	2.0	2	2	2	2

From Table 5, Pareto distribution of 2^{nd} kind provides reasonable fit for submitted products instead of several other lifetime distributions.Using the above information, suppose an experimenter would like to establish a plan when the true average lifetime of the submitted product is 1000 hours with $\beta = 0.05$, r=2, i=j=1, and a=0.7. From Table 1 (two sided group chain sampling plan), he distributes two products to each of two groups and accepts the lot if no defective product is observed in the current lot, and also accept the lot if only one defective product is recorded either in preceding i, current or succeeding j lots during 700 hours of testing. Using the similar design parameters for Table 2 (modified two-sided group chain sampling plan), two products should be allocated to in one group and accept the lot if no defective product is observed in the preceding i, current or succeeding j lots and also accept the lot if only one defective product is observed either in preceding, i, current and succeeding, j, lots during 700 hours. According to Table 3, if a=0.7, the probability of lot acceptance increases 0.1763 to 0.8415 when the mean ratio increases 2 to 12. Furthermore, in Tables 6 and 7, the mean ratios of the proposed plans and sample size are compared with those of the established plans. It is noted that the proposed plans give higher probability of lot acceptance at the same quality levels and ensured more protection to the consumers. Moreover, the proposed plans used more cumulative results (preceding and succeeding lots) of submitted lot over established chain sampling plan based on minimum sample size. It is interesting to note that the proposed plans reduce to ordinary chain sampling plans developed by Mughal et al. [9] with index 2i.

5. Conclusion

In this research article, two sided and modified two sided group chain sampling plans for Pareto distribution of the 2nd kind are introduced that utilizes additional information of preceding and succeeding lots. The algorithms of OC function are developed and tables based on the Binomial model are presented for various designed parameters. Based on real lifetime data, the proposed plans are very useful for the practitioners when the quality and features of products changes over time. Numerical comparisons are provided to show that the proposed plans would offer more accurate and higher probability of lot acceptance over the established plans with minimum sample size. This means that the proposed plans are more appropriate for inspection and economical in the sense of saving cost, time, energy and labor.

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Symbols

g:	Number of groups
r :	Number of testers
n :	Sample size
d:	Number of defective products
i:	Allowable preceding lot
j:	Allowable succeeding lot
α:	Producer's risk (Probability of rejecting a good lot)
β:	Consumer's risk (Probability of accepting a bad lot)
t_0:	Test termination time
μ_0:	Specified average life of a product
μ:	True average life of a product
$P_{(a(TS))}$:	Lot Acceptance Probability of two sided group chain sampling plan
$P_{(a(MTS))}$:	Lot Acceptance Probability of modified two sided group chain sampling plan
(μ/μ_0):	Mean ratio
σ:	Scale parameter of Pareto distribution of the 2nd kind
λ:	Shape parameter of Pareto distribution of the 2nd kind