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A fixed-point approach to a multi-group $SEIRV$ epidemic model

Amelia Bucur[a,*]

[a] *Department of Mathematics and Informatics, Faculty of Sciences, Lucian Blaga University of Sibiu, Sibiu, Romania.*

Abstract

Epidemics was always great problems in the human history and mathematicians have been challenged to bring their contribution to the management of epidemics, by using their abstract concepts in studying and forecasting their evolution. Compartmental models, have been remarkable for analysis the spread of epidemics. This paper has three objectives: to purpose a multi-group $SEIRV$ epidemic model for studying the spread of an epidemics, to present conditions of existence for a solution to the purposed generalized $SEIRV$ model and an example of simulations. The principal conclusion is that, the theory of fixed points can be used for the analysis of epidemics. The results of this paper adapt the results obtained in (Bucur, 2022, in International Journal of Advance Study and Research Work ($IJASRW$) 5(11)) and in (Guran, Bota and Naseem, 2020, in Symmetry 12, 856) to a generalization of the $SEIR$ model.

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1. Introduction and Preliminaries

in computer science, in environmental sciences, in technical sciences, in geometry, theory of chaos, etc. [17, 25]

The theory of fixed points s conditions in which the single functions or multivalued functions admit fixed points. Fixed points was defined as roots of the equality in the following type: $x = f(x)$ or $x \in F(x)$.

A very important method, the method of the sequence of successive approximations [19] which is used to construct a sequence which converge to the fixed point for a contraction, was used in a first fixed-point theorem in complete normed spaces (Stefan Banach, 1922). The remarkable polish mathematicians, Stefan Banach, was gives the first metric fixed-point theorem which is known in the mathematics as the Banach Contraction Principle. This principle gives conditions for the existence and also the uniqueness of a fixed point, when the functions f are contractions.

*Corresponding author
Email address: amelia.bucur@ulbsibiu.ro (Amelia Bucur)

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Also, some specialists also applied aspects of fixed-point theory to study the solutions of some epidemic models i.e.[1, 4, 6, 12, 14, 16, 23, 25]

For a isoperimetric optimal control problem, in [1], the authors presented a fixed point method.

Some scholars created epidemic models for infectious diseases involving fractional operators. Caputo and Fabrizio [4] have recommended a unique fractional derivative operator having the exponential kernel, a nonlocal and nonsingular kernel, which made an optimal analysis dynamics of 2019-nCoV.

In [14] the author applied a fixed point theorem on Banach spaces to epidemics. The assumptions of the fixed point theorem are satisfied by SIS model and SIR model. For the SIS model, the author demonstrated that if the infection rate exceeds the epidemic threshold then a strictly positive epidemic states satisfies a fixed point equation. The model consists of a triplet $(X(\Omega), T, F_\lambda)$, where a $X(\Omega)$ is a Banach space, T is a linear integral operator, and $F_\lambda : X(\Omega) \to X(\Omega)$ is a function associated with a scalar λ playing the role of an infection rate. The model has a fixed point equation describes the stationary solution of a dynamical process.

In [18], the author demonstrated new conditions of existence and uniqueness solutions of the 2019-nCoV models via fractional and fractal-fractional operators, also by using fixed point technics. The paper used the consept of $(\xi - F)$ - contraction which represent an extension of F-contraction for a mathematical model of type $SEIARM$.

The fixed point theory may also be used to find existence conditions for the solutions of an epidemic model, as presented in this paper.

A mathematical model that was validated by its practical application throughout the century is the SIR system. The SIR mathematical model was created by Kermack and McKendrick in a paper published in the year 1927 [10].

The authors created a dynamic model with three compartments, denoted by S (compartment of persons susceptible to the infection), I (compartment of infected persons) and R (compartment of recovered persons). In these compartments take place transfers from S to I and from I to R:

$$\begin{cases} \frac{dS(t)}{dt} = -\beta(S)(t)\frac{I(t)}{N(t)} \\ \frac{dI(t)}{dt} = \beta(S)(t)\frac{I(t)}{N(t)} - \gamma I(t) \\ \frac{dR(t)}{dt} = \gamma I(t). \end{cases} \tag{1.1}$$

which contain a non-linear system of first order differential equations and initial conditions $S(0) = S_0, I(0) = I_0, R(0) = R_0$.

Kermack and McKendrick used the following notations:

- S, I, and R are differentiable functions on the interval $[0, \infty)$, t represents a moment of time;

- β represents the rate of transmission and represents the average number of contacts needed per time unit to infect a person;

- γ is the recovery rate and $1/\gamma$ shows the length of the time interval in which a person becomes infected.

The number of individuals, N, was considered a constant and is applicable only to epidemics for which there are no vaccines available.

Validating the reproduction numbers or the biffurcation parameter of the model can be done by applying tools of mathematical analysis and has the formula: [16]

$$r = \frac{S_0\beta}{N\gamma}. \tag{1.2}$$

If $r > 1$, the SIR system forecasts an epidemic and if $r \leq 1$, then function I is a decreasing function.

In the specialty literature, the number of individuals who become infected per unit of time which was called incidence and the term βSI from (1) was called the disease incidence function [8].

A set of generalizations of the SIR system have been proposed in many papers, in many cases by adding more compartments in (1) or by considering other formulas of the βSI.

In year 1976, Lajmanovic and Yorke [11] created foundation of multi-group models. These specialists suggested the following model with initial conditions:

$$\begin{cases} \frac{dS_k}{dt} = -\sum_{j=1}^{m} S_k I_j \\ \frac{dI_k}{dt} = \sum_{j=1}^{m} \beta_{ji} S_k I_j - \gamma_k I_k, \quad t > 0, k \in \{1, ..., m\}, \\ \frac{dR_k}{dt} = \gamma_k I_k \quad . \end{cases}$$

After this year, a set of generalizations of multi-group models have been introduced in many articles, in many cases by adding more groups, or by considering other forms of $\beta_{ji} S_k I_j$ [15, 20].

Webb, in 1981, included in the SIR epidemics models a spatial factor proposed the following model: [27]

$$\begin{cases} \partial_t S(x,t) - \partial_{xx} S(x,t) = -\beta S(x,t) I(x,t) \\ \partial_t I(x,t) - \partial_{xx} I(x,t) = \beta S(x,t) I(x,t) - \gamma I(x,t), \quad -L < x < L,\ t > 0, \\ \partial_t R(x,t) - \partial_{xx} R(x,t) = \gamma I(x,t) \end{cases} \tag{1.3}$$

with the homogeneous Neumann boundary conditions $S_x(\pm L, t) = I_x(\pm L, t) = R_x(\pm L, t) = 0,\ t \geq 0$, and with initial conditions.

And, after this work, from the period of the twentieth century, until now, from the work of Webb, a set of extensions have been established, in regard to the incidence function $\beta S(x,t) I(x,t)$ or in regard to the spatial diffusion.

After these results, in year 2019, Luo et al. created another type of epidemic model [15]:

$$\begin{cases} \partial_t S_i - \nabla(d_{1i}(x) \nabla S_i) = A_i(x) - \lambda_i(x) S_i - \sum_{j=1}^{n} \beta_{ij}(x) S_i g_{ij}(I_j) \\ \partial_t I_i - \nabla(d_{2i}(x) \nabla I_i) = \sum_{j=1}^{n} \beta_{ij}(x) S_i g_{ij}(I_j) - I_j(\mu_i(x) + \alpha(x) + \gamma_i(x)), \\ \partial_t R_i = \nabla(d_{3i}(x) \nabla I_i) + I_i \gamma_i(x) - R_i \sigma(x) \end{cases} \tag{1.4}$$

in $Q = \Omega \times (0, \infty), i \in \{1, ..., n\}$, with the homogeneous Neumann boundary conditions: $\partial_v S_i = \partial_v I_i = \partial_v R_i = 0$ and initial conditions. The set $\sum = \partial\Omega \times (0, \infty)$ is an open bounded subset of $\mathbb{R}^n$ with boundary $\partial\Omega$ and ∂_v is the outer normal derivative.

In present days, other generalizations and extensions of the SIR model was been created.

An example is the $SEIR$ epidemic disease model. In this mathematical model, the population was divided in four compartments: susceptible-$S(t)$, exposed-$E(t)$, infected-infectious-$I(t)$ and recovered-$R(t)$. t is the time variable and the units are $(1/t)$. The $SEIR$ model, are: [5]

$$\begin{cases} \frac{dS(t)}{dt} = b - \mu S(t) - \beta S(t) \frac{I(t)}{N(t)} \\ \frac{dE(t)}{dt} = \beta S(t) \frac{I(t)}{N(t)} - (\mu + d) E(t) \\ \frac{dI(t)}{dt} = d E(t) - (\gamma + \mu + \alpha) I(t) \\ \frac{dR(t)}{dt} = \gamma I(t) - \mu R(t) \end{cases} \tag{1.5}$$

with the initial conditions $S(0) = S_0, E(0) = E_0, I(0) = I_0$, and $R(0) = R_0$. Where: [5]

- $N = S + E + I + R \leq N_0$;
- μ represents notations for the per-capita natural death rate;
-α is the virus-induced mean fatality rate;
- β is equal to the probability of disease transmission per contact times the number of contacts per unit time;

- d represents the inverse of the incubation period $1/Z$, where Z is the number of days in which the virus is still in the incubation state;

- γ is the invers of the infectious period $1/p$, where p is the number of days in which the infected individual is contagious;

For the parameters $b = \mu = 0, d = \infty$, the $SEIR$ system becomes the SIR system.

After this year, in 2020, Liu and Li created a multi-group $SEIR$ epidemic model using the age structure and the spatial diffusion [13]. Also in the year 2020, in another paper, Xu and Geng introduced a multi-group $SVIR$ epidemic model with vaccination, with reaction-diffusion, homogeneous Neumann boundary conditions and with some initial conditions. [26]

Also based on the $SEIR$ system, in year 2021, Rajapaksha et al. established an extension with five compartments, Susceptible-Exposed-Infected-Recovered-Vaccinated, the $SEIRV$ model, where $V = V(t)$ represents the number of vaccinated people to each moment of time t. [21]

In present, in year 2022, based on Caputo's fractional derivative and Lyapunov functions was introduced the SIR model with fractional time derivative. [24]

The paper has three objectives: to purpose a multi-group $SEIRV$ epidemic model for studying the spread of an epidemics, to present conditions of existence for a solution to the purposed generalized $SEIRV$ model and an example of simulations.

This article is organized as follows. In the next section, I suggest a multi-group $SEIRV$ epidemic model and present conditions of existence for the solution, as well as details about the reproduction number. In section 3, I present, simulations in Maple for particular parameters in model, for two groups of area. Finally, some conclusions are briefly outlined in section 4.

2. Main results

We purpose a generalization of the $SEIRV$ model, a multi-group $SEIRV$ epidemic model for which the total number of individuals varies with time.

The model purpose as generalization of the $SEIRV$ model, with m age groups, as follows:

$$\begin{cases} \frac{dI_k(t)}{dt} = dE_k(t) - \gamma_k I_k(t) - \mu I_k(t) - \alpha I_k(t) + c_1(t)N(t) - c_2(t)N(t) + c_3(t)N(t) - c_4(t)N(t) \\ \frac{dE_k(t)}{dt} = \beta S_k(t)\frac{I_k(t)}{N(t)-V_k(t)} - dE_k(t) - \mu E_k(t) + c_1(t)N(t) - c_2(t)N(t) + c_3(t)N(t) - c_4(t)N(t) \\ \frac{dN(t)}{dt} = bN(t) - \mu N(t) + c_1(t)N(t) - c_2(t)N(t) + c_3(t)N(t) - c_4(t)N(t) \\ \frac{dR_k(t)}{dt} = \gamma_k(I_k(t) - V_k(t)) - \mu R_k(t) + c_1(t)N(t) - c_2(t)N(t) + c_3(t)N(t) - c_4(t)N(t) \\ \frac{dS_k(t)}{dt} = b - \mu S_k(t) - \sum_{j=1}^{m} \beta_{ji} S_k(t)\frac{I_k(t)}{N(t)} + c_1(t)N(t) - c_2(t)N(t) + c_3(t)N(t) - c_4(t)N(t) \\ \frac{dV_k(t)}{dt} = \alpha_k(N(t) - V_k(t)) \end{cases} \tag{2.1}$$

where $k \in \{1, ..., m\}$, $c_i, i = 1, ..., 4$ are coefficients of immigration, type of climate, parameter of the level of the pollution in the studied area, indicator of the quality level of healthcare services, and a_k are rate of vaccinations. t is the time variable. By using elements of statistical mathematics, the estimation of these coefficients in (2.1) may be done by specialists in quality of life, environmental protection, mathematics, sociology, ecology, etc.

In (2.1) we suppose that $S(t) + E(t) + I(t) + R(t) + V(t) = N(t) \leq N_0$ in this case. Initial conditions are $S(0) = S_0, E(0) = E_0, I(0) = I_0, R(0) = R_0$ and $V(0) = V_0$. The initial moment is $t = 0$.

Also, this model is a generalization of the $SENIRV - iepqa$ model [2].

In the following we will demonstrate conditions of existence for solutions from system (2.1).

We write the non-linear system of first order differential equations of our generalization of the $SEIRV$ model in the form:

$$\begin{cases} I_k(t) = I_k(t) - \frac{dI_k(t)}{dt} + dE_k(t) - \gamma_k I_k(t) - \mu I_k(t) - \alpha I_k(t) + c_1(t)N(t) - c_2(t)N(t) + c_3(t)N(t) - c_4(t)N(t) \\ E_k(t) = E_k(t) - \frac{dE_k(t)}{dt} + \beta S_k(t)\frac{I_k(t)}{N(t)-V_k(t)} - dE_k(t) - \mu E_k(t) + c_1(t)N(t) - c_2(t)N(t) + c_3(t)N(t) - c_4(t)N(t) \\ N(t) = N(t) - \frac{dN(t)}{dt} + bN(t) - \mu N(t) + c_1(t)N(t) - c_2(t)N(t) + c_3(t)N(t) - c_4(t)N(t) \\ R_k(t) = R_k(t) - \frac{dR_k(t)}{dt} + \gamma_k(I_k(t) - V_k(t)) - \mu R_k(t) + c_1(t)N(t) - c_2(t)N(t) + c_3(t)N(t) - c_4(t)N(t) \\ S_k(t) = S_k(t) - \frac{dS_k(t)}{dt} + b - \mu S_k(t) - \sum_{j=1}^{m} \beta_{jk} S_k(t)\frac{I_k(t)}{N(t)} + c_1(t)N(t) - c_2(t)N(t) + c_3(t)N(t) - c_4(t)N(t) \\ \frac{dV_k(t)}{dt} = \alpha_k(N(t) - V_k(t)). \end{cases} \quad (2.2)$$

In order to establish existence conditions for the solutions of system (2.2), we used mathematical tools pertaining to the fixed-point theory.

We will write system (2.1) in the same form as system (2.2).

We denote by:

$$T_{1k}(I_k(t), E_k(t), N(t), R_k(t), S_k(t), V_k(t)) =$$
$$= I_k(t) - \frac{dI_k(t)}{dt} + dE_k(t) - \gamma_k I_k(t) - \mu I_k(t) - \alpha I_k(t) + c_1(t)N(t) - c_2(t)N(t) + c_3(t)N(t) - c_4(t)N(t),$$

$$T_{2k}(I_k(t), E_k(t), N(t), R_k(t), S_k(t), V_k(t)) =$$
$$= E_k(t) - \frac{dE_k(t)}{dt} + \beta S_k(t)\frac{I_k(t)}{N(t)-V_k(t)} - dE_k(t) - \mu E_k(t) + c_1(t)N(t) - c_2(t)N(t) + c_3(t)N(t) - c_4(t)N(t),$$

$$T_{3k}(I_k(t), E_k(t), N(t), R_k(t), S_k(t), V_k(t)) =$$
$$= N(t) - \frac{dN(t)}{dt} + bN(t) - \mu N(t) + c_1(t)N(t) - c_2(t)N(t) + c_3(t)N(t) - c_4(t)N(t),$$

$$T_{4k}(I_k(t), E_k(t), N(t), R_k(t), S_k(t), V_k(t)) =$$
$$= R_k(t) - \frac{dR_k(t)}{dt} + \gamma_k(I_k(t) - V_k(t)) - \mu R_k(t) + c_1(t)N(t) - c_2(t)N(t) + c_3(t)N(t) - c_4(t)N(t),$$

$$T_{5k}(I_k(t), E_k(t), N(t), R_k(t), S_k(t), V_k(t)) =$$
$$= S_k(t) - \frac{dS_k(t)}{dt} + b - \mu S_k(t) - \sum_{j=1}^{m} \beta_{jk} S_k(t)\frac{I_k(t)}{N(t)} + c_1(t)N(t) - c_2(t)N(t) + c_3(t)N(t) - c_4(t)N(t),$$

$$T_{6k}(I_k(t), E_k(t), N(t), R_k(t), S_k(t), V_k(t)) = a_k(N(t) - V_k(t)).$$

Let $Pcl(\mathbb{R})$ be the set of all nonempty closed subsets of $\mathbb{R}$.

Theorem 2.1 ([7]). *Let $A \in M_{m,m}(\mathbb{R}_+)$. The following are equivalents:*

(i) A is a matrix which converges to zero;
(ii) $A^n \to 0$ as $n \to \infty$;
(iii) The modulus for every eigen-values of A is lower than 1;
(iv) The matrix $I - A$ is non-singular, with $(I - A)^{-1} = I + A + ... + A^n + ...$.

We give the following theorem, for the hypothesis that $T_{ik} : \mathbb{R} \to Pcl(\mathbb{R})$ for $i \in \{1, ..., 6\}, k \in \{1, ..., m\}$ are contractions. $Pcl(\mathbb{R})$ is the set of all nonempty closed subsets of $\mathbb{R}$, where $\mathbb{R}$ represents the set of real numbers. Examples of conditions for them to be contractions are, for instance, the cases in which the absolute values of the derivatives are lower than 1. This situation is possible when the variations of functions $I_k(t), E_k(t), N(t), R_k(t), S_k(t), V_k(t)$ are low.

Theorem 2.2. *Let $T_{ik} : \mathbb{R} \to Pcl(\mathbb{R})$ for $i \in \{1, ..., 6\}, k \in \{1, ..., m\}$ be contractions and $0 \le a_{ii}1, i \in \{1, ..., 6\}$. Let $(I_k(t_1), E_k(t_1), N(t_1), R_k(t_1), S_k(t_1), V_k(t_1)), (I_k(t_2), E_k(t_2), N(t_2), R_k(t_2), S_k(t_2), V_k(t_2)) \in \mathbb{R}^6$ where t_1 and t_2 are moments from a time interval J.*

If for each $y_{ik} = T_{ik}(I_k(t), E_k(t), N(t), R_k(t), S_k(t), V_k(t)), i \in \{1, ..., 6\}, k \in \{1, ..., m\}$ *there exists* $z_{ik} = T_{ik}(I_k(t), E_k(t), N(t), R_k(t), S_k(t), V_k(t))$ *such that for all* $i \in \{1, ..., 6\}, k \in \{1, ..., m\}$*:*

$$|y_{ik} - z_{ik}| \leq a_{11}|I_k(t_2) - I_k(t_1)|,$$

$$|y_{ik} - z_{ik}| \leq a_{22}|E_k(t_2) - E_k(t_1)|,$$

$$|y_{ik} - z_{ik}| \leq a_{33}|N(t_2) - N(t_1)|,$$

$$|y_{ik} - z_{ik}| \leq a_{44}|R_k(t_2) - R_k(t_1)|,$$

$$|y_{ik} - z_{ik}| \leq a_{55}|S_k(t_2) - S_k(t_1)|,$$

$$|y_{ik} - z_{ik}| \leq a_{66}|V_k(t_2) - V_k(t_1)|.$$

then, the semilinear inclusion system:

$$\begin{cases} I(t) \in T_{1k}(I_k(t), E_k(t), N(t), R_k(t), S_k(t), V_k(t)) \\ E(t) \in T_{2k}(I_k(t), E_k(t), N(t), R_k(t), S_k(t), V_k(t)) \\ N(t) \in T_{3k}(I_k(t), E_k(t), N(t), R_k(t), S_k(t), V_k(t)), \quad k \in \{1, ..., m\} \\ R(t) \in T_{4k}(I_k(t), E_k(t), N(t), R_k(t), S_k(t), V_k(t)) \\ S(t) \in T_{5k}(I_k(t), E_k(t), N(t), R_k(t), S_k(t), V_k(t)) \\ V(t) \in T_{6k}(I_k(t), E_k(t), N(t), R_k(t), S_k(t), V_k(t)), \end{cases}$$

has at least one solution in $\mathbb{R}^{6m}$.

Proof. The theorem is a particular case of theorem 3.11 from [3]. It is demonstrated the same as Theorem 3.11 from [3], with $(u_1, u_2, u_3, u_4, u_5, u_6) = (I_k(t_1), E_k(t_1), N(t_1), R_k(t_1), S_k(t_1), V_k(t_1))$ and $(v_1, v_2, v_3, v_4, v_5, v_6) = (I_k(t_2), E_k(t_2), N(t_2), R_k(t_2), S_k(t_2), V_k(t_2))$ and working with

$$\|u - v\| = \begin{pmatrix} |u_1 - v_1| \\ |u_2 - v_2| \\ |u_3 - v_3| \\ |u_4 - v_4| \\ |u_5 - v_5| \\ |u_6 - v_6| \end{pmatrix}.$$

In this case, the diagonal matrix $A = (a_{ij})$ converges to 0.

The demonstration of the theorem uses elements from the fixed-point theory and results from the fact that $T_k = (T_{1k}, ..., T_{6k}) : \mathbb{R}^6 \to Pcl(\mathbb{R}^6), k \in \{1, ..., m\}$ are multivalued operators A-contraction to the left, thus T_k are MWP operators. The concept of multivalued weakly Picard operator (briefly MWP operator) was introduced by I.A. Rus, A. Petruşel and A. Sîntămărian in [22]. The authors created this concept in connection to the successive approximation technique for the fixed-point set of multivalued operators defined on complete metric space. As $\mathbb{R}^6$ is a Banach space each T_k has at least one fixed point [3], therefore, the conclusion to this theorem is verified. □

3. Simulation

Compartmental mathematical models, such as exponential models, SI, SIS, SIR, $SEIRS$, $MESIR$ models, other generalized SIR models, have been remarkable for studying the spread of epidemics and for their simulations in programs such as MATLAB, GLEAMviz, Maple, C++, Python, etc.

For two area, then $m = 2$, and particular parameters, we have two graphical representations in Maple (figure 1,2).

An example of simulations in Maple, is for $N = 200, \alpha = 0.0015/day, \beta = 1/5day, d = 6, \gamma_1 = (1/5)/day, \gamma_2 = (1/5)/day, b = \mu N = 0.2, c_1 = 0.00001, c_2 = 0.00002, c_3 = 0.00003, c_4 = 0.00002, a_1 = a_2 = 0, \beta_{11} = 0.005, \beta_{21} = 0.003, \beta_{12} = 0.002, \beta_{11} = 0.004, S_1(0) = 177, E_1(0) = 177, I_1(0) = 23, R_1(0) = 0, S_2(0) = 167, E_2(0) = 167, I_2(0) = 33, R_2(0) = 0, V_1(0) = V_1(0) = 0$. The graph show the evolution of the pandemic for a chosen period of time, for example of 30 days.

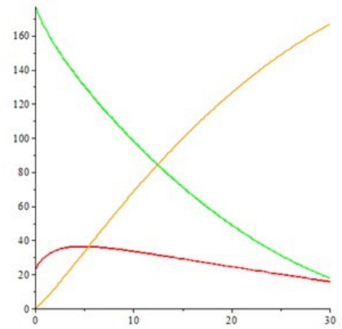

Figure 1: *The red curve for I_1, the green curve for S_1, the orange curve for R_1 (the line of the abscissa axis represents the time, and the ordinates axis shows the values of $I_1(t), S_1(t), R_1(t)$)*

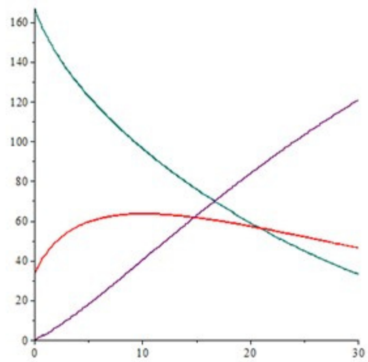

Figure 2: *The red curve for I_2, the azure for S_2, the purple R_2 (the line of the abscissa axis represents the time, and the ordinates axis shows the values of $I_2(t), S_2(t), R_2(t)$)*

In Figure 1, in the period day one-day for, the number of infected persons I_1, increase. It is displayed by the red curve. The susceptible persons have a descending trend in the green curve, while the one for recovered persons has an increasing trend in the orange curve. The curve of infected people has a peak after 4 days, when their number becomes $I_1 = 40$ from 200.

And, in Figure 2, in the period day one-day for, the number of infected persons I_2, increase. It is displayed by the red curve. The susceptible persons have a descending trend in the azure curve, while the one for recovered persons has an increasing trend in the purple curve. The curve of infected people has a peak after 4 days, when their number becomes $I_2 = 40$ from 200.

4. Conclusions

The concepts from the fixed-points theory can be used for the study of epidemics. The epidemic models can be generalized with new parameters and simulations for quickly results.

Future research directions for our generalization would be adding subcompartments to the model, new parameters and the calculation of the reproduction number, study of models that contain fractional derivatives with the technique presented in [9], called "*modified fractional Taylor series method*" to find numerical solutions for fractional epidemic models.

References

[1] I. Abouelkheir, F.El. Kihal, M. Rachik, M. and I. Elmouki, *Time Needed to Control an Epidemic with Restricted Resources in SIR Model with Short-Term Controlled Population: A Fixed Point Method for a Free Isoperimetric Optimal Control Problem*, Math. Comput. **23:64** (2018), 1–19. 1

[2] A. Bucur, *A fixed-point approach to mathematical models in epidemiology*, International Journal of Advance Study and Research Work (IJASRW). **5:11** (2022), 1–13. 2

[3] A. Bucur, L. Guran, L. and A. Petruşel, *Fixed Point Theorems for Multivalued Operators on a Set Endowed with Vector-Valued Metrics and Applications*, Fixed Point Theory **1** (2009), 19–34. 2

[4] M. Caputo and M. Fabrizio, *A new definition of fractional derivative without singular kernel*, Progress in Fractional Differentiation and Applications **1:2** (2015), 73–85. 1

[5] J.M. Carcione, J.E. Santos, C. Bagaini, C. and J.A. Ba, *Simulation of a COVID-19 Epidemic Based on a Deterministic SEIR Model*, Front. Public Health. **8**, (2020), :230. 1, 1

[6] G.Giordano, F. Blanchini, R. Bruno, et al., *Modelling the COVID-19 epidemic and implementation of population-wide interventions in Italy*, Nat Med. **26** (2020), 855–860. 1

[7] L. Guran, M.-F. Bota, A. Naseem, *Fixed Point Problems on Generalized Metric Spaces in Perov's Sense*, Symmetry **12** (2020), :856. 2.1

[8] W.H. Hamer, *On epidemic disease in England*, The Lancet **167:4305** (1906), 569–574. 1

[9] A. Khalouta and A. Kadem, *Numerical solutions of the fractional SIS epidemic model via a novel technique*, Journal of Prime Research in Mathematics **16:2** (2020), 44–55. 4

[10] W.O. Kermack, A.G. McKendrick and G.T. Walker, G.T., *A contribution to the mathematical theory of epidemics*, Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character. **115:772**, (1927), 700–721. 1

[11] A. Lajmanovich and J.A. Yorke, *A deterministic model for gonorrhea in a nonhomogeneous population*, Mathematical Biosciences **28:3** (1976), 221–236. 1

[12] R.W.Leggett, *A Fixed Point Theorem with Application to an Infectious Disease Model*, Journal of Mathematical Analysis and Applications **76**, (1980), 91–97. 1

[13] P.Liu, and H.-X. Li, *Global behavior of a multi-group SEIR epidemic model with age structure and spatial diffusion*, Mathematical Biosciences and Engineering **17:6**, (2020), 7248-7273. 1

[14] A.R. Lucas, *A fixed point theorem for a general epidemic model*, J. Math. Anal. Appl. **404** (2013), 135–149. 1

[15] Y. Luo, S. Tang, Z. Teng and I. Zhang, *Global dynamics in a reaction-diffusion multi-group SIR epidemic model with nonlinear incidence*, Nonlinear Analysis. Real World Applications **50** (2019), 365–385. 1

[16] F.J. Narsingani, M.B. Prajapati and P.H. Bhathawala, *Fixed Point Analysis of Kermack Mckendrick SIR Model*, Kalpa Publications in Computing **2** (2017), 13–19. 1, 1

[17] V.V. Nalawade and U.P. Dolhare, *Importance of fixed points in mathematics: a survey*, International Journal of Applied and Pure Science and Agriculture **2:12** (2016), 131–140. 1

[18] S.K. Panda, *Applying fixed point methods and fractional operators in the modelling of novel coronavirus 2019-nCoV/SARS-CoV-2*, Results in Physics **19** (2020), 103–433. 1

[19] C.É. Picard, *Mémoire sur la theorie des equations aux dérivées partielles et la methode des approximations successives*, J. Math. Pures et Appl. **6** (1890), 145–210. 1

[20] Z. Qiu, Y. Sun, X. He, et al. *Application of genetic algorithm combined with improved SEIR model in predicting the epidemic trend of COVID-19, China*, Sci Rep. **12** (2022), :8910. 1

[21] R.N.U. Rajapaksha, M.S.D. Wijesinghe, S.P. Jayasooriya, B.I. Gunawardana and W.P.C. Weerasinghe, *An Extended Susceptible-Exposed-Infected-Recovered (SEIR) Model with Vaccination for Forecasting the COVID-19 Pandemic in Sri Lanka*, MedRxiv (2021). 1

[22] I.A. Rus, A. Petruşel and A. Sîntămărian, *Data dependence of the fixed point set of multivalued weakly Picard operators*, Studia Univ. "Babeş-Bolyai", Mathematica. **XLVI:2** (2001), 111–121. 2

[23] A.S. Shaikh, I.N.Shaikh and K.S. Nisar, *A mathematical model of COVID-19 using fractional derivative: outbreak in India with dynamics of transmission and control*, Adv. Differ. Equ. (2020), :373. 1

[24] M.R. Sidi Ammi, M. Tahiri, M. Tilioua et al., *Global analysis of a time fractional order spatio-temporal SIR model*, Sci Rep **12** (2022), :5751. 1

[25] A. Tomar and M.C. Joshi (editors), *Fixed Point Theory and its Applications to Real Word Problem*, New York: Nova Science Publishers, (2021). 1

[26] J.Xu and Y. Geng, *Dynamics of a Diffusive Multigroup SVIR Model with Nonlinear Incidence*, Hindawi, (2020), article ID 8847023, 15 pages. 1

[27] G.F. Webb, *A Reaction-Diffusion Model for a Deterministic Diffusive Epidemic*, J. of Mathematical Analysis and Applications, **84** (1981), 150–161. 1