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On edge irregularity strength of certain families of snake graph

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Abstract

Edge irregular mapping or vertex mapping $\beta : V(U) \longrightarrow \{1, 2, 3, ..., s\}$ is a mapping of vertices in such a way that all edges have distinct weights. We evaluate weight of any edge by using equation $wt_\beta(cd) = \beta(c)+\beta(d)$, $\forall c, d \in V(U)$ and $cd \in E(U)$. Edge irregularity strength denoted by $es(U)$ is a minimum positive integer used to label vertices to form edge irregular labeling. The aim of this paper is to determine the exact value of edge irregularity strength of different families of snake graph.

Keywords: Edge irregularity strength, irregular assignment, irregularity strength, pendant edge, snake graphs.
2010 MSC: 05C78.

1. Introduction and Preliminaries

In this paper, we consider finite, simple and undirected graphs. The procedure of assignment of non-negative integers to the elements of a graph U is termed as labeling. Vertex set $V(U)$ and edge set $E(U)$ are the elements of a graph U. If we label vertices or edges, this labeling is categorized as vertex labeling or edge labeling. If we label both vertices and edges, this labeling is termed as total labeling. Now a days, graph labeling is widely used in various fields of life, including DNA sequence analysis, where it helps study genetic information and relationships, as well as in image and video processing for tasks like object detection, tracking and scene analysis. The latest Gallian survey [14] shows that much effort has been done on graph labeling. In 1988, Chartrand et al.[12] introduced edge labeling for a graph U. We call this labeling as irregular assignments because all vertices have distinct weights. Irregularity strength $s(U)$ is a minimum

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positive integer which is used to form irregular labeling. In 1998, Amar et al. [2] proved that the irregularity strength of any tree with no vertices of degree two is its number of pendant vertices. In 2011, Kalkowski et al. [18] gave a new upper bound for the irregularity strength of graphs. Similarly, results regarding irregularity strength of graphs can be seen in papers [9], [13] and [20]. Vertex irregular mapping or edge mapping $\beta : E(U) \longrightarrow \{1, 2, 3, ..., s\}$ is a mapping of edges in such a way that all vertices have distinct weights. Weight of any vertex can be calculated by using the equation $wt_\beta(c) = \Sigma\beta(cd)$, $\forall c, d \in V(U)$ and $cd \in E(U)$.

In 2007, Baca et al. [11] developed a new labeling known as edge irregular total labeling as a result of Chartrand's research. Edge irregular total labeling for a graph U is a mapping $\beta : E(U) \cup V(U) \longrightarrow \{1, 2, 3, ..., s\}$ in such a way that the total edge weight is different for all edges. We can evaluate total edge weight by using the relation $wt_\beta(cd) = \beta(c) + \beta(d) + \beta(cd)$, $\forall c, d \in V(U)$ and $cd \in E(U)$. Total edge irregularity strength denoted by $tes(U)$ is a minimum positive integer used to label edges to form edge irregular total labeling. More results were deduced by many researchers as a result of this inspiration. In 2012, Ahmad et al. determined exact value of total edge irregularity strength of the strong product of two paths P_n and P_m in their paper [6], while in 2014, they found total edge irregularity strength of product of two cycles C_n and C_m in their paper [4]. In 2012, Mushayt et al. [8] calculated exact value of total edge irregularity strength of hexagonal grid graphs. In 2014, Baca et al. [10] obtained useful results regarding total edge irregularity strength of generalized prism. Many authors contributed to the study of total edge irregularity strength in their papers [15],[17],[19],[21] and [22].

Edge irregularity and vertex irregularity were both new labels developed by Marzuki based on the previously improved motivation in [11], which were categorized as total labels with complete irregularity. Total irregularity strength for a graph U is denoted as $ts(U)$. Results related to irregular total labeling were developed in papers [11] and [20].

Because of the challenge to the previous results, Ahmed et al. developed a new concept of edge irregularity strength denoted by $es(U)$ in [3], which was a minimum positive integer used to label vertices to form edge irregular labeling. Inspired by this, many researchers found edge irregularity strength of various graphs. In 2016, Ahmad et al. [5] investigated exact value of edge irregularity strength of different families of toeplitz graph. In 2017, Mushayt et al. [7] took product of certain families of graphs with P_2 and determined their exact value of edge irregularity strength. In the same year, Imran et al. [16] gave results on edge irregularity strength of friendship graphs, cycle chains, caterpillars, star graphs and kite graphs. Ahmad et al. [1] computed edge irregularity strength of some chain graphs and the join of two graphs, and introduced a conjecture and open problems for researchers to research further. Tarawneh et al. [25] estimated edge irregularity strength of corona graphs of path P_m with P_2, P_m with K_1 and S_m with P_m, as well as the edge irregularity strength of the corona product of a cycle with isolated vertices in their paper [23]. Additionally, they explored edge irregularity strength of various graphs in papers [26] and [27]. By the motivation of previous results, Zhang et al. [28] introduced some new families of comb graph, such as comb graph Ca_n, Cd_n, Ce_n, Cf_n and Cg_n, and found their exact value of edge irregularity strength in 2020. Furthermore, in 2021, Tarawneh et al. [24] obtained edge irregularity strength of some classes of plane graphs.

Theorem 1.1. *[3] Let U be a simple graph with maximum degree $\triangle{=}\triangle(U)$, then* $es(U) \geq max\{\lceil \frac{|E(U)|+1}{2} \rceil, \triangle(U)\}$.

Definition 1.2. An edge of a graph is categorized as pendant edge if one of its vertices has degree one.

Definition 1.3. To obtain a triangular snake graph, let's consider a path graph P_m with $m \geq 2$. If we replace each edge of the path graph with a triangle C_3, we get the triangular snake graph T_m. It is formed by vertex set $V(T_m) = \{a_z; 1 \leq z \leq m-1\} \bigcup \{b_z; 1 \leq z \leq m\}$ and edge set $E(T_m) = \{b_zb_{z+1}; 1 \leq z \leq m-1\} \bigcup \{a_zb_z; 1 \leq z \leq m-1\} \bigcup \{a_zb_{z+1}; 1 \leq z \leq m-1\}$. It has $(2m-1)$ vertices and $3(m-1)$ edges.

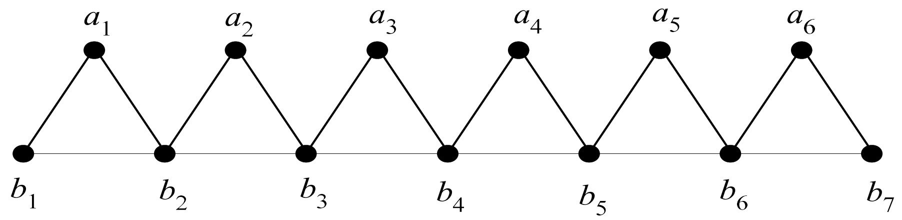

Figure 1: Triangular snake graph T_7.

Definition 1.4. Triangular snake graph T_m with pendant edges is formed by vertex set $V(T_m) = \{a_z, b_z; 1 \leq z \leq m-1\} \bigcup \{c_z, d_z; 1 \leq z \leq m\}$ and edge set $E(T_m) = \{a_z b_z; 1 \leq z \leq m-1\} \bigcup \{b_z c_z; 1 \leq z \leq m-1\} \bigcup \{b_z c_{z+1}; 1 \leq z \leq m-1\} \bigcup \{c_z c_{z+1}; 1 \leq z \leq m-1\} \bigcup \{c_z d_z; 1 \leq z \leq m\}$. It has $(5m-4)$ edges and $(4m-2)$ vertices.

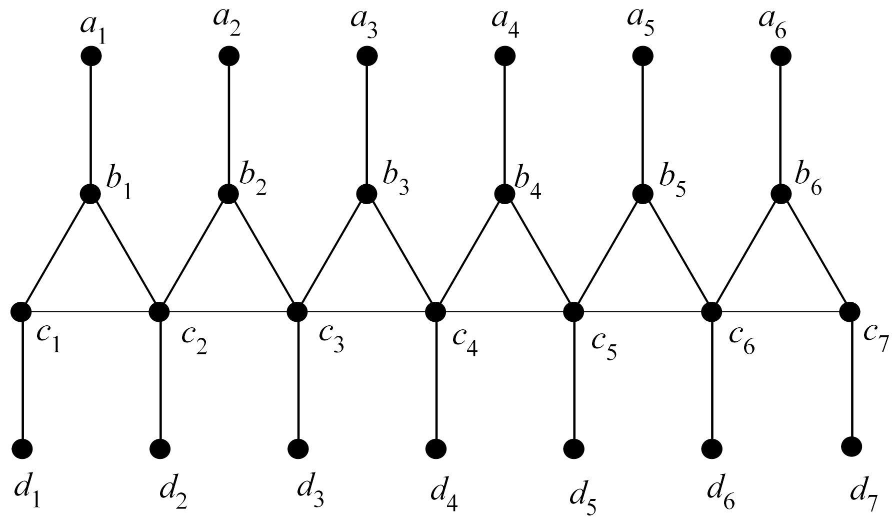

Figure 2: Triangular snake graph T_7 with pendant edges.

Definition 1.5. In order to obtain the alternate triangular snake graph AT_m, let's consider a path graph P_m with $m \geq 2$. If we join P_i and P_{i-1} (alternately) to a new vertex a_i, so that each alternating edge of a path is replaced by triangle C_3, we get the alternate triangular snake graph AT_m. It is formed by vertex set $V(AT_m) = \{a_z; 1 \leq z \leq m-2\} \bigcup \{b_z; 1 \leq z \leq m\}$ and edge set $E(AT_m) = \{b_z b_{z+1}; 1 \leq z \leq m-1\} \bigcup \{a_z b_z; 1 \leq z \leq m-2\} \bigcup \{a_z b_{z+2}; 1 \leq z \leq m-2\}$. It has $(2m-2)$ vertices and $(3m-5)$ edges.

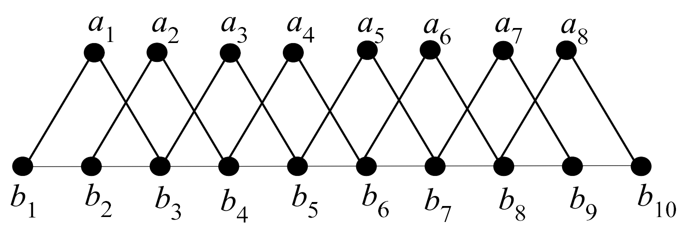

Figure 3: Alternate triangular snake graph AT_{10}.

Definition 1.6. Alternate triangular snake graph AT_m with pendant edges is formed by vertex set $V(AT_m) = \{a_z, b_z; 1 \le z \le m-2\} \bigcup \{c_z, d_z; 1 \le z \le m\}$ and edge set $E(AT_m) = \{a_z b_z; 1 \le z \le m-2\} \bigcup \{b_z c_z; 1 \le z \le m-2\} \bigcup \{b_z c_{z+2}; 1 \le z \le m-2\} \bigcup \{c_z c_{z+1}; 1 \le z \le m-1\} \bigcup \{c_z d_z; 1 \le z \le m\}$. It has $(4m-4)$ vertices and $(5m-7)$ edges.

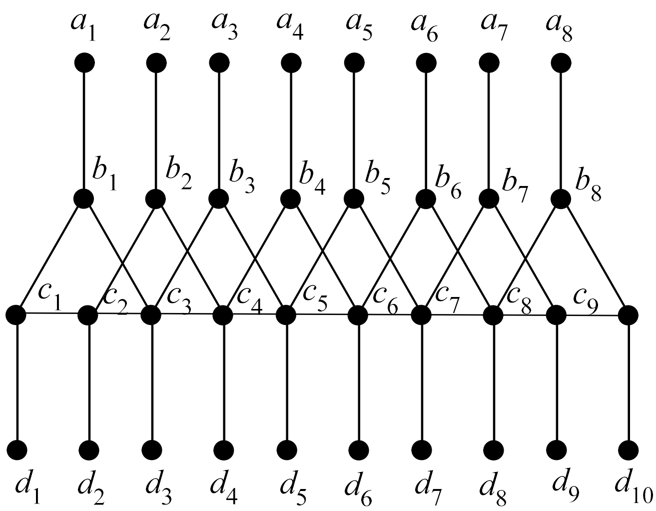

Figure 4: Alternate triangular snake graph AT_{10} with pendant edges.

2. Main results

Theorem 2.1. *Let T_m be a triangular snake graph, then $es(T_m) = 2m-1$.*

Proof. Let T_m be a triangular snake graph. We have to show that $es(T_m) = 2m-1$. From Theorem 1.1, we get lower bound $es(T_m) \ge 2m-1$. To establish the equality, it is enough to prove the existence of an optimal edge irregular $(2m-1)$-labeling. For this, define a vertex labeling $\beta : V(T_m) \to \{1, 2, 3, ..., 2m-1\}$ such that:

$$\beta(a_z) = 2z,\ 1 \le z \le m-1.$$

$$\beta(b_z) = 2z-1,\ 1 \le z \le m.$$

Now we evaluate weights for all edges as follows:

$$w_t(b_z b_{z+1}) = 4z,\ 1 \le z \le m-1.$$

$$w_t(a_z b_z) = 4z-1,\ 1 \le z \le m-1.$$

$$w_t(a_z b_{z+1}) = 4z+1,\ 1 \le z \le m-1.$$

On the basis of the above calculations, we can see that all vertex labels are at most $2m-1$, and all edges have distinct weights. The labeling β provides the upper bound on $es(T_m)$, i.e $es(T_m) \le 2m-1$. Combining with the lower bound, we conclude that $es(T_m) = 2m-1$. This completes the proof. □

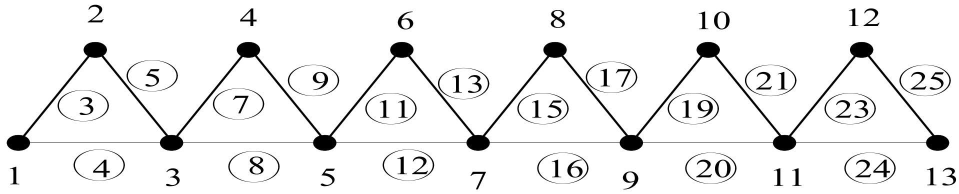

Figure 5: Irregular labeling on triangular snake graph T_7.

Theorem 2.2. *Let T_m be a triangular snake graph with pendant edges, then $es(T_m) = \lceil \frac{5m-3}{2} \rceil$.*

Proof. Let T_m be a triangular snake graph with pendant edges. We have to show that $es(T_m) = \lceil \frac{5m-3}{2} \rceil$. From Theorem 1.1, we get lower bound $es(T_m) \geq \lceil \frac{5m-3}{2} \rceil$. To establish the equality, it is enough to prove the existence of an optimal edge irregular $\lceil \frac{5m-3}{2} \rceil$-labeling. For this, define a vertex labeling $\beta : V(T_m) \rightarrow \{1, 2, 3, ..., \lceil \frac{5m-3}{2} \rceil\}$ such that:

$$\beta(a_z) = \begin{cases} 1, & \text{if } z = 1 \\ \frac{5z+2}{2}, & \text{if } z \text{ is even} \\ \frac{5z-1}{2}, & \text{if } z > 1, \text{ odd.} \end{cases}$$

$$\beta(b_z) = \begin{cases} 1, & \text{if } z = 1 \\ \frac{5z+2}{2}, & \text{if } z \text{ is even} \\ \frac{5z+3}{2}, & \text{if } z > 1, \text{ odd.} \end{cases}$$

$$\beta(c_z) = \begin{cases} 2, & \text{if } z = 1 \\ \frac{5z-4}{2}, & \text{if } z \text{ is even} \\ \frac{5z-5}{2}, & \text{if } z > 1, \text{ odd.} \end{cases}$$

$$\beta(d_z) = \begin{cases} 4, & \text{if } z = 1, 2 \\ \frac{5z-6}{2}, & \text{if } z > 2, \text{ even} \\ \frac{5z-5}{2}, & \text{if } z > 1, \text{ odd.} \end{cases}$$

Now we evaluate weights for all edges as follows:

$$w_t(a_z b_z) = \begin{cases} 2, & \text{if } z = 1 \\ 5z+1, & \text{if } z > 1, \text{ odd} \\ 5z+2, & \text{if } z \text{ is even.} \end{cases}$$

$$w_t(b_z c_z) = \begin{cases} 3, & \text{if } z = 1 \\ 5z-1, & \textit{otherwise.} \end{cases}$$

$$w_t(b_z c_{z+1}) = \begin{cases} 4, & \text{if } z = 1 \\ 5z+2, & \text{if } z > 1, \text{ odd} \\ 5z+1, & \text{if } z \text{ is even.} \end{cases}$$

$$w_t(c_z c_{z+1}) = \begin{cases} 5, & \text{if } z = 1 \\ 5z-2, & \textit{otherwise.} \end{cases}$$

$$w_t(c_z d_z) = \begin{cases} z+5, & \text{if } z = 1, 2 \\ 5z-5, & otherwise. \end{cases}$$

On the basis of the above calculations, we can see that all vertex labels are at most $\lceil \frac{5m-3}{2} \rceil$, and all edges have distinct weights. The labeling β provides the upper bound on $es(T_m)$, i.e $es(T_m) \leq \lceil \frac{5m-3}{2} \rceil$. Combining with the lower bound, we conclude that $es(T_m) = \lceil \frac{5m-3}{2} \rceil$. This completes the proof. □

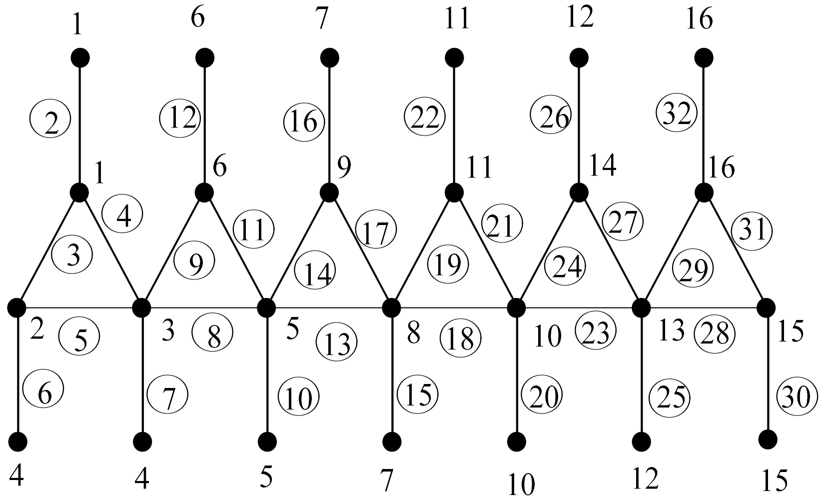

Figure 6: Irregular labeling on triangular snake graph T_7 with pendant edges.

Theorem 2.3. *Let AT_m be an alternate triangular snake graph, then $es(AT_m) = \frac{3m-4}{2}$.*

Proof. Let AT_m be an alternate triangular snake graph. We have to show that $es(AT_m) = \frac{3m-4}{2}$. From Theorem 1.1, we get lower bound $es(AT_m) \geq \frac{3m-4}{2}$. To establish the equality, it is enough to prove the existence of an optimal edge irregular $\frac{3m-4}{2}$-labeling. For this, define a vertex labeling $\beta : V(AT_m) \to \{1, 2, 3, ..., \frac{3m-4}{2}\}$ such that:

$$\beta(a_z) = \begin{cases} 1, & \text{if } z = 1 \\ \frac{3z+2}{2}, & \text{if } z \text{ is even} \\ \frac{3z+3}{2}, & \text{if } z > 1, \text{ odd.} \end{cases}$$

$$\beta(b_z) = \begin{cases} z, & \text{if } z = 1, 2 \\ \frac{3z-4}{2}, & \text{if } z > 2, \text{ even} \\ \frac{3z-3}{2}, & \text{if } z > 1, \text{ odd.} \end{cases}$$

Now we evaluate weights for all edges as follows:

$$w_t(b_z b_{z+1}) = \begin{cases} 2z+1, & \text{if } z = 1, 2 \\ 3z-2, & otherwise. \end{cases}$$

$$w_t(a_z b_z) = \begin{cases} 4z-2, & \text{if } z = 1, 2 \\ 3z, & \text{if } z > 1, \text{ odd} \\ 3z-1, & \text{if } z > 2 \text{ , even.} \end{cases}$$

$$w_t(a_zb_{z+2}) = \begin{cases} 4, & \text{if } z = 1 \\ 3z+3, & \text{if } z > 1, \text{ odd} \\ 3z+2, & \text{if } z \text{ is even.} \end{cases}$$

On the basis of the above calculations, we can see that all vertex labels are at most $\frac{3m-4}{2}$, and all edges have distinct weights. The labeling β provides the upper bound on $es(AT_m)$, i.e $es(AT_m) \leq \frac{3m-4}{2}$. Combining with the lower bound, we conclude that $es(AT_m) = \frac{3m-4}{2}$. This completes the proof. □

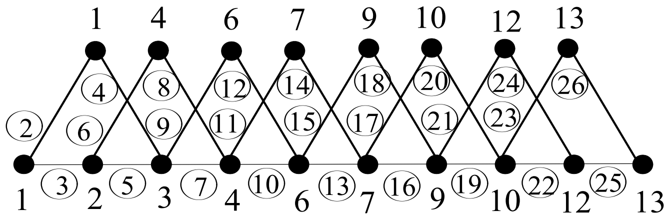

Figure 7: Irregular labeling on alternate triangular snake graph AT_{10}.

Theorem 2.4. *Let AT_m be an alternate triangular snake graph with pendant edges, then $es(AT_m) = \frac{5m-6}{2}$.*

Proof. Let AT_m be an alternate triangular snake graph with pendant edges. We have to show that $es(AT_m) = \frac{5m-6}{2}$. From Theorem 1.1, we get lower bound $es(AT_m) \geq \frac{5m-6}{2}$. To establish the equality, it is enough to prove the existence of an optimal edge irregular $\frac{5m-6}{2}$-labeling. For this, define a vertex labeling $\beta : V(AT_m) \to \{1, 2, 3, ..., \frac{5m-6}{2}\}$ such that:

$$\beta(a_z) = \begin{cases} \frac{5z}{2}, & \text{if } z = 2, 4 \\ \frac{5z-2}{2}, & \text{if } z > 4 \text{ , even} \\ \frac{7z-5}{2}, & \text{if } z = 1, 3 \\ \frac{5z-1}{2}, & \text{if } z > 3, \text{ odd.} \end{cases}$$

$$\beta(b_z) = \begin{cases} 4, & \text{if } z = 2 \\ \frac{5z}{2}, & \text{if } z > 2 \text{ , even} \\ \frac{7z-5}{2}, & \text{if } z = 1, 3 \\ \frac{5z+3}{2}, & \text{if } z > 3, \text{ odd.} \end{cases}$$

$$\beta(c_z) = \begin{cases} \frac{5z-1}{2}, & \text{if } z = 1, 3 \\ \frac{5z-3}{2}, & \text{if } z > 3 \text{ , odd} \\ 3, & \text{if } z = 2 \\ \frac{5z-6}{2}, & \text{if } z > 2, \text{ even.} \end{cases}$$

$$\beta(d_z) = \begin{cases} 2, & \text{if } z = 1 \\ \frac{5z-5}{2}, & \text{if } z > 1 \text{ , odd} \\ \frac{3z}{2}, & \text{if } z = 2, 4 \\ \frac{5z-6}{2}, & \text{if } z > 4, \text{ even.} \end{cases}$$

Now we evaluate weights for all edges as follows:

$$w_t(a_z b_z) = \begin{cases} 2, & \text{if } z = 1 \\ \frac{11z-4}{2}, & \text{if } z = 2, 4 \\ 5z+1, & \text{if } z \geq 3, \text{ odd} \\ 5z-1, & \text{if } z \geq 6 \text{ , even.} \end{cases}$$

$$w_t(b_z c_z) = \begin{cases} 3, & \text{if } z = 1 \\ 5z, & \text{if } z > 1, \text{ odd} \\ 5z-3, & \text{if } z \text{ is even.} \end{cases}$$

$$w_t(b_z c_{z+2}) = \begin{cases} 8, & \text{if } z = 1 \\ 8z-5, & \text{if } 2 \leq z \leq 3 \\ 5z+2, & \text{if } z > 2, \text{ even} \\ 5z+5, & \text{if } z > 3 \text{ , odd.} \end{cases}$$

$$w_t(c_z c_{z+1}) = \begin{cases} 5, & \text{if } z = 1 \\ 4z+2, & \text{if } 2 \leq z \leq 3 \\ 5z-2, & \textit{otherwise.} \end{cases}$$

$$w_t(c_z d_z) = \begin{cases} 4z, & \text{if } z = 1, 3 \\ \frac{7z-2}{2}, & \text{if } z = 2, 4 \\ 5z-4, & \text{if } z \geq 5, \text{ odd} \\ 5z-6, & \text{if } z \geq 6 \text{ , even.} \end{cases}$$

On the basis of the above calculations, we can see that all vertex labels are at most $\frac{5m-6}{2}$, and all edges have distinct weights. The labeling β provides the upper bound on $es(AT_m)$, i.e $es(AT_m) \leq \frac{5m-6}{2}$. Combining with the lower bound, we conclude that $es(AT_m) = \frac{5m-6}{2}$. This completes the proof. □

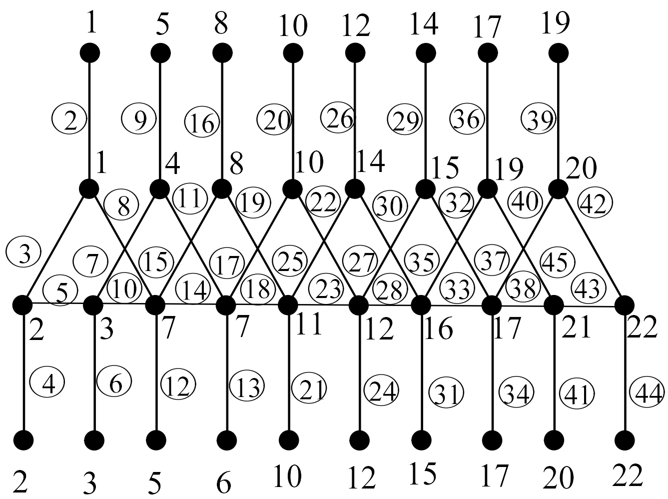

Figure 8: Irregular labeling on alternate triangular snake graph AT_{10} with pendant edges.

3. Conclusion

In this paper, we obtained accurate value of edge irregularity strength of triangular snake graph with and without pendant edges and alternate triangular snake graph with and without pendant edges. Numerous applications, including network design and optimization, rely on these findings. Further advancing the field of graph theory research, the approach employed in this work will be utilized as a base for investigating the edge irregularity strengths of other complicated graphs. Moving forward, a promising future direction will be to find edge irregularity strength of shadow graph, splitting graph, jewel graph, jellyfish graph, 4-Pan graph and deep neural network. Finding edge irregularity strength of above mentioned graphs will definitely potentially lead to valuable findings in graph theory.

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References

[1] A. Ahmad, A. Gupta, and R. Simanjuntak, *Computing the edge irregularity strengths of chain graphs and the join of two graphs*, Electronic Journal of Graph Theory and Applications, **6:1** (2018), 201. 1

[2] D. Amar and O. Togni, *Irregularity strength of trees*, Discrete mathematics, **190:1** (1998), 15–38.

[3] A. Ahmad, O. B. S. Al-Mushayt, and M. Bača, *On edge irregularity strength of graphs*, Applied Mathematics and Computation, **243** (2014), 607–610.

[4] A. Ahmad, M. Bača and M. K. Siddiqui, *On edge irregular total labeling of categorical product of two cycles*, Theory of computing systems, **54** (2014), 1–12.

[5] A. Ahmad, M. Bača and M. F. Nadeem, *On edge irregularity strength of Toeplitz graphs*, UPB Scientific Bulletin, Series A: Applied Mathematics and Physics, **78:4** (2016), 155–162. 1

[6] A. Ahmad, M. Baca, Y. Bashir and M. K. Siddiqui, *Total edge irregularity strength of strong product of two paths*, Ars combinatoria, **106** (2012), 449-459. 1, 1.1

[7] O. S. Al-Mushayt, *On edge irregularity strength of products of certain families of graphs with path P2.* Ars combinatoria, **135** (2017), 323–334.

[8] O. B. S. Al-Mushayt, A. Ahmad and M. K. Siddiqui, *On the total edge irregularity strength of hexagonal grid graphs*, Australas. J Comb., **53** (2012), 263–272. 1

[9] T. Bohman and D. Kravitz, *On the irregularity strength of trees*, Journal of graph theory, **45:4** (2004), 241–254. 1

[10] M. Baca, M. K. Siddiqui, *Total edge irregularity strength of generalized prism*, Applied mathematics and computation, **235** (2014), 168–173.

[11] M. Bača, S. Jendrol, M. Miller, and J. Ryan, *On irregular total labellings*, Discrete Mathematics, **307:11** (2007), 1378-–1388. 1

[12] G. Chartrand, M. S. Jacobson, J. Lehel, O. R. Oellermann, S. Ruiz and F. Saba, *Irregular networks*, *Congr. Numer.*,**64** (1988), 187–192. 1

[13] A. Frieze, R. J. Gould, M. Karoński and F. Pfender, *On graph irregularity strength*, Journal of graph theory, **41:2** (2002), 120–137. 1

[14] J. A. Gallian, (2022). *A Dynamic Survey of Graph Labeling*, The Electronic Journal of Combinatorics, 1000. https://doi.org/10.37236/11668 1

[15] K. M. M. Haque, *Irregular total labellings of generalized Petersen graphs*, Theory of computing systems, **50** (2012), 537–544. 1

[16] M. Imran, A. Aslam, S. Zafar and W. Nazeer, *Further results on edge irregularity strength of graphs*, Indonesian Journal of Combinatorics, **1:2** (2017), 82–91. 1

[17] J. Ivančo, S. Jendrol, *Total edge irregularity strength of trees*, Discussiones mathematicae graph theory, **26:3** (2006), 449–456. 1

[18] M. Kalkowski, M. Karoński and F. Pfender,*A new upper bound for the irregularity strength of graphs*, SIAM journal on discrete mathematics, **25:3** (2011), 1319–1321.

[19] J. Miškuf and R. Soták, *Total edge irregularity strength of complete graphs and complete bipartite graphs*, Discrete mathematics, **310:3** (2010), 400–407. 1

[20] P. Majerski and J. Przybyło, *On the irregularity strength of dense graphs*, SIAM Journal on Discrete Mathematics, **28:1** (2014), 197–205.

[21] M. K. Siddiqui, A. Ahmad, M. F. Nadeem and Y. Bashir, *Total edge irregularity strength of the disjoint union of sun graphs*, International Journal of Mathematics and Soft Computing, **3:1** (2013), 21–27. 1

[22] F. Salama, *On total edge irregularity strength of polar grid graph*, Journal of Taibah University for Science, **13:1** (2019), 912–916. 1

[23] I. Tarawneh, R. Hasni and A. Ahmad, *On the edge irregularity strength of corona product of cycle with isolated vertices*, AKCE international journal of graphs and combinatorics, **13:3** (2016), 213–217. 1

[24] I. Tarawneh, R. Hasni, A. Ahmad and M. A. Asim, *On the edge irregularity strength for some classes of plane graphs*, AIMS Math, **6** (2021), 2724–2731. 1

[25] I. Tarawneh, R. Hasni and A. Ahmad, *On the edge irregularity strength of corona product of graphs with paths*, Appl. Math. E-Notes, **16** (2016), 80–87. 1

[26] I. Tarawneh, R. Hasni and M. A. Asim, *On the edge irregularity strength of disjoint union of star graph and subdivision of star graph*, Ars Comb., **141** (2018), 93–100. 1

[27] I. Tarawneh, R. Hasni and A. Ahmad, *On the edge irregularity strength of grid graphs*. AKCE International Journal of Graphs and Combinatorics, **17:1** (2020), 414–418. 1
1

[28] X. Zhang, M. Cancan, M. F. Nadeem and M. Imran, *Edge irregularity strength of certain families of comb graph*, Proyecciones (Antofagasta), **39:4** (2020), 787–797.