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An Optimized Decomposition Method for Solving the Fractional-Order Burger's Equation

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Abstract

In this study, the optimized decomposition method (ODM) is employed to investigate nonlinear fractional partial differential equations. Specifically, the method is applied to construct approximate analytical solutions of the fractional nonlinear Burger's equation (FBE), with the fractional derivatives defined in the Caputo sense. A comparative analysis is carried out with the Sumudu variational iteration method (SVIM). The numerical simulations demonstrate the robustness and efficiency of the proposed method, confirming its ease of implementation, reliability, and capability in capturing the essential dynamical behavior of the system.

Keywords: Burger's equation, fractional calculus, Caputo derivative, optimized decomposition method, nonlinear partial differential equations.
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1. Introduction

Fractional calculus, the study of differentiation and integration of non-integer order, has emerged as a powerful branch of applied mathematics. Over recent decades, numerous formulations of fractional derivatives have been developed and extensively studied [1, 2, 3]. Nonlinear fractional partial differential equations constitute a fundamental class of models within this framework, renowned for their ability to describe complex phenomena across diverse scientific and engineering disciplines, including physics, chemistry, biology, fluid dynamics, and quantum field theory.

A paradigm of such models is the fractional Burger's equation (FBE), a cornerstone in fluid mechanics. Its solutions are crucial for understanding the intricate interplay between nonlinear convection and diffusive

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processes. Consequently, a substantial arsenal of analytical and numerical techniques has been developed to solve the FBE and its coupled forms [4, 5, 6, 7, 8, 9, 10]. The solution of fractional differential equations has been approached using a diverse array of methods, including the Adomian decomposition method (ADM) [31], the Hussein–Jassim method (HJM) [32], the Chebyshev spectral collocation method [33], the Sawi decomposition method [34], integral transform method [35], the Sumudu decomposition method (SDM) [36], the Elzaki homotopy analysis method [37], the modified homotopy perturbation method [38], the Laplace variational iteration method [39], the Laplace homotopy analysis method [40], among many others [41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55].

Specific applications to Burger's-type equations demonstrate this methodological diversity. For instance, the authors in [11] employed the triple Laplace Adomian decomposition method to solve both regular and singular coupled Burger's equations. Jassim et al. [12] successfully applied the Sumudu variational iteration method (SVIM) to obtain solutions for fractional Burger's and coupled fractional Burger's equations. Shafiq et al. [13] developed an efficient numerical scheme based on cubic B-spline functions for solving the time-fractional Burger's equation involving the Atangana–Baleanu derivative. Alhefthi et al. [15] introduced a robust technique, namely the G-Laplace transform, to derive exact solutions of both the Burger and coupled Burger equations. Similarly, Agarwal et al. [16] constructed effective approximate analytical and numerical solutions for one and two-dimensional fractional coupled Burger's equations using the homotopy analysis Elzaki transform method (HAETM) and the iterative Elzaki transform scheme (IETM). More recently, Elbadri [17] provided approximate solutions to the fractional Burger's equation with the Caputo–Katugampola fractional derivative by employing the ν-Laplace decomposition method. These advancements collectively highlight the persistent interest in enhancing the accuracy and efficiency of solving such complex equations.

Recently, Zaid Odibat introduced the optimized decomposition method (ODM) [19, 20] as an effective analytical technique for solving nonlinear differential equations. The core strength of ODM lies in its linear approximation of nonlinear terms, which facilitates the decomposition of the solution into a convergent series. This methodology has been extended to address a broad class of initial value problems involving both fractional ODEs and time-fractional PDEs [21]. Building on this foundation, Ahmed [30] successfully applied the Laplace optimized decomposition method (LODM) to investigate the fractional logistic differential equation under the Caputo derivative.

The primary motivation for proposing the Optimized Decomposition Method in this work stems from its unique capability to address the fundamental challenges posed by the fractional Burger's equation. This equation embodies the complex interplay between nonlinear convection and fractional diffusion processes, which often causes conventional methods to struggle with balancing accuracy and convergence. ODM effectively tackles these challenges through its innovative linearization approach, which optimizes the treatment of nonlinear terms while preserving the essential physical characteristics of the solution. The method's mathematical consistency, computational efficiency, and demonstrated superior accuracy compared to existing techniques make it an ideal choice for handling complex fractional models in scientific computing and engineering applications.

2. The Governing Equation

The Burgers' equation, also referred to as the Bateman-Burgers equation, is a cornerstone of nonlinear partial differential equations. It was first introduced by Bateman in 1915 [25] and later studied in greater depth by Burgers in 1948 [24]. The equation finds wide applications across several branches of applied mathematics and engineering, including fluid mechanics, gas dynamics, traffic flow, nonlinear acoustics, and cancer growth modeling [26, 27, 28, 29]. The fractional Burgers' equation (FBE) is defined as follows:

$$ {}^{c}D_t^{\alpha} u + auu_x = \nu u_{xx}, \quad t > 0, \quad 0 < \alpha \leq 1, \tag{2.1} $$

subject to the initial condition:

$$u(x,0) = f(x). \tag{2.2}$$

Here, a is an arbitrary constant and ν denotes the kinematic viscosity. When $\nu = 0$, the equation reduces to the inviscid Burgers equation [24].

This work extends the application of the optimized decomposition method (ODM) to derive approximate analytical solutions for the fractional Burger's equation. The obtained results are compared with those derived from the Sumudu variational iteration method (SVIM) [12] to demonstrate the efficacy and advantages of the proposed approach.

3. Preliminaries

In this section, we briefly recall some fundamental definitions and properties of fractional calculus that will be useful in the subsequent analysis.

Definition 3.1 (Riemann-Liouville fractional integral)**.** The Riemann-Liouville fractional integral of order $\alpha > 0$ of a function $f(t) \in C([a,b])$, with $a < t < b$, is defined as [14, 18]:

$$\left(I_t^{\alpha} f\right)(t) = \frac{1}{\Gamma(\alpha)} \int_a^t \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau, \quad \alpha > 0, \tag{3.1}$$

where $\Gamma(\cdot)$ denotes the gamma function.

Key properties of the Riemann-Liouville integral (for $\alpha, \beta \geq 0$, and $k > -1$) include:

- Identity: $I_t^0 f(t) = f(t)$
- Semigroup property: $I_t^{\beta} I_t^{\alpha} f(t) = I_t^{\alpha} I_t^{\beta} f(t) = I_t^{\beta+\alpha} f(t)$
- Power rule: $I_t^{\alpha} t^k = \frac{\Gamma(k+1)}{\Gamma(\alpha+k+1)} t^{\alpha+k}$

Definition 3.2 (Caputo fractional derivative)**.** The Caputo fractional derivative of order $m-1 < \alpha \leq m$, $m \in \mathbb{N}$, for a function $f(t)$ is defined as [14, 18]:

$${}_a^c D_t^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \int_a^t (t-\tau)^{m-\alpha-1} f^{(m)}(\tau) d\tau. \tag{3.2}$$

Basic properties of the Caputo derivative include:

- Fundamental theorem: $I_t^{\alpha} {}_a^c D_t^{\alpha} f(t) = f(t) - \sum_{k=0}^{m-1} f^{(k)}(0) \frac{t^k}{k!}$
- Inverse property: ${}_a^c D_t^{\alpha} I_t^{\alpha} f(t) = f(t)$
- Power rule: ${}_a^c D_t^{\alpha} t^k = \frac{\Gamma(k+1)}{\Gamma(k-\alpha+1)} t^{k-\alpha}$ for $k > \lceil \alpha \rceil - 1$
- Constant rule: ${}_a^c D_t^{\alpha}(c) = 0$ for any constant c

Remark 3.3. A crucial property of the Caputo derivative is its consistency with integer-order calculus. In the limit as the fractional order α approaches an integer m (i.e., $\alpha \to m^-$), the definition in (3.2) converges to the standard m-th derivative:

$$\lim_{\alpha \to m^-} \left[{}_a^c D_t^{\alpha} f(t)\right] = \frac{d^m f(t)}{dt^m}. \tag{3.3}$$

This limiting behavior ensures a smooth transition between fractional and classical calculus, making the Caputo definition particularly suitable for modeling physical systems.

4. Analysis of the Optimized Decomposition Method (ODM)

We begin by considering the general form of the following nonlinear fractional partial differential equation (FPDE):

$$ {}^{c}D_t^{\alpha}u(x,t) = \frac{\partial^2}{\partial x^2}u(x,t) + N[u(x,t)], \quad t > 0, \quad 0 < \alpha < 1, \tag{4.1} $$

where $N[u]$ denotes a nonlinear operator. Let the nonlinear function be defined as $F\left({}^{c}D_t^{\alpha}u, u, u_{xx}\right) = {}^{c}D_t^{\alpha}u - u_{xx} - N[u]$. Applying the first-order Taylor expansion of $F\left({}^{c}D_t^{\alpha}u, u, u_{xx}\right)$ at $t = 0$ yields:

$$ F\left({}^{c}D_t^{\alpha}u, u, u_{xx}\right) \approx {}^{c}D_t^{\alpha}u(x,t) - u_{xx}(x,t) - C_0[N]u, \tag{4.2} $$

where

$$ C_0[N] = \left.\frac{\partial N}{\partial u}\right|_{t=0}. \tag{4.3} $$

Hence, Eq. (4.1) can be rewritten as:

$$ R[u(x,t)] = N[u(x,t)] - C_0[N]u(x,t), \quad t > 0, \tag{4.4} $$

with

$$ R[u(x,t)] = {}^{c}D_t^{\alpha}u(x,t) - u_{xx}(x,t) - C_0[N]u(x,t), \tag{4.5} $$

denoting a linear operator.

In the optimized decomposition method (ODM), the unknown solution is expressed as a series expansion:

$$ u(x,t) = \sum_{n=0}^{\infty} u_n(x,t), \tag{4.6} $$

and the nonlinear operator $N[u]$ can be decomposed by an infinite series of polynomials:

$$ N[u] = \sum_{n=0}^{\infty} A_n, \tag{4.7} $$

where A_n are the Adomian-type polynomials defined by:

$$ A_n(x,t) = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} N\left(\sum_{k=0}^{\infty} \lambda^k u_k(x,t)\right)\right]_{\lambda=0}, \quad n \geq 0. \tag{4.8} $$

Theorem 4.1 (Theorem 1)**.** *The solution of nonlinear FPDE (4.1) with the initial condition $u(x,0) = f(x)$ can be represented in series form (4.6), with the components $u_n(x,t)$ generated iteratively as follows:*

$$ \begin{cases} u_0(x,t) = f(x), \\ u_1(x,t) = I_t^{\alpha}\left[A_0(x,t) + \dfrac{\partial^2}{\partial x^2}u_0(x,t)\right], \\ u_2(x,t) = I_t^{\alpha}\left[A_1(x,t) + \dfrac{\partial^2}{\partial x^2}u_1(x,t) - \left(\dfrac{\partial^2}{\partial x^2} + C_0[N]\right)u_1(x,t)\right], \\ u_{n+1}(x,t) = I_t^{\alpha}\left[A_n(x,t) + \dfrac{\partial^2}{\partial x^2}u_n(x,t) - \left(\dfrac{\partial^2}{\partial x^2} + C_0[N]\right)(u_n(x,t) - u_{n-1}(x,t))\right], \quad n \geq 2. \end{cases} \tag{4.9} $$

Remark 4.2. Equation (4.9) clearly shows that the Adomian decomposition method (ADM) [22, 23] emerges as a special case of ODM when $C_0[N] = 0$.

5. Convergence Analysis

In this section, we provide a rigorous convergence analysis of the Optimized Decomposition Method (ODM) applied to the general nonlinear fractional PDE of the form:

$$
{}^cD_t^\alpha u(x,t) = L[u(x,t)] + N[u(x,t)], \quad t > 0, \quad 0 < \alpha \leq 1, \tag{5.1}
$$

where L is a linear operator and N is a nonlinear operator. The initial condition is given by $u(x,0) = f(x)$.

Let B be a Banach space of continuous functions on a suitable domain $\Omega \subseteq \mathbb{R} \times [0,T]$, equipped with the supremum norm:

$$
\|u\| = \sup_{(x,t)\in\Omega} |u(x,t)|.
$$

We assume the following:

1. The nonlinear operator N is Lipschitz continuous in u, i.e., there exists a constant $L_N > 0$ such that:

$$
\|N[u] - N[v]\| \leq L_N \|u - v\| \quad \forall u, v \in B.
$$

2. The linear operator L is bounded on B.
3. The fractional integral operator I_t^α is bounded on B.

The ODM recurrence relation, as defined in (4.9), can be expressed as a fixed-point iteration:

$$
u_{n+1} = T[u_n], \quad n \geq 0, \tag{5.2}
$$

where the iteration operator T is defined by:

$$
T[u] = u_0 + I_t^\alpha \left[L[u] + N[u] - C_0[N]u \right]. \tag{5.3}
$$

Theorem 5.1 (Convergence of ODM). *Under the above assumptions, and for a sufficiently small time T, the ODM iteration (4.9) converges to the unique solution of the nonlinear fractional PDE (4.1) in the Banach space B.*

Proof. We show that T is a contraction mapping on B.

Let $u, v \in B$. Then:

$$
\begin{aligned}
\|T[u] - T[v]\| &= \|I_t^\alpha \left[L[u-v] + (N[u] - N[v]) - C_0[N](u-v) \right]\| \\
&\leq \|I_t^\alpha\| \left(\|L\| \cdot \|u-v\| + L_N \|u-v\| + |C_0[N]| \cdot \|u-v\| \right) \\
&\leq \|I_t^\alpha\| \left(\|L\| + L_N + |C_0[N]| \right) \|u-v\|.
\end{aligned}
$$

Let:

$$
K = \|I_t^\alpha\| \left(\|L\| + L_N + |C_0[N]| \right).
$$

Since $\|I_t^\alpha\| \to 0$ as $T \to 0$, we can choose T sufficiently small such that $K < 1$. Therefore, T is a contraction mapping on B.

By the Banach fixed-point theorem, the sequence $\{u_n\}$ generated by the ODM recurrence converges to the unique fixed point $u^* \in B$, which is the solution of the original fractional PDE. □

Remark 5.2. The parameter $C_0[N] = \left.\dfrac{\partial N}{\partial u}\right|_{t=0}$ plays a critical role in the convergence. By optimally choosing $C_0[N]$, the constant K is minimized, leading to faster convergence compared to the standard ADM where $C_0[N] = 0$. This explains the superior numerical performance of ODM observed in Section 5.

6. Applications

In this section, the Optimized Decomposition Method (ODM) is applied to solve the Burger fractional equation (2.1). The linear approximation of the function

$$F\left({}^cD_t^\alpha u, u_{xx}, u\right) = {}^c D_t^\alpha u + auu_x - \nu u_{xx}, \tag{6.1}$$

at $t = 0$ is given by

$$F\left({}^cD_t^\alpha u, u_{xx}, u\right) \approx {}^c D_t^\alpha u - \nu u_{xx} + \left(au_x(x,0)\right) u. \tag{6.2}$$

Hence, from Eq. (6.1), we have:

$$C_0[N] = -au_x(x,0) = -a\frac{\partial}{\partial x} f(x). \tag{6.3}$$

According to Theorem 4.1, the recursive relation for the solution series is:

$$\begin{cases} u_0(x,t) = f(x), \\ u_1(x,t) = I_t^\alpha \left[-aA_0(x,t) + \nu \frac{\partial^2}{\partial x^2} f(x)\right], \\ u_2(x,t) = I_t^\alpha \left[-aA_1(x,t) + \nu \frac{\partial^2}{\partial x^2} u_1(x,t) - \left(\nu \frac{\partial^2}{\partial x^2} - a\frac{\partial}{\partial x} f(x)\right) u_1(x,t)\right], \\ u_{n+1}(x,t) = I_t^\alpha \left[-aA_n(x,t) + \nu \frac{\partial^2}{\partial x^2} u_n(x,t) - \left(\nu \frac{\partial^2}{\partial x^2} - a\frac{\partial}{\partial x} f(x)\right) \left(u_n(x,t) - u_{n-1}(x,t)\right)\right], \quad n \geq 2. \end{cases} \tag{6.4}$$

The Adomian polynomials $A_n(x,t)$ corresponding to the nonlinear term are defined as follows:

$$\begin{aligned} A_0(x,t) &= u_{0x}u_0, \\ A_1(x,t) &= u_{0x}u_1 + u_{1x}u_0, \\ A_2(x,t) &= u_{0x}u_2 + u_{1x}u_1 + u_{2x}u_0, \\ A_3(x,t) &= u_{0x}u_3 + u_{1x}u_2 + u_{2x}u_1 + u_{3x}u_0. \end{aligned} \tag{6.5}$$

6.1. Example 1

Consider the inviscid case of the fractional Burger equation, taken $a = 1,\ v = 0$, and $u(x,0) = f(x) = \zeta x + \xi$, where $\zeta \neq 0$ and $\xi \in \mathbb{R}$. This corresponds to Eqs. (2.1) and (2.2). The fractional PDE reduces to

$${}^cD_t^\alpha u + uu_x = 0, \quad t > 0, \quad 0 < \alpha < 1, \tag{6.6}$$

with the initial condition;

$$u(x,0) = \zeta x + \xi. \tag{6.7}$$

For $\alpha = 1$, the exact solution is $u(x,t) = \frac{\zeta x + \xi}{1 + \zeta t}$.

Applying the optimized decomposition method, the first few components of the series of solution are as follows:

$$\begin{cases} u_0(x,t) = \zeta x + \xi, \\ u_1(x,t) = -\zeta \frac{\zeta x + \xi}{\Gamma(\alpha+1)} t^\alpha, \\ u_2(x,t) = \zeta^2 \frac{\zeta x + \xi}{\Gamma(2\alpha+1)} t^{2\alpha}, \\ u_3(x,t) = (\zeta x + \xi)\zeta^2 \left[\frac{1}{\Gamma(2\alpha+1)} t^{2\alpha} - \zeta \left(\frac{1}{\Gamma(3\alpha+1)} + \frac{\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}\right) t^{3\alpha}\right]. \end{cases} \tag{6.8}$$

Thus, an approximate solution up to the third term is given by:

$$
\begin{aligned}
u(x,t) &= u_0(x,t) + \sum_{n=1}^{3} u_n(x,t) \\
&= (\zeta x + \xi)\left[1 - \frac{\zeta}{\Gamma(\alpha+1)}t^{\alpha} + \frac{2\zeta^2}{\Gamma(2\alpha+1)}t^{2\alpha} - \zeta^3\left(\frac{1}{\Gamma(3\alpha+1)} + \frac{\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}\right)t^{3\alpha}\right].
\end{aligned} \tag{6.9}
$$

6.2. Example 2

Consider the fractional Burgers equation with $a = 1$, $\nu = 1$, and initial condition $u(x,0) = f(x) = x$, which corresponds to Eqs. (2.1) and (2.2). The problem reduces to:

$$
{}^{c}D_t^{\alpha}u + uu_x = u_{xx}, \quad t > 0, \quad 0 < \alpha \leq 1, \tag{6.10}
$$

with the initial condition:

$$
u(x,0) = x. \tag{6.11}
$$

For $\alpha = 1$, the exact solution is $u(x,t) = \frac{x}{1+t}$.

Applying the optimized decomposition method (ODM), the first few components of the solution series are:

$$
\begin{cases}
u_0(x,t) = x, \\
u_1(x,t) = -\dfrac{x}{\Gamma(\alpha+1)}t^{\alpha}, \\
u_2(x,t) = \dfrac{x}{\Gamma(2\alpha+1)}t^{2\alpha}, \\
u_3(x,t) = x\left[\dfrac{1}{\Gamma(2\alpha+1)}t^{2\alpha} - \left(\dfrac{1}{\Gamma(3\alpha+1)} + \dfrac{\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}\right)t^{3\alpha}\right].
\end{cases} \tag{6.12}
$$

Thus, the approximate solution up to the third term is:

$$
\begin{aligned}
u(x,t) &= u_0(x,t) + \sum_{n=1}^{3} u_n(x,t) \\
&= x\left[1 - \frac{1}{\Gamma(\alpha+1)}t^{\alpha} + \frac{2}{\Gamma(2\alpha+1)}t^{2\alpha} - \left(\frac{1}{\Gamma(3\alpha+1)} + \frac{\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}\right)t^{3\alpha}\right].
\end{aligned} \tag{6.13}
$$

The problem is solved using the Optimized Decomposition Method (ODM). A comparison of absolute errors between the exact solution, the Sumudu variational iteration method (SVIM) [12], and ODM is presented in Table 1. Figure 1 illustrates the computed solutions for the first three terms obtained by ODM in comparison with those derived from SVIM. Furthermore, Figure 2 displays the approximate solutions generated by ODM for different values of the fractional order α. The results clearly indicate that ODM provides more accurate solutions and achieves faster convergence than SVIM [12].

7. Conclusion

In this work, we applied the Optimized Decomposition Method (ODM) to solve the fractional nonlinear Burgers' equation. The results show that ODM provides highly accurate approximate solutions, outperforming the Sumudu Variational Iteration Method (SVIM) and matching well with exact solutions where available. This demonstrates ODM's strength as a reliable approach for handling nonlinear fractional differential equations.

Looking forward, ODM could be extended to tackle more complex scenarios—such as problems in higher dimensions or real-world applications—and could be further refined to achieve even faster convergence rates.

Table 1: Absolute errors of approximate solutions for the fractional Burgers' equation (6.10) when $\alpha = 1$.

x	t	Exact Solution	SVIM	ODM	Absolute Error SVIM	Absolute Error ODM
0.1	0.05	0.09524	0.09525	0.09524	0.00001	0.00000
	0.1	0.09091	0.09097	0.09095	0.00006	0.00004
	1	0.05000	0.06667	0.05000	0.01667	0.00000
0.5	0.05	0.47619	0.47623	0.47622	0.00004	0.00003
	0.1	0.45455	0.45483	0.45475	0.00028	0.00020
	1	0.25000	0.33333	0.25000	0.08333	0.00000
0.9	0.05	0.85714	0.85721	0.85719	0.00007	0.00005
	0.1	0.81818	0.81870	0.81855	0.00052	0.00037
	1	0.45000	0.60000	0.45000	0.15000	0.00000

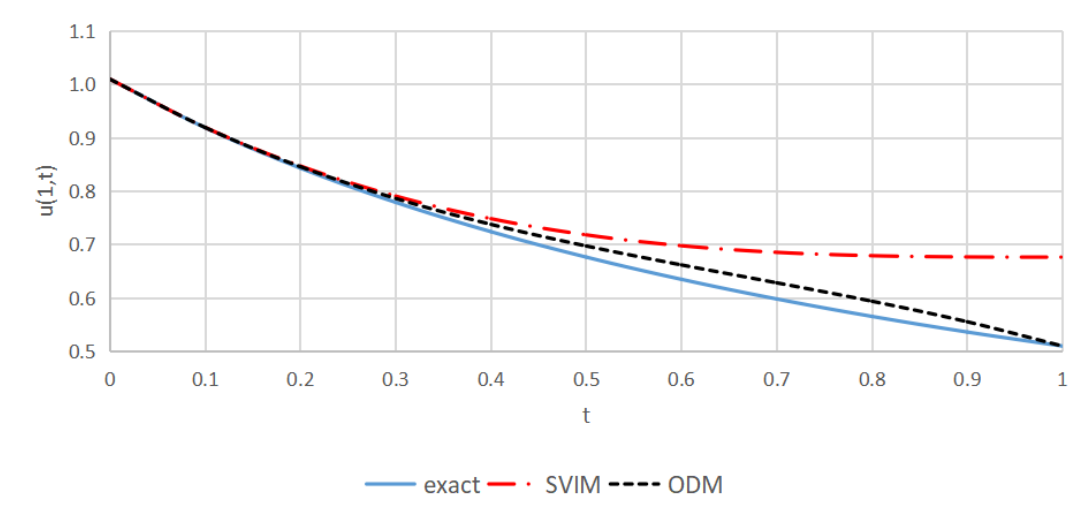

Figure 1: Comparison of ODM (3-term approximation) and SVIM (2-term approximation) solutions with the exact solution for $\alpha = 1$ at $x = 1$. The superior convergence of ODM is evident, with the 3-term ODM solution nearly overlapping the exact solution, while SVIM shows significant deviation, particularly for $t > 0.5$. This demonstrates the accelerated convergence achieved through the optimized linearization in ODM.

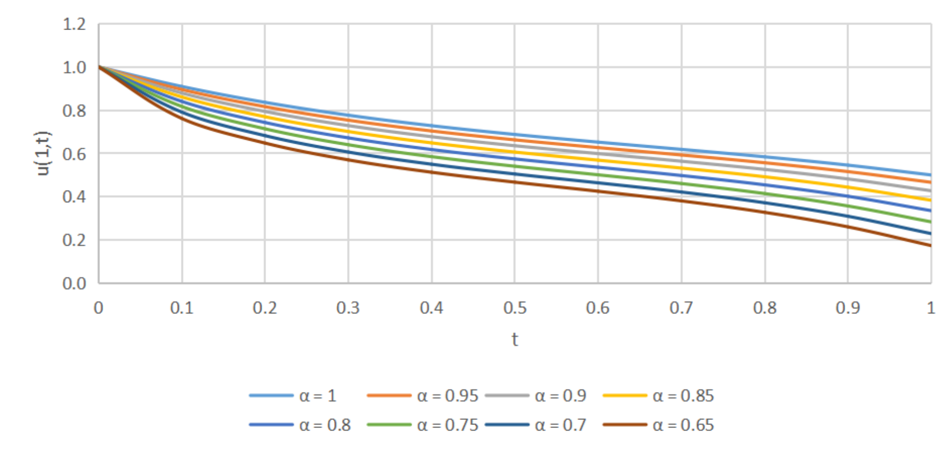

Figure 2: Behavior of ODM solutions for the fractional Burgers' equation (6.10) with varying α values. The figure demonstrates the continuous transition from fractional to classical dynamics, highlighting the effect of fractional differentiation on temporal behavior and wave propagation characteristics.

References

[1] A. A. Kilbas, H. M. Srivastava, and J. J. Trujillo, *Theory and Applications of Fractional Differential Equations*, vol. 204. Elsevier, 2006. 1

[2] M. Caputo and M. Fabrizio, "A new definition of fractional derivative without singular kernel," *Progress in Fractional Differentiation and Applications*, vol. 1, no. 2, pp. 73–85, 2015, doi: 10.12785/pfda/010201. 1

[3] A. Atangana and D. Baleanu, "New fractional derivatives with non-local and nonsingular kernel: Theory and application to heat transfer model," *Thermal Science*, vol. 20, pp. 763–769, 2016, doi: 10.2298/TSCI160111018A. 1

[4] K. M. Saad and A. I. Al-Sharif, "Analytical study for time and time-space fractional Burgers' equation," *Advances in Difference Equations*, 2017, Art. no. 300, doi: 10.1186/s13662-017-1358-0. 1

[5] J. Biazar and H. Aminikhah, "Exact and numerical solutions for non-linear Burger's equation by VIM," *Mathematical and Computer Modelling*, vol. 49, pp. 1394–1400, 2009. 1

[6] M. Inc, "The approximate and exact solutions of the space- and time-fractional Burgers equations with initial conditions by variational iteration method," *Journal of Mathematical Analysis and Applications*, vol. 345, pp. 476–484, 2008. 1

[7] M. Dehghan, A. Hamidi, and M. Shakourifar, "The solution of coupled Burgers' equations using Adomian–Padé technique," *Applied Mathematics and Computation*, vol. 189, pp. 1034–1047, 2007. 1

[8] C. A. J. Fletcher, "Generating exact solutions of the two-dimensional Burgers' equation," *International Journal for Numerical Methods in Fluids*, vol. 3, pp. 213–216, 1983. 1

[9] R. Abazari and A. Borhanifar, "A numerical study of solution of the Burgers' equations by a differential transformation method," *Computers & Mathematics with Applications*, vol. 59, pp. 2711–2722, 2010. 1

[10] M. J. Ablowitz and P. A. Clarkson, *Solitons, Nonlinear Evolution Equation and Inverse Scattering.* Cambridge University Press, 1991. 1

[11] H. Eltayeb and I. Bachar, "A note on singular two-dimensional fractional coupled Burgers' equation and triple Laplace Adomian decomposition method," *Boundary Value Problems*, 2020, Art. no. 129, doi: 10.1186/s13661-020-01426-0. 1

[12] H. K. Jassim and S. A. Khafif, "SVIM for solving Burger's and coupled Burger's equations of fractional order," *Progress in Fractional Differentiation and Applications*, vol. 7, no. 1, pp. 73–78, 2021, doi: 10.18576/pfda/070107. 1, 2, 6.2

[13] M. Shafiq et al., "Numerical solutions of time fractional Burgers' equation involving Atangana–Baleanu derivative via cubic B-spline functions," *Results in Physics*, vol. 34, Art. no. 105244, 2022, doi: 10.1016/j.rinp.2022.105244. 1

[14] S. Kazem, "Exact solution of some linear fractional differential equations by Laplace transform," *International Journal of Nonlinear Science*, vol. 16, pp. 3–11, 2013. 3.1, 3.2

[15] R. K. Alhefthi and H. Eltayeb, "The solution of coupled Burgers' equation by G-Laplace transform," *Symmetry*, vol. 15, Art. no. 1764, 2023, doi: 10.3390/sym15091764. 1

[16] G. Agarwal et al., "Two analytical approaches for space- and time-fractional coupled Burger's equations via Elzaki transform," *Progress in Fractional Differentiation and Applications*, vol. 8, no. 1, pp. 177–190, 2022, doi: 10.18576/pfda/080111. 1

[17] M. Elbadri, "An approximate solution of a time fractional Burgers' equation involving the Caputo–Katugampola fractional derivative," *Partial Differential Equations in Applied Mathematics*, 2023, doi: 10.1016/j.padiff.2023.100560. 1

[18] A. Ahmed, "Hussein–Jassim method for solving fractional ordinary delay differential equations," *Open Journal of Mathematical Analysis*, vol. 9, no. 1, pp. 37–43, 2023, doi: 10.30538/psrp-oma2025.0152. 3.1, 3.2

[19] Z. Odibat, "An optimized decomposition method for nonlinear ordinary and partial differential equations," *Physica A*, Art. no. 123323, 2019, doi: 10.1016/j.physa.2019.123323. 1

[20] Z. Odibat, "The optimized decomposition method for a reliable treatment of IVPs for second order differential equations," *Physica Scripta*, vol. 96, 2021. 1

[21] M. Laoubi, Z. Odibat, and B. Maayah, "Effective optimized decomposition algorithms for solving nonlinear fractional differential equations," *Journal of Computational and Nonlinear Dynamics*, vol. 18, 2023, doi: 10.1115/1.4056254. 1

[22] G. Adomian, *Solving Frontier Problems of Physics: The Decomposition Method*. Kluwer Academic Publishers, 1994. 4.2

[23] G. Adomian, "A review of the decomposition method in applied mathematics," *Journal of Mathematical Analysis and Applications*, vol. 135, no. 2, pp. 501–544, 1988. 4.2

[24] J. M. Burgers, "A mathematical model illustrating the theory of turbulence," *Advances in Applied Mechanics*, vol. 1, pp. 171–199, 1948. 2, 2

[25] H. Bateman, "Some recent researches on the motion of fluids," *Monthly Weather Review*, vol. 43, no. 4, pp. 163–170, 1915. 2

[26] S. Albeverio, A. Korshunova, and O. Rozanova, "A probabilistic model associated with the pressureless gas dynamics," *Bulletin des Sciences Mathématiques*, vol. 137, no. 7, pp. 902–922, 2013. 2

[27] Y. Çenesiz, D. Baleanu, A. Kurt, and O. Tasbozan, "New exact solutions of Burgers' type equations with conformable derivative," *Waves in Random and Complex Media*, vol. 27, no. 1, pp. 103–116, 2017. 2

[28] A. Kurt, Y. Çenesiz, and O. Tasbozan, "On the solution of Burgers' equation with the new fractional derivative," *Open Physics*, vol. 13, no. 1, 2015. 2

[29] M. Senol, O. Tasbozan, and A. Kurt, "Numerical solutions of fractional Burgers' type equations with conformable derivative," *Chinese Journal of Physics*, vol. 58, pp. 75–84, 2019. 2

[30] A. Ahmed, "Laplace optimized decomposition method for solving fractional order logistic growth in a population," *International Journal of Nonlinear Analysis and Applications*, 2025, doi: 10.22075/ijnaa.2024.32062.4755. 1

[31] H. Jafari and V. Daftardar-Gejji, "Solving a system of nonlinear fractional differential equations using Adomian decomposition," *Journal of Computational and Applied Mathematics*, vol. 196, no. 2, pp. 644–651, 2006, doi: 10.1016/j.cam.2005.10.017. 1

[32] H. K. Jassim and M. Abdulshareef Hussein, "A new approach for solving nonlinear fractional ordinary differential equations," *Mathematics*, vol. 11, no. 7, Art. no. 1565, 2023, doi: 10.3390/math11071565. 1

[33] V. M. Tripathi, H. M. Srivastava, H. Singh, C. Swarup, and S. Aggarwal, "Mathematical analysis of non-isothermal reaction–diffusion models arising in spherical catalyst and spherical biocatalyst," *Applied Sciences*, vol. 11, no. 21, Art. no. 10423, 2021, doi: 10.3390/app112110423. 1

[34] M. Higazy, S. Aggarwal, and T. A. Nofal, "Sawi decomposition method for Volterra integral equation with application," *Journal of Mathematics*, vol. 2020, pp. 1–13, 2020, doi: 10.1155/2020/6687134. 1

[35] S. Aggarwal and K. Bhatnagar, "Dualities between Laplace transform and some useful integral transforms," *International Journal of Engineering and Advanced Technology*, vol. 9, no. 1, pp. 936–941, 2019, doi: 10.35940/ijeat.a9433.109119. 1

[36] P. Cui and H. K. Jassim, "Local fractional Sumudu decomposition method to solve fractal PDEs arising in mathematical physics," *Fractals*, vol. 32, no. 04, 2024, doi: 10.1142/s0218348x24400292. 1

[37] S. A. Sachit and H. K. Jassim, "Solving fractional PDEs by Elzaki homotopy analysis method," in *International Conference of Computational Methods in Sciences and Engineering (ICCMSE 2021)*, vol. 2611, Art. no. 040074, 2023, doi: 10.1063/5.0115742. 1

[38] D. Baleanu and H. K. Jassim, "A modification fractional homotopy perturbation method for solving Helmholtz and coupled Helmholtz equations on Cantor sets," *Fractal and Fractional*, vol. 3, no. 2, Art. no. 30, 2019, doi: 10.3390/fractalfract3020030. 1

[39] H. Jafari and H. Jassim, "Local fractional Laplace variational iteration method for solving nonlinear partial differential equations on Cantor sets within local fractional operators," *Journal of Zankoy Sulaimani - Part A*, vol. 16, no. 4, pp. 49–57, 2014, doi: 10.17656/jzs.10345. 1

[40] M. A. Abdoon, M. Elbadri, A. B. M. Alzahrani, M. Berir, and A. Ahmed, "Analyzing the inverted fractional Rössler system through two approaches: numerical scheme and LHAM," *Physica Scripta*, vol. 99, no. 11, Art. no. 115220, 2024, doi: 10.1088/1402-4896/ad7f01. 1

[41] D. Baleanu and H. K. Jassim, "Approximate analytical solutions of Goursat problem within local fractional operators," *Journal of Nonlinear Science and Applications*, vol. 9, pp. 4829–4837, 2016. 1

[42] D. Baleanu and H. K. Jassim, "Exact solution of two-dimensional fractional partial differential equations," *Fractal and Fractional*, vol. 4, no. 21, pp. 1–9, 2020. 1

[43] D. Baleanu and H. K. Jassim, "Approximate solutions of the damped wave equation and dissipative wave equation in fractal strings," *Fractal and Fractional*, vol. 3, no. 26, pp. 1–12, 2019. 1

[44] H. K. Jassim and H. A. Kadhim, "Fractional Sumudu decomposition method for solving PDEs of fractional order," *Journal of Applied and Computational Mechanics*, vol. 7, no. 1, pp. 302–311, 2021. 1

[45] H. K. Jassim, "A new approach to find approximate solutions of Burger's and coupled Burger's equations of fractional order," *TWMS Journal of Applied and Engineering Mathematics*, vol. 11, no. 2, pp. 415–423, 2021. 1

[46] L. K. Alzaki and H. K. Jassim, "The approximate analytical solutions of nonlinear fractional ordinary differential equations," *International Journal of Nonlinear Analysis and Applications*, vol. 12, no. 2, pp. 527–535, 2021. 1

[47] H. K. Jassim, H. Ahmad, A. Shamaoon, C. Cesarano, and C. Cesarano, "An efficient hybrid technique for the solution of fractional-order partial differential equations," *Carpathian Mathematical Publications*, vol. 13, no. 3, pp. 790–804, 2021. 1

[48] H. K. Jassim, "New approaches for solving Fokker Planck equation on Cantor sets within local fractional operators,"

Journal of Mathematics, vol. 2015, Art. no. 1–8, 2015. 1

[49] P. Agarwal, A. A. El-Sayed, and J. Tariboon, "Vieta–Fibonacci operational matrices for spectral solutions of variable-order fractional integro-differential equations," *Journal of Computational and Applied Mathematics*, vol. 382, Art. no. 113063, Jan. 2021, doi: 10.1016/j.cam.2020.113063. 1

[50] A. A. El-Sayed, D. Baleanu, and P. Agarwal, "A novel Jacobi operational matrix for numerical solution of multi-term variable-order fractional differential equations," *Journal of Taibah University for Science*, vol. 14, no. 1, pp. 963–974, Jan. 2020, doi: 10.1080/16583655.2020.1792681. 1

[51] X. Zhang, P. Agarwal, Z. Liu, and H. Peng, "The general solution for impulsive differential equations with Riemann–Liouville fractional-order q a (1,2)," *Open Mathematics*, vol. 13, no. 1, Dec. 2015, doi: 10.1515/math-2015-0073. 1

[52] S. K. Ntouyas, P. Agarwal, and J. Tariboon, "On Pólya-Szegö and Chebyshev types inequalities involving the Riemann–Liouville fractional integral operators," *Journal of Mathematical Inequalities*, no. 2, pp. 491–504, 2016, doi: 10.7153/jmi-10-38. 1

[53] S. Rashid, K. T. Kubra, S. Sultana, P. Agarwal, and M. S. Osman, "An approximate analytical view of physical and biological models in the setting of Caputo operator via Elzaki transform decomposition method," *Journal of Computational and Applied Mathematics*, vol. 413, Art. no. 114378, Oct. 2022, doi: 10.1016/j.cam.2022.114378. 1

[54] İ. O. Kıymaz, P. Agarwal, S. Jain, and A. Çetinkaya, "On a New Extension of Caputo Fractional Derivative Operator," *Advances in Real and Complex Analysis with Applications*, pp. 261–275, 2017, doi: 10.1007/978-981-10-4337-6_11. 1

[55] J. CHOI, P. AGARWAL, and S. JAIN, "Certain Fractional Integral Operators and Extended Generalized Gauss Hypergeometric Functions," *Kyungpook mathematical journal*, vol. 55, no. 3, pp. 695–703, Sep. 2015, doi: 10.5666/kmj.2015.55.3.695. 1