Fermi, Bose and Vicious Walk Configurations
on the Directed Square Lattice

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Abstract

Inui and Katori introduced Fermi walk configurations which are non-crossing subsets of the directed random walks between opposite corners of a rectangular $l \times w$ grid. They related them to Bose configurations which are similarly defined except that they include multisets. Bose configurations biject to vicious walker watermelon configurations. It is found that the maximum number of walks in a Fermi configuration is $lw + 1$ and the number of configurations corresponding to this number of walks is a w-dimensional Catalan number $C_{l,w}$. Product formulae for the numbers of Fermi configurations with lw and $lw - 1$ walks are derived. We also consider generating functions for the numbers of n−walk configurations as a function of l and w. The Bose generating function is rational with denominator $(1 - z)^{lw+1}$. The Fermi generating function is found to have a factor $(1 + z)^{l+w}$ and the complementary factor $Q_{l,w}^{Fermi}(z)$ is related to the numerator of the Bose generating function which is a generalized Naryana polynomial introduced by Sulanke. Recurrence relations for the numbers of Fermi walks and for the coefficients of the polynomial $Q_{l,w}^{Fermi}(z)$ are obtained.

Keywords: Lattice paths, enumerative combinatorics, Fermi walks, interacting random walks, flows, d-dimensional Naryana and Catalan numbers.
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