Available online at http://jprm.sms.edu.pk/
Journal of Prime Research in Mathematics, 17(1) (2021), 40–47

Journal of Prime Research in Mathematics

Reversed degree-based topological indices for Benzenoid systems

Abdul Jalil M. Khalaf[a], Abaid ur Rehman Virk[b,*], Ashaq Ali[c], Murat Cancan[d]

[a]Department of Mathematics, Faculty of Computer Science and Mathematics University of Kufa, Najaf, Iraq.
[b]Department of Mathematics, University of Sialkot, Sialkot, Pakistan.
[c]Department of Mathematics and Statistics, The University of Lahore, Lahore, Pakistan.
[d]Faculty of Education, Van Yznc Yil University, Van, Turkey.

Abstract

Topological indices are numerical values that correlate the chemical structures with physical properties. In this article, we compute some reverse topological indices namely reverse Atom-bond connectivity index and reverse Geometric-arithmetic index for four different types of Benzenoid systems.

Keywords: Topological indices, Reverse topological indices, Benzenoid systems.
2010 MSC: 15A09, 47A10, 65F40, 65F15.

1. Introduction

Chemical graph theory is an effective branch of mathematics that provides us tools to gain information about chemical structures [1, 2]. A topological index is a numeric quantity, that best describe the topology of chemical graph.[3, 4].

Estrada [5] introduced the idea of Atom-bond connectivity index. Actually, Atom-bond connectivity index is the amended version of the first genuine degree-based topological index, that was put forward by Milan Randić in 1975, in his seminal paper [6], On characterization of molecular branching named as Branching index but after some period of time it was named as connectivity index. Now a days it in known as Randić index. Atom-bond connectivity index can be abbreviated as ABC index. It is defined as

$$ABC(G) = \sum_{uv\epsilon E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u.d_v}}.$$

*Corresponding author
Email addresses: abduljaleel.khalaf@uokufa.edu.iq (Abdul Jalil M. Khalaf), abaid.math@gmail.com (Abaid ur Rehman Virk), arhamusman2001@gmail.com (Ashaq Ali), m_cencen@yahoo.com (Murat Cancan)

Received : 2 February 2021*; Accepted:* 29 May 2021*; Published Online:* 10 June 2021.

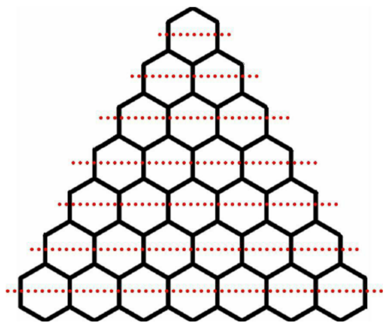

Figure 1: Triangular benzenoid system

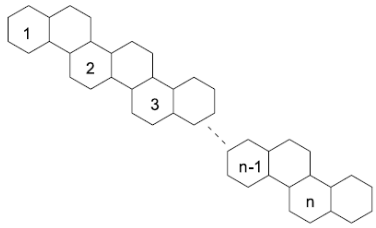

Figure 2: Zigzag benzenoid system

For explanation of further properties of ABC index, we refer [6].

Another degree based topological index that utilizes the difference between the geometric and arithmetic means [7, 8] was invented in [9] as

$$GA(G) = \sum_{uv\epsilon E(G)} \frac{\sqrt{d_u.d_v}}{\frac{1}{2}[d_u + d_v]}.$$

Motivated by the idea of reverse topological indices [10], the idea of reverse Atom-bond connectivity index and reverse Geometric-arithmetic index was put forward in [11] as

$$CABC(G) = \sum_{uv\epsilon E(G)} \sqrt{\frac{c_u + c_v - 2}{c_u.c_v}},$$

and

$$CGA(G) = \sum_{uv\epsilon E(G)} \frac{\sqrt{c_u.c_v}}{\frac{1}{2}[c_u + c_v]},$$

where, $c_u = \Delta(G) - d_G(v) + 1$. For more about topological indices one can find [12, 13, 14].

In this paper, we aim to compute some reverse topological indices namely reverse Atom-bond connectivity index and reverse Geometric-arithmetic index for four different types of Benzenoid Systems.

2. Computational Results

In this section, we compute reversed Atom-bond connectivity index (CABC) and reverse Geometric-arithmetic index (CGA) for Triangular benzenoid system T_n (Figure 1), Zigzag benzenoid system Z_n (Figure 2), Rhombic benzenoid system R_n (Figure 3) and Hourglass benzenoid system X_p (Figure 4).

2.1. Triangular Benzenoid System T_n

Let T_n be a Triangular benzenoid system where n is the number of hexagons in graph and total quantity of hexagons in T_n is $\frac{1}{2}n(n+1)$ The vertex and edge set of T_n are

$$V(T_n) = n^2 + 4n + 1,$$

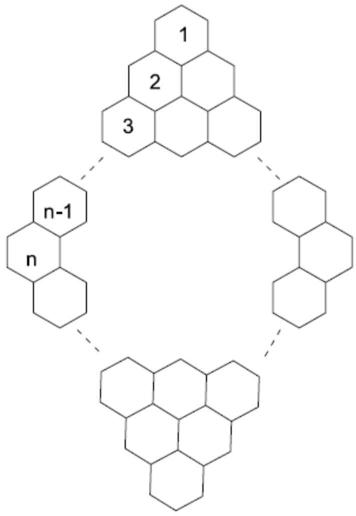

Figure 3: Rhombic benzenoid system

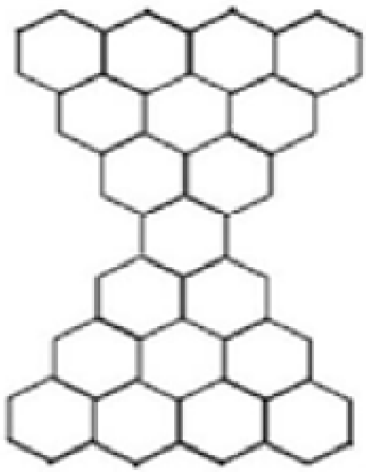

Figure 4: Hourglass benzenoid System

and

$$E(T_n) = \frac{3}{2}n(n+3)$$

respectively.

For the Triangular benzenoid system T_n there are following three types of edges in edge set of T_n;

$$\begin{aligned} E_1(T_n) &= \{uv\epsilon E(T_n); d_u = 2, d_v = 2\} \;\; ; \;\; |E_1(T_n)| = 6, \\ E_2(T_n) &= \{uv\epsilon E(T_n); d_u = 2, d_v = 3\} \;\; ; \;\; |E_2(T_n)| = 6(n-1), \\ E_3(T_n) &= \{uv\epsilon E(T_n); d_u = 3, d_v = 3\} \;\; ; \;\; |E_3(T_n)| = \frac{3}{2}n(n-1). \end{aligned}$$

The reverse edge set of T_n is given as,

$$\begin{aligned} CE_1(T_n) &= \{uv\epsilon E(T_n); d_u = 2, d_v = 2\} \;\; ; \;\; |CE_1(T_n)| = 6, \\ CE_2(T_n) &= \{uv\epsilon E(T_n); d_u = 2, d_v = 1\} \;\; ; \;\; |CE_2(T_n)| = 6(n-1), \\ CE_3(T_n) &= \{uv\epsilon E(T_n); d_u = 1, d_v = 1\} \;\; ; \;\; |CE_3(T_n)| = \frac{3}{2}n(n-1). \end{aligned}$$

Theorem 2.1. *Let T_n be the Triangular benzenoid System, then we have*

1. $CABC(T_n) = \frac{6}{\sqrt{2}}n$,
2. $CGA(T_n) = \frac{3}{2}n^2 + \frac{1}{2}(8\sqrt{2}-3)n + 2(3-2\sqrt{2})$.

Proof. Using the reverse edge partition of T_n we have following computations for our results;

1.

$$\begin{aligned} CABC(T_n) &= \sum_{uv\epsilon E(T_n)} \sqrt{\frac{c_u(T_n)+c_v(T_n)-2}{c_u(T_n).c_v(T_n)}} \\ &= \sum_{uv\epsilon E_1(T_n)} \sqrt{\frac{c_u(T_n)+c_v(T_n)-2}{c_u(T_n).c_v(T_n)}} + \sum_{uv\epsilon E_2(T_n)} \sqrt{\frac{c_u(T_n)+c_v(T_n)-2}{c_u(T_n).c_v(T_n)}} \\ &+ \sum_{uv\epsilon E_3(T_n)} \sqrt{\frac{c_u(T_n)+c_v(T_n)-2}{c_u(T_n).c_v(T_n)}} \\ &= (6)\sqrt{\frac{2+2-2}{2.2}} + [6(n-1)]\sqrt{\frac{2+1-2}{2.1}} + \left[\frac{3}{2}n(n-1)\right]\sqrt{\frac{1+1-2}{1.1}} \\ &= \frac{6}{\sqrt{2}}n. \end{aligned}$$

2.

$$\begin{aligned} CGA(T_n) &= \sum_{uv\epsilon E(T_n)} \frac{\sqrt{c_u(T_n).c_v(T_n)}}{\frac{1}{2}[c_u(T_n)+c_v(T_n)]} \\ &= \sum_{uv\epsilon E_1(T_n)} \frac{\sqrt{c_u(T_n).c_v(T_n)}}{\frac{1}{2}[c_u(T_n)+c_v(T_n)]} + \sum_{uv\epsilon E_2(T_n)} \frac{\sqrt{c_u(T_n).c_v(T_n)}}{\frac{1}{2}[c_u(T_n)+c_v(T_n)]} \\ &+ \sum_{uv\epsilon E_3(T_n)} \frac{\sqrt{c_u(T_n).c_v(T_n)}}{\frac{1}{2}[c_u(T_n)+c_v(T_n)]} \\ &= (6)\frac{\sqrt{2.2}}{\frac{1}{2}[2+2]} + [6(n-1)]\frac{\sqrt{2.1}}{\frac{1}{2}[2+1]} + \left[\frac{3}{2}n(n-1)\right]\frac{\sqrt{1.1}}{\frac{1}{2}[1+1]} \\ &= \frac{3}{2}n^2 + \frac{1}{2}(8\sqrt{2}-3)n + 2(3-2\sqrt{2}). \end{aligned}$$

□

2.2. Zigzag benzenoid system Z_p

Zigzag benzenoid system is denoted by Z_p, where p is the number of rows in graph of Z_p and each row consists of two hexagons as shown in Figure 2. For the Zigzag Benzenoid System Z_p there are following three types of edges;

$$E_1(Z_p) = \{uv\epsilon E(Z_p); d_u = 2, d_v = 2\} \quad ; \quad |E_1(Z_p)| = 2p + 4,$$
$$E_2(Z_p) = \{uv\epsilon E(Z_p); d_u = 2, d_v = 3\} \quad ; \quad |E_2(Z_p)| = 4p,$$
$$E_3(Z_p) = \{uv\epsilon E(Z_p); d_u = 3, d_v = 3\} \quad ; \quad |E_3(Z_p)| = 4p - 3.$$

The maximum edge degree in edge set of T_n is 3 so the reverse edge set of Z_p is given as,

$$CE_1(Z_p) = \{uv\epsilon E(Z_p); d_u = 2, d_v = 2\} \quad ; \quad |CE_1(Z_p)| = 2p + 4,$$
$$CE_2(Z_p) = \{uv\epsilon E(Z_p); d_u = 2, d_v = 1\} \quad ; \quad |CE_2(Z_p)| = 4p,$$
$$CE_3(Z_p) = \{uv\epsilon E(Z_p); d_u = 1, d_v = 1\} \quad ; \quad |CE_3(Z_p)| = 4p - 3.$$

Theorem 2.2. *Let Z_p be the graph of Zigzag benzenoid system, then we have*

1. $CABC(Z_p) = 3\sqrt{2}p + 2\sqrt{2}$,
2. $CGA(Z_p) = \frac{2}{3}(9 + 4\sqrt{2})p + 1$.

Proof. Using the reverse edge partition of Z_p we have following computations for our results;

1.

$$\begin{aligned}
CABC(Z_p) &= \sum_{uv\epsilon E(Z_p)} \sqrt{\frac{c_u(Z_p) + c_v(Z_p) - 2}{c_u(Z_p).c_v(Z_p)}} \\
&= \sum_{uv\epsilon E_1(Z_p)} \sqrt{\frac{c_u(Z_p) + c_v(Z_p) - 2}{c_u(Z_p).c_v(Z_p)}} + \sum_{uv\epsilon E_2(Z_p)} \sqrt{\frac{c_u(Z_p) + c_v(Z_p) - 2}{c_u(Z_p).c_v(Z_p)}} \\
&\quad + \sum_{uv\epsilon E_3(Z_p)} \sqrt{\frac{c_u(Z_p) + c_v(Z_p) - 2}{c_u(Z_p).c_v(Z_p)}} \\
&= (2p + 4)\sqrt{\frac{2 + 2 - 2}{2.2}} + (4p)\sqrt{\frac{2 + 1 - 2}{2.1}} + (4p - 3)\sqrt{\frac{1 + 1 - 2}{1.1}} \\
&= 3\sqrt{2}p + 2\sqrt{2}.
\end{aligned}$$

2.

$$\begin{aligned}
CGA(Z_p) &= \sum_{uv\epsilon E(Z_p)} \frac{\sqrt{c_u(Z_p).c_v(Z_p)}}{\frac{1}{2}[c_u(Z_p) + c_v(Z_p)]} \\
&= \sum_{uv\epsilon E_1(Z_p)} \frac{\sqrt{c_u(Z_p).c_v(Z_p)}}{\frac{1}{2}[c_u(Z_p) + c_v(Z_p)]} + \sum_{uv\epsilon E_2(Z_p)} \frac{\sqrt{c_u(Z_p).c_v(Z_p)}}{\frac{1}{2}[c_u(Z_p) + c_v(Z_p)]} \\
&\quad + \sum_{uv\epsilon E_3(Z_p)} \frac{\sqrt{c_u(Z_p).c_v(Z_p)}}{\frac{1}{2}[c_u(Z_p) + c_v(Z_p)]} \\
&= (2p + 4)\frac{\sqrt{2.2}}{\frac{1}{2}[2 + 2]} + (4p)\frac{\sqrt{2.1}}{\frac{1}{2}[2 + 1]} + (4p - 3)\frac{\sqrt{1.1}}{\frac{1}{2}[1 + 1]} \\
&= \frac{2}{3}(9 + 4\sqrt{2})p + 1.
\end{aligned}$$

□

2.3. Rhombic benzenoid system R_n

Take another benzenoid system in which hexagons are arranged to form a rhombic shape R_n, in which there are n rows of n hexagons as given in Figure 3. The vertex and edge set of Rhombic Benzenoid System R_n is given as,

$$V(R_n) = 2n(n+2)$$

and edge set is

$$E(R_n) = 3n^2 + 4n - 1$$

respectively.

There are following type of edges in Rhombic benzenoid system R_n;

$$\begin{aligned}
E_1(R_n) &= \{uv\epsilon E(R_n); d_u = 2, d_v = 2\} \;\; ; \;\; |E_1(R_n)| = 6,\\
E_2(R_n) &= \{uv\epsilon E(R_n); d_u = 2, d_v = 3\} \;\; ; \;\; |E_2(R_n)| = 8(n-1),\\
E_3(R_n) &= \{uv\epsilon E(R_n); d_u = 3, d_v = 3\} \;\; ; \;\; |E_3(R_n)| = 3n^2 - 4n + 1.
\end{aligned}$$

The maximum edge degree in edge set of R_n is 3, then the reverse edge set of R_n is given as,

$$\begin{aligned}
CE_1(R_n) &= \{uv\epsilon E(R_n); d_u = 2, d_v = 2\} \;\; ; \;\; |CE_1(R_n)| = 6,\\
CE_2(R_n) &= \{uv\epsilon E(R_n); d_u = 2, d_v = 1\} \;\; ; \;\; |CE_2(R_n)| = 8(n-1),\\
CE_3(R_n) &= \{uv\epsilon E(R_n); d_u = 1, d_v = 1\} \;\; ; \;\; |CE_3(R_n)| = 3n^2 - 4n + 1.
\end{aligned}$$

Theorem 2.3. *Let R_n be the graph of Rhombic benzenoid system R_n, then we have*

1. $CABC(R_n) = \frac{8}{\sqrt{2}}n - \sqrt{2}$,
2. $CGA(R_n) = 3n^2 + \frac{4}{3}(4\sqrt{2} - 3) + \frac{1}{3}(21 - 16\sqrt{2})$.

Proof. Using the reverse edge partition of R_n we have following computations for our results;

1.

$$\begin{aligned}
CABC(R_n) &= \sum_{uv\epsilon E(R_n)} \sqrt{\frac{c_u(R_n) + c_v(R_n) - 2}{c_u(R_n).c_v(R_n)}}\\
&= \sum_{uv\epsilon E_1(R_n)} \sqrt{\frac{c_u(R_n) + c_v(R_n) - 2}{c_u(R_n).c_v(R_n)}} + \sum_{uv\epsilon E_2(R_n)} \sqrt{\frac{c_u(R_n) + c_v(R_n) - 2}{c_u(R_n).c_v(R_n)}}\\
&\quad + \sum_{uv\epsilon E_3(R_n)} \sqrt{\frac{c_u(R_n) + c_v(R_n) - 2}{c_u(R_n).c_v(R_n)}}\\
&= (6)\sqrt{\frac{2+2-2}{2.2}} + 8(n-1)\sqrt{\frac{2+1-2}{2.1}} + (3n^2 - 4n + 1)\sqrt{\frac{1+1-2}{1.1}}\\
&= \frac{8}{\sqrt{2}}n - \sqrt{2}.
\end{aligned}$$

2.

$$\begin{aligned}
CGA(R_n) &= \sum_{uv\epsilon E(R_n)} \frac{\sqrt{c_u(R_n).c_v(R_n)}}{\frac{1}{2}[c_u(R_n) + c_v(R_n)]}\\
&= \sum_{uv\epsilon E_1(R_n)} \frac{\sqrt{c_u(R_n).c_v(R_n)}}{\frac{1}{2}[c_u(R_n) + c_v(R_n)]} + \sum_{uv\epsilon E_2(R_n)} \frac{\sqrt{c_u(R_n).c_v(R_n)}}{\frac{1}{2}[c_u(R_n) + c_v(R_n)]}
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{uv\epsilon E_3(R_n)}\frac{\sqrt{c_u(R_n).c_v(R_n)}}{\frac{1}{2}[c_u(R_n)+c_v(R_n)]}\\
=&(6)\frac{\sqrt{2.2}}{\frac{1}{2}[2+2]}+8(n-1)\frac{\sqrt{2.1}}{\frac{1}{2}[2+1]}+(3n^2-4n+1)\frac{\sqrt{1.1}}{\frac{1}{2}[1+1]}\\
=&3n^2+\frac{4}{3}(4\sqrt{2}-3)+\frac{1}{3}(21-16\sqrt{2}).
\end{aligned}
$$

□

2.4. Hourglass benzenoid system X_n

Let X_n be a Hourglass benzenoid system which is obtained from two copies of a triangular benzenoid T_n by overlapping their external hexagons. The vertex and edge sets of X_n are

$$V(X_n)=2(n^2+4n-2)$$

and

$$E(X_n)=3n^2+9n-4$$

respectively.

For the Triangular benzenoid system X_n there are following three types of edges in edge set of X_n;

$$
\begin{aligned}
E_1(X_n)&=\{uv\epsilon E(X_n); d_u=2, d_v=2\} \;;\; |E_1(X_n)|=8,\\
E_2(X_n)&=\{uv\epsilon E(X_n); d_u=2, d_v=3\} \;;\; |E_2(X_n)|=4(3n-4),\\
E_3(X_n)&=\{uv\epsilon E(X_n); d_u=3, d_v=3\} \;;\; |E_3(X_n)|=3n^2-3n+4.
\end{aligned}
$$

The maximum edge degree in edge set of X_n is 4, so the reverse edge set of X_n is given as,

$$
\begin{aligned}
CE_1(X_n)&=\{uv\epsilon E(X_n); d_u=2, d_v=2\} \;;\; |CE_1(X_n)|=8,\\
CE_2(X_n)&=\{uv\epsilon E(X_n); d_u=2, d_v=1\} \;;\; |CE_2(X_n)|=4(3n-4),\\
CE_3(X_n)&=\{uv\epsilon E(X_n); d_u=1, d_v=1\} \;;\; |CE_3(X_n)|=3n^2-3n+4.
\end{aligned}
$$

Theorem 2.4. *Let X_n be the graph of Hourglass Benzenoid System X_n, then we have*

1. $CABC(X_n)=6\sqrt{2}n-4\sqrt{2}$,
2. $CGA(X_n)=3n^2+(8\sqrt{2}-3)n+\frac{4}{3}(8\sqrt{2}+9)$.

Proof. Using the reverse edge partition of X_n we have following computations for our results;

1.

$$
\begin{aligned}
CABC(X_n)=&\sum_{uv\epsilon E(X_n)}\sqrt{\frac{c_u(X_n)+c_v(X_n)-2}{c_u(X_n).c_v(X_n)}}\\
=&\sum_{uv\epsilon E_1(X_n)}\sqrt{\frac{c_u(X_n)+c_v(X_n)-2}{c_u(X_n).c_v(X_n)}}+\sum_{uv\epsilon E_2(X_n)}\sqrt{\frac{c_u(X_n)+c_v(X_n)-2}{c_u(X_n).c_v(X_n)}}\\
&+\sum_{uv\epsilon E_3(X_n)}\sqrt{\frac{c_u(X_n)+c_v(X_n)-2}{c_u(X_n).c_v(X_n)}}\\
=&(8)\sqrt{\frac{2+2-2}{2.2}}+[4(3n-4)]\sqrt{\frac{2+1-2}{2.1}}+[3n^2-3n+4]\sqrt{\frac{1+1-2}{1.1}}\\
=&6\sqrt{2}n-4\sqrt{2}.
\end{aligned}
$$

2.

$$
\begin{aligned}
CGA(X_n) &= \sum_{uv\epsilon E(X_n)} \frac{\sqrt{c_u(X_n).c_v(X_n)}}{\frac{1}{2}[c_u(X_n)+c_v(X_n)]} \\
&= \sum_{uv\epsilon E_1(X_n)} \frac{\sqrt{c_u(X_n).c_v(X_n)}}{\frac{1}{2}[c_u(X_n)+c_v(X_n)]} + \sum_{uv\epsilon E_2(X_n)} \frac{\sqrt{c_u(X_n).c_v(X_n)}}{\frac{1}{2}[c_u(X_n)+c_v(X_n)]} \\
&\quad + \sum_{uv\epsilon E_3(X_n)} \frac{\sqrt{c_u(X_n).c_v(X_n)}}{\frac{1}{2}[c_u(X_n)+c_v(X_n)]} \\
&=(8)\frac{\sqrt{2.2}}{\frac{1}{2}[2+2]} + [4(3n-4)]\frac{\sqrt{2.1}}{\frac{1}{2}[2+1]} + [3n^2-3n+4]\frac{\sqrt{1.1}}{\frac{1}{2}[1+1]} \\
&=3n^2 + (8\sqrt{2}-3)n + \frac{4}{3}(8\sqrt{2}+9).
\end{aligned}
$$

□

3. Conclusion

With the help of topological index, we can assign a single number to a chemical structure. In quantitative structure activity/ property relationship, knowledge of topological indices plays an important rule. In this article, we compute some reverse topological indices for four benzenoid systems, namely Triangular benzenoid system T_n, Zigzag benzenoid system Z_p, Rhombic benzenoid system R_n and Hourglass benzenoid system X_n.

References

[1] Brckler, F. M., Dolic, T., Graovac, A., & Gutman, I. (2011). On a class of distance-based molecular structure descriptors. *Chemical physics letters, 503*(4-6), 336-338. 1

[2] Gonzalez-Diaz, H., Vilar, S., Santana, L., & Uriarte, E. (2007). Medicinal chemistry and bioinformatics-current trends in drugs discovery with networks topological indices. *Current Topics in Medicinal Chemistry, 7*(10), 1015-1029. 1

[3] Numan, M., Butt, S. I., Butt, S. I., & Taimur, A. (2021). Super cyclic antimagic covering for some families of graphs. *Open Journal of Mathematical Sciences, 5*(1), 27-33. 1

[4] Ajmal, M., Nazeer, W., Munir, M., Kang, S. M., & Jung, C. Y. (2017). The M-polynomials and topological indices of generalized prism network. *International Journal of Mathematical Analysis, 11*(6), 293-303. 1

[5] Estrada, E. (2008). Atombond connectivity and the energetic of branched alkanes. *Chemical Physics Letters, 463*(4-6), 422-425. 1

[6] Randic, M. (1975). Graph theoretical approach to local and overall aromaticity of benzenenoid hydrocarbons. *Tetrahedron, 31*(11-12), 1477-1481. 1

[7] Gutman, I., & Polansky, O. E. (2012). *Mathematical concepts in organic chemistry.* Springer Science & Business Media. 1

[8] Caporossi, G., Gutman, I., Hansen, P., & Pavlovic, L. (2003). Graphs with maximum connectivity index. *Computational Biology and Chemistry, 27*(1), 85-90. 1

[9] Nagesh, H. M & Girish, V. R. (2020). On the entire Zagreb indices of the line graph and line cut-vertex graph of the subdivision graph. *Open Journal of Mathematical Sciences, 4*(1), 470-475. 1

[10] Kulli, V. R. (2018). Reverse Zagreb and reverse hyper-Zagreb indices and their polynomials of rhombus silicate networks. *Annals of Pure and Applied Mathematics, 16*(1), 47-51. 1

[11] Virk. A. R., Rehman, M. A & Nazeer, W. (2019). New definition of atomic bond connectivity index to overcome deficiency of structure sensitivity and abruptness in existing definition. *Sci Inquiry Rev, 3*(4), 1-20. 1

[12] Gutman, I., & Das, K. C. (2004). The first Zagreb index 30 years after. *MATCH Commun. Math. Comput. Chem, 50*(1), 83-92. 1

[13] Shao, Z., Virk, A. R., Javed, M. S., Rehman, M. A. & Farahani, M. R. (2019).Degree based graph invariants for the molecular graph of Bismuth Tri-Iodide, *Eng. Appl. Sci. Lett.,2*(1),01-11. 1

[14] Virk, A. R., Jhangeer, M. N., & Rehman, M. A. (2018). Reverse Zagreb and reverse hyper-Zagreb indices for silicon carbide $Si_2C_3 - I[r,s]$ and $Si_2C_3 - II[r,s]$. *Eng. Appl. Sci. Lett.,1*(2), 37-48. 1