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On Metric Dimension of Chemical Networks

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Abstract

Metric Dimension of any graph G is termed as the minimum number of basis element in the resolving set. Let $G = (V, E)$ be any connected graph and length of the shortest path between s and h is known as distance, denoted by $d(s, h)$ in G. Let $B = \{b_1, b_2, ..., b_q\}$ be any ordered subset of V and representation $r(u|B)$ with respect to B is the $q-tuple$ $(d(u, b_1), d(u, b_2), d(u, b_3), ..., d(u, b_q)\}$, here B is called the resolving set or the locating set if every vertex of G is uniquely represented by distances from the vertices of B or if distinct vertices of G have distinct representations with respect to B. Any resolving set containing minimum cardinality is named as basis for G and its cardinality is the metric dimension of G is denoted by $dim(G)$. We investigated metric dimension of Polythiophene Network, Backbone Network, Hex-derive Network and $Nylone_{6,6}$.

Keywords: Graphs, Distance, Resolving sets, Metric dimension, Chemical network.
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1. Introduction

The name of the first mathematician who introduced the terminology of metric dimension of a connected graph is Slater in [1]. Same concept was also studied by Melter et al. independently in [2]. Another mathematician named F. Simon Raj et al. also studied the metric dimensions of various chemical networks as Star of David network $SD(n)$ in [3]. We need to study the concept of distance of graphs to get the clear concept about metric dimension of connected graphs clearly.

Suppose a connected graph G, then **distance in graph** is length of the shortest path between any two vertices s and h and it can be denoted as $d(s, h)$. Here is an example of ladder graph G given in $fig3.1$ in which different paths exist from vertex p to u but two of them are the shortest paths. First is p to q and then move to u and second path is from p to t and then move to u and both have length 2. So the distance between p and u is 2 $i.e$ $d(p, u) = 2$.

The maximum value of distance $d(p, u)$ is called **eccentricity** of the vertex .

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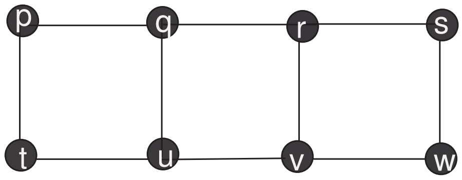

Figure 1: *LadderGraph*

Let assume a connected graph G and $L = \{l_1, l_2, ..., l_k\}$ is an ordered subset of set of vertices of G $i.e$ $V(G)$. The representation $r(m|L)$ of any fixed vertex m of graph G w.r.t L is the $k - tuple$ $(d(m, l_1), d(m, l_2), d(m, l_3), ..., d(m, l_k)\}$, where L is said to be a resolving set or locating set in [4, 1], if each vertex of graph G is uniquely determined by its distances from the vertices of L. **Basis** for the graph G can be defined as, " It is the resolving set containing least number of vertices. Minimum cardinality of the resolving set is considered as the **metric dimension** of graph G, which is labeled as $dim(G)$ in [5]. An adequate literature related to metric basis is discussed in [6, 2, 7].
Bounded metric dimension can be defined as, If the metric dimension of a connected graph changes when the number of vertices changes in the graph and remain finite when the number of vertices become infinite then it is called $boundedmetricdimension$.
Unbounded metric dimension can be defined as, If the metric dimension of a connected graph changes when the number of vertices changes in the graph, and becomes infinite when number of vertices is infinite then it is known as $unboundedmetricdimension$.
If the metric dimension remains unchanged for all number of vertices in a connected graph G, then it is said to be **constant metric dimension**. Path graph is the only graph which has metric dimension equal to 1 in [8] and metric dimension of cycles is 2 for $n \geq 3$.
It will not be wrong to say that metric dimension is the most important field of graph theory. It has lot of uses in various field of life, for instance in pattern recognition, network theory, image processing, optimization and robot navigation etc. The implementation of metric dimension is in space navigation. A work place can be represented by vertex and edge represents the link between the workplaces. We are to place minimum robots at certain vertices in such way that they can trace each and every vertex exactly one time, this problem of placing robots is solved by using the concept of metric dimension. In chemistry, many chemical compounds exists which have one chemical formula but different chemical structures but chemists choose only that one which expresses the best physical and chemical properties of compound. For this, chemists need such mathematical labeling of that chemical compound which gives different labeling to distinct compounds. Therefore, mathematical representation of different chemical compounds has many importance for chemists in drug discovery. A chemical compound structure is expressed by a graph which is labeled in such way that vertex of graph express the atom and edge of graph express bond types, mentioned in [9, 8]. So, theoretic description of graphs is discussed in the papers [10, 8, 7].

2. Polythiophene Network

Polythiophenes are five membered rings with one heteroatom together with their benzo and other carbocylic. The order of $PLY_{(m)}$ is $5m$ and the size of $PLY_{(m)}$ is $6m - 1$. Polythiophene are used in electronic devices like water purification devices, biosensers and light emitting diodes and in hydrogen storage.

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Theorem 1.1. For $G \cong PLY_{(m)}$; $m = 1$ then G has metric dimension 2.

Proof: $PLY_{(1)}$ is a cycle with 5 vertices as it is not path so its metric dimension is not 1 [9] and as it is C_3 so have metric dimension 2.

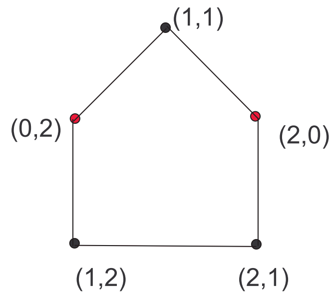

Figure 2: $PLY_{(1)}$

Theorem 1.2. For $G \cong PLY_{(m)}$; $m \geq 2$ then G has metric dimension is 2.

Proof: Let W = $(V_{1,5}, V_{\frac{n}{5},4})$
then
$V_{i,1} = (3i-2, \frac{3n}{5} - 3i + 2)$, for $1 \leq i \leq \frac{n}{5}$, $j = 1$
$V_{i,2} = (3i-1, \frac{3n}{5} + 1 - 3i)$, for $1 \leq i \leq \frac{n}{5}$, $j = 2$
$V_{i,3} = (3i-3, \frac{3n}{5} + 2 - 3i)$, for $1 \leq i \leq \frac{n}{5}$, $j = 3$
$V_{i,4} = (3i-1, \frac{3n}{5} - 3i)$, for $1 \leq i \leq \frac{n}{5}$, $j = 4$
$V_{i,5} = (3i-2, \frac{3n}{5} + 1 - 3i)$, for $1 \leq i \leq \frac{n}{5}$, $j = 5$.
Because Representation of each vertices with respect to W is unique.
$\Rightarrow$ W is resolving set of G and $|W| = 2$
Because G is not a path so metric dimension of G $\cong PLY_{(m)}$ is 2.
where n is numbers of vertices.

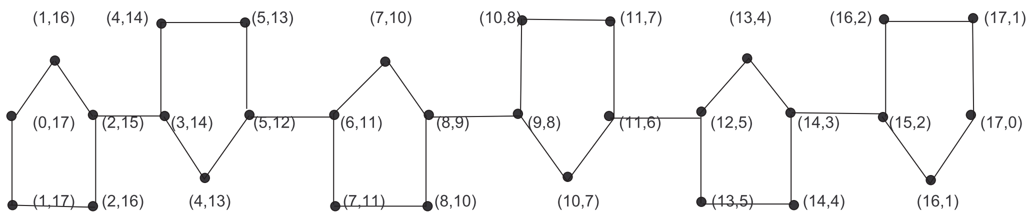

Figure 3: $PLY_{(6)}$

1.1. Backbone DNA Network

DNA consists of two strands that wind around each other like a twisted ladder.Each strand has a backbone made of deoxyribose sugar and a phosphate group.4-bases attached to each sugar are adenine,cytosine,guanine,thyaine.Both ends of DNA have a number.i-e one end is $\acute{5}$ and the other is $\acute{3}.\acute{\acute{5}}$ end having a phosphate group attached to $\acute{\acute{5}}$ carbon of ribose ring and $\acute{\acute{3}}$ end is usally unmodified.The order of $BS_{DNA}(m)$ is $7m - 2$ and the size of $BS_{DNA}(m)$ is $8m - 3$.DNA having genetic material essential for living things.DNA functions include replication,encoding informatiom,mutation or recombination and gene expression.

Theorem 1.3. *For $G \cong BS_{DNA}(m)$; $m = 1$ then G has metric dimension* 2.

Proof. $BS_{DNA}(1)$ is a cycle with five vertices as it is not path so its metric dimension is not 1 [9] and as it is C_5 so we have metric dimension 2. □

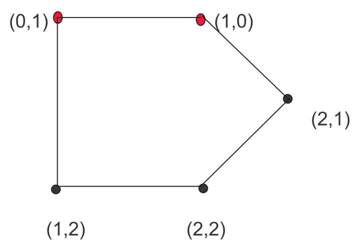

Figure 4: $BS_{DNA}(1)$

Theorem 1.4. *For $G \cong BS_{DNA}(m)$; $m \geq 1$ then G has metric dimension* 2.

Proof. Let W $= (V_{1,4}, V_{\frac{n+2}{7},5})$

then
$u_{i,1} = (4i-3, \frac{4(n+2)}{7} + 2 - 4i)$, for $1 \leq i \leq \frac{n+2}{7}$, $j = 1$
$u_{i,2} = (4i-2, \frac{4(n+2)}{7} + 2 - 4i)$, for $1 \leq i \leq \frac{n+2}{7}$, $j = 2$
$u_{i,3} = (4i-2, \frac{4(n+2)}{7} + 1 - 4i)$, for $1 \leq i \leq \frac{n+2}{7}$, $j = 3$
$u_{i,4} = (4i-4, \frac{4(n+2)}{7} + 1 - 4i)$, for $1 \leq i \leq \frac{n+2}{7}$, $j = 4$
$u_{i,5} = (4i-3, \frac{4(n+2)}{7} - 4i)$, for $1 \leq i \leq \frac{n+2}{7}$, $j = 5$
$u_{i,6} = (4i-2, \frac{4(n+2)}{7} - 1 - 4i)$, for $1 \leq i \leq \frac{n+2}{7}$, $j = 6$
$u_{i,7} = (4i-1, \frac{4(n+2)}{7} - 2 - 4i)$, for $1 \leq i \leq \frac{n+2}{7}$, $j = 7$
Because Representation of each vertices with respect to W is unique.
$\Rightarrow$ W is resolving set of G and $|W| = 2$ Because G is not a path so metric dimension of G $\cong BS_{DNA}(m)$ is 2.
where n is numbers of vertices. □

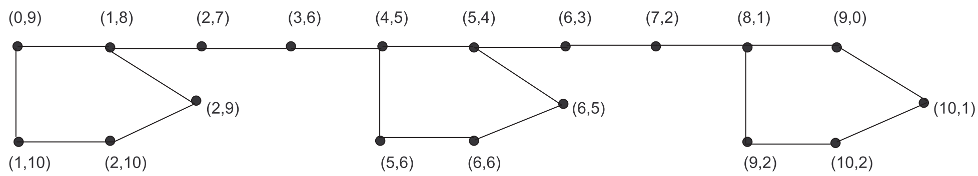

Figure 5: $BS_{DNA}(3)$

1.2. Hex-Derived Network $HDN1_{(n)}$

Hex-derived network $HDN1_{(n)}$ have simple structure. Just by adding extra node in every triangular face of hexagonal mesh. Total number of vertices and edges in $HDN1_{(n)}$ is $9n^2 - 15n + 7$ and $27n^2 - 51n + 24$ respectively. We named it mesh network. Mesh network is useful in minimizing the cost of computer networking and enhancing their performance and reliability.

Theorem 1.5. *For $G \cong HDN1_{(n)}$; $n = 2$ then dim(G) = 2.*

Proof. $HDN1_{(2)}$ is a cycle contains faces in which every face contains C_3 as it is not a path so its metric dimension is not 1 [9] and as it is C_3 so we have metric dimension 2. □

Theorem 1.6. *For $G \cong HDN1_{(n)}$; $n \geq 2$ then dim(G) > 2.*

Proof. Suppose on contrary $HDN_(n)$ has D as its resolving set with cardinality 2. Let $D = (c, a_i)$ be a resolving set.

$$r\left(\frac{a_i}{D}\right) = \begin{cases} (1,0), & for\ some\ i \in \mathbb{N}, \\ (1,2), & for\ some\ i \in \mathbb{N}. \end{cases} \tag{1.1}$$

which is contradiction.
Let $D = (c, b_i)$

$$r\left(\frac{a_i}{D}\right) = \begin{cases} (1,1), & for\ some\ i \in \mathbb{N}, \\ (1,2), & for\ some\ i \in \mathbb{N}. \end{cases} \tag{1.2}$$

which is contradiction.
Let $D = (a_i, a_{\acute{i}})$ where$a_i \neq a_{\acute{i}}$

$$r\left(\frac{a_i}{D}\right) = \begin{cases} (0,2), & for\ some\ i \in \mathbb{N}, \\ (2,0), & for\ some\ i \in \mathbb{N}, \\ (2,2), & for\ some\ i \in \mathbb{N} \end{cases} \tag{1.3}$$

which is contradiction.
Let $D = (b_j, b_{\acute{j}})$ where$b_i \neq b_{\acute{i}}$

$$r\left(\frac{a_i}{D}\right) = \begin{cases} (1,1), & for\ some\ i \in \mathbb{N}, \\ (1,2), & for\ some\ i \in \mathbb{N}, \\ (2,1), & for\ some\ i \in \mathbb{N}, \\ (2,2), & for\ some\ i \in \mathbb{N}. \end{cases} \tag{1.4}$$

which is contradiction.
Let $D = (a_i, b_j)$

$$r\left(\frac{a_i}{D}\right) = \begin{cases} (0,1), & for\ some\ i \in \mathbb{N}, \\ (2,1), & for\ some\ i \in \mathbb{N}, \\ (2,2), & for\ some\ i \in \mathbb{N}. \end{cases} \tag{1.5}$$

Similarly, there is no resolving set with two basis element. Hence dim(G) = 2 for $n \geq 2$. □

Theorem 1.7. *For $G \cong HDN1_{(n)}$; $n \geq 2$ then dim(G) > 3.*

Proof. Suppose on contrary $HDN_{(n)}$ has D as its resolving set with cardinality 3. Let $D = (c, a_i, a_{\acute{i}})$ be a resolving set. where$a_i \neq a_{\acute{i}}$

$$r\left(\frac{a_i}{D}\right) = \begin{cases} (1,0,2), & for\ some\ i \in \mathbb{N}, \\ (1,2,0), & for\ some\ i \in \mathbb{N}, \\ (1,2,2), & for\ some\ i \in \mathbb{N}. \end{cases} \tag{1.6}$$

which is contradiction.
Let $D = (c, b_j, b_{\acute{j}})$ be a resolving set where $b_j \neq b_{\acute{j}}$

$$r\left(\frac{a_i}{D}\right) = \begin{cases} (1,1,2), & for\ some\ i \in \mathbb{N}, \\ (1,1,1), & for\ some\ i \in \mathbb{N}, \\ (1,2,1), & for\ some\ i \in \mathbb{N}, \\ (1,2,2), & for\ some\ i \in \mathbb{N}. \end{cases} \tag{1.7}$$

which is contradiction.
Let $D = (c, a_i, b_j)$ be a resolving set.

$$r\left(\frac{a_i}{D}\right) = \begin{cases} (1,0,1), & for\ some\ i \in \mathbb{N}, \\ (1,2,1), & for\ some\ i \in \mathbb{N}, \\ (1,2,2), & for\ some\ i \in \mathbb{N}, \\ (1,0,2), & for\ some\ i \in \mathbb{N}. \end{cases} \tag{1.8}$$

which is contradiction.
Let $W = (a_i, a_{\acute{i}}, b_j)$, where $i \neq \acute{i} \quad i, \acute{i}, j \in \mathbb{N}$

$$r\left(\frac{a_m}{D}\right) = r\left(\frac{a_n}{D}\right) \quad m \neq n \quad for\ some\ m, n \in \mathbb{N} \tag{1.9}$$

which is contradiction.
Let $W = (a_i, b_j, b_{\acute{j}})$, where $j \neq \acute{j} \quad i, j, \acute{j} \in \mathbb{N}$

$$r\left(\frac{a_o}{D}\right) = r\left(\frac{a_p}{D}\right) \quad o \neq p \quad for\ some\ o, p \in \mathbb{N} \tag{1.10}$$

which is contradiction.
Let $W = (a_i, a_{i'}, a_{i''})$, where $i \neq i' \neq i'' \quad i, \acute{i}, i'' \in \mathbb{N}$

$$r\left(\frac{a_s}{D}\right) = r\left(\frac{a_t}{D}\right) \quad s \neq t \quad for\ some\ s, t \in \mathbb{N} \tag{1.11}$$

which is contradiction.
Let $W = (b_j, b_{j'}, b_{j''})$, where $j \neq j' \neq j'' \quad j, \acute{j}, j'' \in \mathbb{N}$

$$r\left(\frac{a_u}{D}\right) = r\left(\frac{a_v}{D}\right) \quad u \neq v \quad for\ some\ u, v \in \mathbb{N} \tag{1.12}$$

which is contradiction.
Similarly, there is no resolving set with three basis element. Hence dim(G) > 3 for $n \geq 2$. □

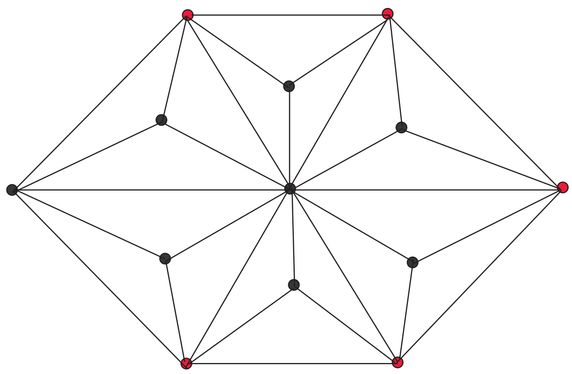

Figure 6: $HDN1_{(2)}$

Theorem 1.8. *For $G \cong HDN1_{(n)}$; $n \geq 2$ then dim(G) > 4.*

Proof. Suppose on contrary $HDN(n)$ has W as its resolving set with cardinality 4.
Let $W = (c, a_i, b_j, b_{j'})$, where $j \neq j' \quad i, j, j' \in \mathbb{N}$

$$r\left(\frac{a_k}{W}\right) = r\left(\frac{a_l}{W}\right) \quad k \neq l \quad for\ some\ k, l \in \mathbb{N} \tag{1.13}$$

which is contradiction.
Let $W = (c, a_i, a_{i'}, b_j)$, where $i \neq i' \quad i, i', j \in \mathbb{N}$

$$r\left(\frac{a_m}{W}\right) = r\left(\frac{a_n}{W}\right), \quad m \neq n \quad for\ some\ m, n \in \mathbb{N} \tag{1.14}$$

which is contradiction.
Let $W = (c, a_i, a_{i'}, a_{i''})$, where $i \neq i', \neq i'' \quad i, i', i'' \in \mathbb{N}$

$$r\left(\frac{a_o}{W}\right) = r\left(\frac{a_p}{W}\right), \quad o \neq p \quad for\ some\ o, p \in \mathbb{N} \tag{1.15}$$

which is contradiction.
Let $W = (c, b_j, b_{j'}, b_{j''})$, where $j \neq j', \neq j'' \quad j, j', j'' \in \mathbb{N}$

$$r\left(\frac{a_q}{W}\right) = r\left(\frac{a_r}{W}\right), \quad q \neq r \quad for\ some\ q, r \in \mathbb{N} \tag{1.16}$$

□

which is contradiction.
Let $W = (a_i, b_j, b_{j'}, b_{j''})$, *where* $j \neq j', \neq j'' \quad i, j, j', j'' \in \mathbb{N}$

$$r\left(\frac{a_s}{W}\right) = r\left(\frac{a_t}{W}\right), \quad s \neq t \quad for\ some\ s, t \in \mathbb{N} \tag{1.17}$$

which is contradiction.
Let $W = (a_i, a_{i'}, b_j, b_{j'})$, *where* $i \neq i'$ *and* $j \neq j' \quad i, i', j, j' \in \mathbb{N}$

$$r\left(\frac{a_u}{W}\right) = r\left(\frac{a_v}{W}\right), \quad u \neq v \quad for\ some\ u, v \in \mathbb{N} \tag{1.18}$$

which is contradiction.
Let $W = (a_i, a_{i'}, a_{i''}, b_j)$, *where* $i \neq i' \neq i'' \quad i, i', i'', j \in \mathbb{N}$

$$r\left(\frac{a_w}{W}\right) = r\left(\frac{a_x}{W}\right), \quad w \neq x \quad for\ some\ w, x \in \mathbb{N} \tag{1.19}$$

which is contradiction.
Let $W = (a_i, a_{i'}, a_{i''}, a_{i'''})$, *where* $i \neq i' \neq i'' \neq i''' \quad i, i', i'', i''' \in \mathbb{N}$

$$r\left(\frac{a_j}{W}\right) = r\left(\frac{a_z}{W}\right), \quad j \neq z \quad for\ some\ j, z \in \mathbb{N} \tag{1.20}$$

which is contradiction.
Let $W = (b_j, b_{j'}, b_{j''}, b_{j'''})$, *where* $j \neq j' \neq j'' \neq j''' \quad j, j', j'', j''' \in \mathbb{N}$

$$r\left(\frac{a_s}{W}\right) = r\left(\frac{a_t}{W}\right), \quad s \neq t \quad for\ some\ s, t \in \mathbb{N} \tag{1.21}$$

which is contradiction.
Similarly, there is no resolving set with three basis element. Hence dim(G) > 4 *for* $n \geq 2$.

Theorem 1.9. *For* $G \cong HDN1_{(h)}$; $n \geq 2$ *then G has metric dimension* 5.

Proof. Let W $= (V^1_{h-1}W^1_{h-1}, X^1_{h-1}, Y^1_{h-1}, Z^1_{h-1})$
then for $1 \leq s, j \leq m$ and $m \in \mathbb{N}$
we have

$$\lambda\left(\frac{a_s^{3j-k}}{W}\right) = \begin{cases} (h-s, h-1+j, h-2+s+j, h-2+s+j, h-1+s), & k=2, \\ (h-s, h-1+j, h-1+s+j, h-1+s+j, h-1+s), & k=1, \\ (h-1-s, h-1+j, h-1+s+j, h-1+s+j, h-1+s), & k=0. \end{cases} \tag{1.22}$$

$$\lambda\left(\frac{b_s^{3j-k}}{W}\right)=\begin{cases}(h-s,h-j,h-1+s,h-2+s+j,h-2+s+j), & k=2,\\ (h-s,h-j,h-1+s,h-1+s+j,h-1+s+j), & k=1,\\ (h-1-s,h-1-j,h-1+s,h-1+s+j,h-1+s+j), & k=0.\end{cases} \tag{1.23}$$

$$\lambda\left(\frac{c_s^{3j-k}}{W}\right)=\begin{cases}(h-1+j,h-s,h-j,h-1+s,h-2+s+j), & k=2,\\ (h-1+j,h-s,h-j,h-1+s,h-1+s+j), & k=1,\\ (h-1+j,h-1-s,h-1-j,h-1+s,h-1+s+j), & k=0.\end{cases} \tag{1.24}$$

$$\lambda\left(\frac{d_s^{3j-k}}{W}\right)=\begin{cases}(h-1+s,h-2+s+j,h-2+s+j,h-1+j,h-s), & k=2,\\ (h-1+s,h-1+s+j,h-1+s+j,h-1+j,h-j), & k=1,\\ (h-1+s,h-1+s+j,h-1+s+j,h-1+j,h-1-s), & k=0.\end{cases} \tag{1.25}$$

$$\lambda\left(\frac{e_s^{3j-k}}{W}\right)=\begin{cases}(h-2+s+j,h-2+s+j,h-1+s,h-j,h-s), & k=2,\\ (h-1+s+j,h-1+s+j,h-1+s,h-j,h-s), & k=1,\\ (h-1+s+j,h-1+s+j,h-1+s,h-1-j,h-1-s), & k=0.\end{cases} \tag{1.26}$$

$$\lambda\left(\frac{f_s^{3j-k}}{W}\right)=\begin{cases}(h-2+s+j,h-1+s,t-j,h-s,h-1+j), & k=2,\\ (h-1+s+j,h-1+s,h-j,h-s,h-1+j), & k=1,\\ (h-1+s+j,h-1+s,h-1-j,h-1-s,h-1+j), & k=0.\end{cases} \tag{1.27}$$

For $j=1$, $1\leq s\leq m$, $m\in\mathbb{N}$

$$U_s^j=(h-1,h-1+s,h-1+s,h-1+s,h-1) \tag{1.28}$$

$$V_s^j=(h-1-s,h-1,h-1+s,h-1+s,h-1+s) \tag{1.29}$$

$$W_s^j=(h-1,h-1-s,h-1,h-1+s,h-1+s) \tag{1.30}$$

$$X_s^j=(h-1+s,h-1,h-1-s,h-1,h-1+s) \tag{1.31}$$

$$Y_s^j=(h-1+s,h-1+s,h-1,h-1-s,h-1) \tag{1.32}$$

$$Z_s^j=(h-1+s,h-1+s,h-1+s,h-1,h-1-s) \tag{1.33}$$

For Centre point

$$O=(h-1,h-1,h-1,h-1,h-1) \tag{1.34}$$

Because Representation of each vertices with respect to W is unique.
⇒ W is resolving set of G and $|W|=5$ Because G is not a path so metric dimension of $G\cong HDN1_{(h)}$ is 5. where h is numbers of vertices. □

1.3. Nylone$_{6,6}$

Nylone$_{6,6}$ is a man-made synthetic fiber.Nylone$_{6,6}$ is a polyamide containing total 12 carbon atoms in each repeating unit and made by polcondensation of adipic acid methylenediamine.Properties of Nylone$_{6,6}$ are toughness good appearance,resistance,excellent abration resistant,high tensile strength, resistant to photo degradation,reduces moisture senstivity,stable in nature and resistant to head.Nylone$_{6,6}$ used in nylone rope,hosiery dress socks,swimwear,shorts,track pants and bed spreads.

Theorem 1.10. *For $G\cong NL_{6,6}(n)$; $n=1$ then G has metric dimension* 2.

Proof. Let W $=(V_{11},V_{21})$

$$r\left(\frac{V_{ij}}{W}\right)=\begin{cases}(i+j-2,i+j-1)\ ;i=1, & 1\leq j\leq 3,\\ (i+j-2,i+j-3)\ ;2\leq i\leq n, & x\leq j\leq t,\end{cases} \tag{1.35}$$

$$r\left(\frac{V_{ij}}{W}\right)=\begin{cases}(i+j-2,i+j-1)\ ;1\leq j\leq 6, & 4\leq j\leq 6,\\ (i+j-2,i+j-3)\ ;7\leq i\leq n+1, & 4\leq j\leq 6,\end{cases} \tag{1.36}$$

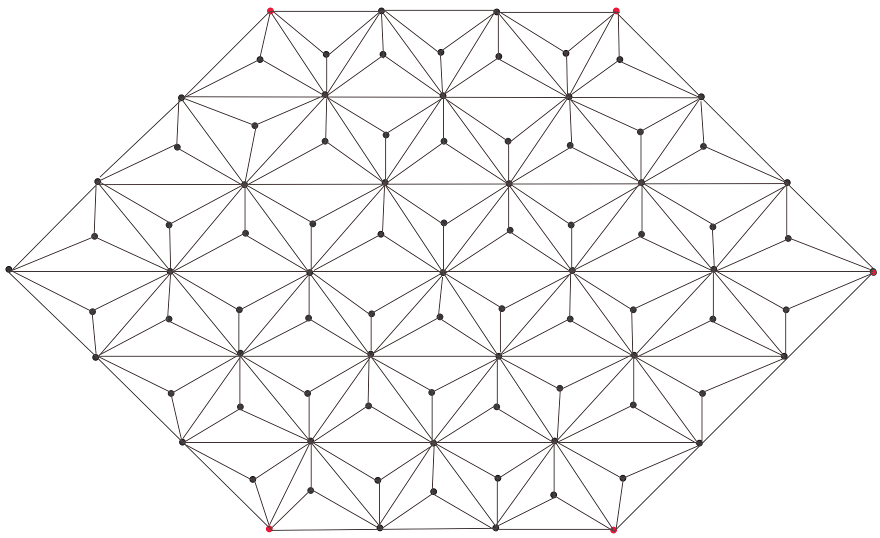

Figure 7: $HDN1_{(4)}$

$$r\left(\frac{V_{ij}}{W}\right) = \begin{cases} (i+j-2, i+j-1)\ ; 2 \le j \le 7, & 7 \le j \le 9, \\ (i+j-2, i+j-3)\ ; 8 \le i \le n+2, & 7 \le j \le 9, \end{cases} \tag{1.37}$$

$$r\left(\frac{V_{ij}}{W}\right) = \begin{cases} (i+j-2, i+j-1)\ ; 3 \le j \le 8, & j = 10, \\ (i+j-2, i+j-3)\ ; 9 \le i \le n+2, & j = 10, \end{cases} \tag{1.38}$$

Because Representation of each vertices with respect to W is unique.
$\Rightarrow$ W is resolving set of G and $|W| = 2$ Because G is not a path so metric dimension of $G \cong NL_{6,6}(n)$ is 2. where n is numbers of vertices. □

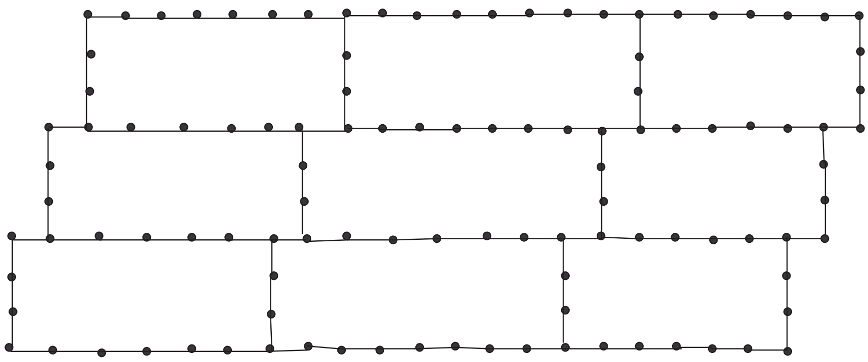

Figure 8: $NL_{6,6}(21)$

Conclusion and Open Problems

In the foregoing section we investigated the Polythiophene network $PLY_{(n)}$, Backbone network BS_{DNA}, Hex-derive network $HDN1_{(n)}$ and Nylone$_{6,6}$ network then determined the metric dimension of Polythiophene network $PLY_{(n)}$,Backbone network BS_{DNA} ,Hex-derive network $HDN1_{(n)}$ and Nylone$_{6,6}$ network. We have prove that $dim(PLY_{(n)}) = 2$,

$dim(BS_{DNA}) = 2$,
$dim(HDN1_{(n)}) = 5$,
$dim(NL_{6,6}) = 2$.
We close this section by raising the following open problem.
Open Problem 1. Determine the metric dimension of line graph of Polythiophene network $PLY_{(n)}$.
Open Problem 2. Determine the metric dimension of line graph of Backbone network BS_{DNA}.
Open Problem 3. Determine the metric dimension of line graph of Hex-derive network $HDN1_{(n)}$ for n subdivision.
Open Problem 4. Determine the metric dimension of line graph of Nylone$_{6,6}$ network $NL_{6,6}$.

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References

[1] Tomescu, I., & Imran, M. (2011). Metric dimension and R-sets of connected graphs. *Graphs and Combinatorics, 27*(4), 585-591. 1, 1
[2] Melter, F. H., & Harary, F. (1976). On the metric dimension of a graph. *Ars Combinatoria, 2*(1976), 191-5. 1, 1
[3] Godsil, C. D., & McKay, B. D. (1978). A new graph product and its spectrum. *Bulletin of the Australian Mathematical Society, 18*(1), 21-28. 1
[4] Javaid, I., Rahim, M. T., & Ali, K. (2008). Families of regular graphs with constant metric dimension. *Utilitas mathematica, 75*(1), 21-33. 1
[5] Imran, M., Baig, A. Q., Bokhary, S. A. U. H., & Javaid, I. (2012). On the metric dimension of circulant graphs. *Applied mathematics letters, 25*(3), 320-325. 1
[6] Baskoro, E. T., Salman, A. N. M., Saputro, S. W., & Suprijanto, D. On metric dimension of regular bipartite graphs. *Bulletin Mathématique de laSociété des Sciences Mathématiques de Roumanie, 54*(102), 15-28. 1
[7] Sebő, A., & Tannier, E. (2004). On metric generators of graphs. *Mathematics of Operations Research, 29*(2), 383-393. 1
[8] Chartrand, G., Eroh, L., Johnson, M. A., & Oellermann, O. R. (2000). Resolvability in graphs and the metric dimension of a graph. *Discrete Applied Mathematics, 105*(1-3), 99-113. 1
[9] Cameron, P. J., & Designs, J. V. L. (1991). Graphs, Codes and Their Links. *London Mathematical Society Student Texts, 22.* 1, 1, 1.3, 1.2
[10] Buczkowski, P., Chartrand, G., Poisson, C., & Zhang, P. (2003). On k-dimensional graphs and their bases. *Periodica Mathematica Hungarica, 46*(1), 9-15. 1
[11] S. Khuller, B. Raghavachari, A. Rosenfeld, Localization in graphs, Technical report CS-TR-3326, University of Maryland at College Park,1994.
[12] Melter, R. A., & Tomescu, I. (1984). Metric bases in digital geometry. *Computer vision, graphics, and image Processing, 25*(1), 113-121.
[13] Bermond, J. C., Comellas, F., & Hsu, D. F. (1995). Distributed loop computer-networks: a survey. *Journal of parallel and distributed computing, 24*(1), 2-10.
[14] P.J.Slater. (1988). Dominating and references sets in graphs. *Journal of Mathematical Physics, 22*(4), 445-455.
[15] Slater, P. J. (1975). Leaves of trees. *Congr. Numer, 14*, 549-559.
[16] Feng, T. Y. (1981). A survey of interconnection networks. *Computer, 14*(12), 12-27.
[17] Tomescu, I., & Javaid, I. (2007). On the metric dimension of the Jahangir graph. *Bulletin mathématique de la Société des Sciences Mathématiques de Roumanie,* 371-376.
[18] Wilkov, R. (1972). Analysis and design of reliable computer networks. *IEEE Transactions on Communications, 20*(3), 660-678.
[19] Wu, C. L., & Feng, T. Y. (1980). On a class of multistage interconnection networks. *IEEE transactions on Computers, 100*(8), 694-702.
[20] van Lint, R., Wilson, R., & Designs, G. (1991). *Codes, and Their Links.* University Press, Cambridge.
[21] Hillis, W. D. (1989). *The connection machine.* MIT press.
[22] Hwang, K., & Ghosh, J. (1987). Hypernet: A communication-efficient architecture for constructing massively parallel computers. *IEEE Transactions on Computers, 100*(12), 1450-1466.