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Journal of Prime Research in Mathematics, 19(1) (2023), 44–54

Journal of Prime Research in Mathematics

# Zagreb Connection Numbers on Different Networks

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**Abstract**

The first Zagreb index formulated in its approximate form for -electron energy in 1972 and second Zagreb index formulated in 1975 for branching of molecules. Some modification of these indices was proposed in three different ways naming as novel modification, connection indices and leap Zagreb indices. In this paper we proposed and calculated connection indices for Honey Comb Network and triangular benezoid structures. Furthermore as an extension of our work we also formulated connection indices for line graph of subdivision of Honey Comb and triangular benezoid networks.

*Keywords:* Connection numbers, Honey Comb, Triangular benezoid, Line graph.

## 1. Introduction and Preliminaries

Physicochemical modeling plays an important and vital role to resolve issues in theoretical chemistry. A topological index is a calculated numerical value, which is obtained from the molecular graph (a graph, which is used to represents a chemical compound in which vertices relate to the atoms while edges relate to the covalent bonds between atoms) of a chemical compound in such a way that this value must remain same under graph isomorphism. It is necessary to be well related to at least one physicochemical property of the considered chemical compound. There is popular trend in theoretical chemistry to use the topological indices for prediction in the specific properties of chemical compounds, see [1, 2, 3, 4].

The first Zagreb index $M_1$, in an approximate formula of total $\pi$-electron energy, and the second Zagreb index $M_2$, which is helpful for the study of molecular branching, are among the most studied topological indices [5, 6]. These topological indices are given below:

$$M_1 = \sum_{u \in V} d_u^2 \tag{1.1}$$

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*Received :* 20 April 2022; *Accepted:* 10 June 2023; *Published Online:* 17 July 2023.

$$M_2 = \sum_{uv \in E} (d_u.d_v) \tag{1.2}$$

Where, $V(G)$ is the vertex set of a graph $G$, $E(G)$ is the edge set of $G$ and $d_u$ denotes degree of the vertex $u$. The first and second Zagreb indices have been studied extensively, see the recent survey papers [7, 8, 9, 10].

Corresponding to the following connection-number-based version of the Zagreb indices were put forwarded independently. There are three novel and modified form of Zagreb connection indices are defined. These modified versions are called as first Zagreb connection number ($ZC_1$), second Zagreb connection number ($ZC_2$) and more modified type of first Zagreb as first modified connection number ($ZC_1^*$).

$$ZC_1 = \sum_{u \in V} \tau_u^2 \tag{1.3}$$

$$ZC_2 = \sum_{uv \in E} (\tau_u.\tau_v) \tag{1.4}$$

$$ZC_1^* = \sum_{uv \in E} (\tau_u + \tau_v) \tag{1.5}$$

The index ($ZC_1^*$)did not study explicitly till 2016. Recently, this index has been reconsidered independently. Ali and Trinajstić verified the chemical applicability of ($ZC_1^*$) and they found that this topological index correlates well with the entropy and acentric factor of octane isomers. It was also calculated that graphs having maximum and minimum ($ZC_1^*$) values among all molecular trees with $n$ vertices by them [11, 12, 13]. Naji et al.formulated several bounds on the topological indices ($ZC_1$), ($ZC_2$), ($ZC_1^*$), and they also derived formulas for calculating these three topological indices of join of graphs. More work on the topological indices ($ZC_1$), ($ZC_2$), ($ZC_1^*$) can be found in [14, 15, 16, 17].

The main objective of this paper to calculate the first Zagreb connection number ($ZC_1$), second Zagreb connection number ($ZC_2$) and more modified type of first Zagreb as first modified connection number ($ZC_1^*$) of different networks such as honeycomb, triangular benezoid with the line graph of subdivision of these networks. This will be helpful for us to understand the structure and properties of chemical compounds.

## 2. Main results

### *2.1. Honeycomb Network*

**Honeycomb Network** is a chemical graph based on hexagons. Number of designs in which hexagons offers to build HC network. If $HC_1$ is a hexagonal honeycomb network , we will add six hexagons to the exterior edges of $HC_1$ in order to obtain the honeycomb network $HC_2$. Consequently, "when we add a layer of hexagon on the boundary of $HC_{n-1}$, we get honeycomb network $HC_n$." The, order of $HC_n$ is $6n^2$ and the size of $HC_n$ is $9n^2$-$3n$. Honeycomb Network is very useful network. This network used in navigation, computer graphics, image processing and cell phone.

### *2.2. Results for Honeycomb Network*

**Theorem 2.1.** *The first Zägreb connection index of a Honeycomb Network for all* $n \geq 2$ *is:-*
$ZC_1(HC_n)= 216n^2 - 240n + 36 \ \forall \ n \geq 2$

*Proof.* Let G=$HC_n$ be a "Honeycomb Network" for all $n \geq 2$. The order of $HC_n$ is $6n^2$. The connection numbers of different vertices is given below in table

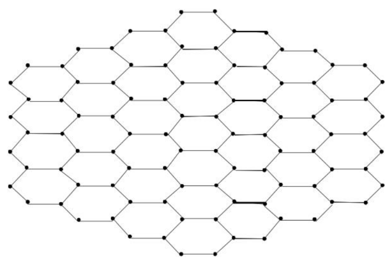

Figure 1: Honeycomb Network $n = 4$

Table 1: Connection Numbers of a graph G based on vertices.

| $(\tau_u)$, | No. of V(G) |
|---|---|
| 3 | 12 |
| 4 | $12n - 18$ |
| 6 | $6n^2 - 12n + 6$ |

"The first *Zägreb* connection index of a graph G is define as"

$$ZC_1(G) = \sum_{u \in V(G)} (\tau_u)^2$$

For Honeycomb Network $\forall\ n \geq 2$

Now,

$ZC_1(HC_n) = 12(3)^2 + (12n - 18)(4)^2 + (6n^2 - 12n + 6)(6)^2$

$ZC_1(HC_n) = 108 + 192n - 288 + 216n^2 - 432n + 216$

$ZC_1(HC_n) = 216n^2 - 240n + 36$

□

**Theorem 2.2.** *The Second Zägreb connection index of a Honeycomb Network for all $n \geq 2$ is:-*

$ZC_2(HC_n) = 324n^2 - 420n + 102\ \forall\ n \geq 2$

*Proof.* Let G= $HC_n$ is a graph of Honeycomb Network for all $n \geq 2$. The size of $HC_n$ is $9n^2 - 3n$. An edge connecting the vertices having connection numbers $\tau_u$ and $\tau_v$ respectively.

The connection numbers of different edges is given below in table

Table 2: Connection Numbers of a graph G based on edges

| $(\tau_u, \tau_v)$,where $uv \in E(G)$ | No. of E(G) |
|---|---|
| $(3, 3)$ | 6 |
| $(3, 4)$ | 12 |
| $(4, 4)$ | $12n - 24$ |
| $(4, 6)$ | $6n - 6$ |
| $(6, 6)$ | $9n^2 - 21n + 12$ |

"The second *Zägreb* connection index of a graph G is define as"

$$ZC_2(G) = \sum_{(u,v)} (\tau_u.\tau_v)$$

For Honeycomb Network $\forall\ n \geq 2$

$ZC_2(HC_n) = 9(6) + 12(12) + 16(12n - 24) + 24(6n - 6) + 36(9n^2 - 21n + 12)$

After some calculation we get $ZC_2(HC_n)$ = $324n^2$ - 402 + 102

□

**Theorem 2.3.** *The First modified Zägreb connection index of Honeycomb Network for all* $n \geq 2$

$$ZC_1^*(HC_n) = 108n^2 - 96n + 12$$

*Proof.* Let G= $HC_n$ is a graph of Honeycomb Network for all $n \geq 2$.

"The connection numbers of different edges is given below in table."

Table 3: Connection Numbers of a graph G based on edge.

| $(\tau_u, \tau_v)$,where $uv \in E(G)$ | No. of E(G) |
|---|---|
| $(3, 3)$ | 6 |
| $(3, 4)$ | 12 |
| $(4, 4)$ | $12n - 24$ |
| $(4, 6)$ | $6n - 6$ |
| $(6, 6)$ | $9n^2 - 21n + 12$ |

$ZC_1^*(HC_n) = (3+3)(6) + (3+4)(12) + (4+4)(12n-24) + (4+6)(6n-6) + (6+6)(9n^2-21n+12)$

After doing some calculations we get

$ZC_1^*(HC_n) = 108n^2 - 96n + 12$ □

*2.3. Results for Line Graph of Honeycomb Network*

**Theorem 2.4.** *The first Zägreb connection index of Line graph of a Honeycomb Network for all* $n \geq 2$ *is:-*

$ZC_1[L(HC_n)]$= $1296n^2 - 1836n + 480$ $\forall$ $n \geq 2$

*Proof.* Let G= $L[HC_n]$ is a line graph of Honeycomb Network for all $n \geq 2$. The order of a $L[HC_n]$ is $9n^2 - 3n$. Generally, in $L[HC_n]$ there are 6 verices which have connection number equals to 4. There are total 12 vertices in $L[HC_n]$ which has connection number equals to 6 and $12n - 24$ vertices which has connection number equal to 7 and $6n - 6$ vertices which shows connection number equals to 10. In the end there are $9n^2 - 21n + 12$ vertices which has connection number equals to 12 respectively.

In this table we have shown all the connection numbers of $L[HC_n]$

Table 4: Connection Numbers of a graph G based on edge.

| $(\tau_u)$, | No. of V(G) |
|---|---|
| 4 | 6 |
| 6 | 12 |
| 7 | $12n - 24$ |
| 10 | $6n - 6$ |
| 12 | $9n^2 - 21n + 12$ |

"The first *Zägreb* connection index is define as"

$$ZC_1(G) = \sum_{u \in V(G)} (\tau_u)^2$$

For Line graph of Honeycomb Network $\forall$ $n \geq 2$

$ZC_1[L(HC_n)]$=$6(4)^2 + 12(6)^2 + (12n-24)(7)^2 + (6n-6)(10)^2 + (9n^2-21n+12)(12)^2$

After doing some calculation we get

$ZC_1[L(HC_n)]= 1296n^2 - 1836n + 480$ □

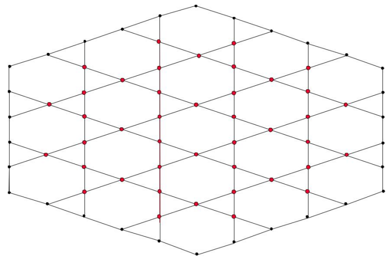

Figure 2: Line Graph of Honeycomb Network for $n = 3$

**Theorem 2.5.** *The Second Zägreb connection index of a Line graph of Honeycomb Network for all $n \geq 2$ is:-*

$ZC_2[L(HC_n)] = 2592n^2 - 4044n + 1242 \ \forall \ n \geq 2$

*Proof.* Let G= $L[HC_n]$ is a line graph of Honeycomb Network for all $n \geq 2$. The Size of $[L(HC_n)]$ is $18n^2 - 12n$. In this Edges connecting the vertices having connection number $\tau_u$ and $\tau_v$. These types of edges are represents in the form of $\tau_u$ and $\tau_v$ respectively.

Table 5: Connection Numbers of a Graph G based on edges

| $(\tau_u, \tau_v)$,where $uv \in E(G)$ | No. of E(G) |
|---|---|
| $(7, 7)$ | $12n - 30$ |
| $(12, 12)$ | $18n^2 - 48n + 30$ |
| $(4, 6)$ | 12 |
| $(6, 7)$ | 12 |
| $(6, 10)$ | 12 |
| $(7, 10)$ | $12n - 24$ |
| $(10, 12)$ | $12n - 12$ |

$ZC_2[L(HC_n)]= (7 \times 7)(12n - 30) + (12 \times 12)(18n^2 - 48n + 30) + (4 \times 6)(12) + (6 \times 7)(12)(6 \times 10)(12) + (7 \times 10)(12n - 24) + (10 \times 12)(12n - 12)$

After doing some calculation we get

$ZC_2[L(HC_n)]= 2592n^2 - 4044n + 1242$

□

**Theorem 2.6.** *The First modified Zägreb connection index of a Line graph of Honeycomb Network for all $n \geq 2$ is:-*

$ZC_1^*[L(HC_n)] = 432n^2 - 516n + 96 \ \forall \ n \geq 2$

*Proof.* Let G= $L[HC_n]$ is a line graph of Honeycomb Network for all $n \geq 2$. In this theorem, we add the connection numbers for every particular edge respectively.

"The connection numbers of $[L(HC_n)]$ of different edges are given in above table."

According to the definition of first modified *Zägreb* connection index is

$$ZC_1^*(G) = \sum_{(u,v)} (\tau_u + \tau_v)$$

$ZC_1^*[L(HC_n)]= (7+7)(12n-30) + (12+12)(18n^2 - 48n+30) + (4+6)(12) + (6+7)(12) + (6+10)(12) + (7 + 10)(12n - 24) + (10 + 12)(12n - 12)$

After doing some calculation we get
$ZC_1^*[L(HC_n)] = 432n^2 - 516n + 96 \ \forall \ n \geq 2$ □

*2.4. Results for Line Graph of Sub-Division of Honeycomb Network*

**Theorem 2.7.** *The first Zägreb connection index of Line graph of Sub-Division of Honeycomb Network for all $n \geq 2$ is:-*
$ZC_1[L(SHC_n)] = 648n^2 - 672n + 72 \ \forall \ n \geq 2$

*Proof.* Let G= $L[S(HC_n)]$ is a line graph of Sub-Division of Honeycomb Network for all $n \geq 2$. The order of line graph of a sub-division for $HC_n$ is $6n(3n-1)$.

"The connection numbers of $L[S(HC_n)]$ for different vertices are given below in table."

Table 6: Connection Numbers of a graph G based on edge.

| $(\tau_u)$, | No. of V(G) |
|---|---|
| 2 | 12 |
| 3 | $12n - 12$ |
| 5 | $12n - 24$ |
| 6 | $18n^2 - 30n + 12$ |

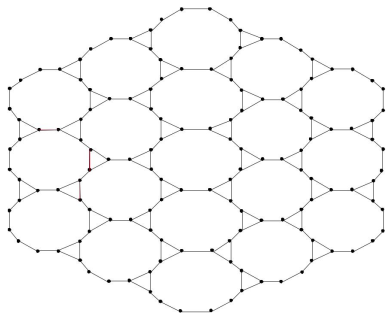

Figure 3: : Line Graph of Sub-Division of Honeycomb Network for $n = 3$

"The first *Zägreb* connection index is define as"

$$ZC_1(G) = \sum_{u \in V(G)} (\tau_u)^2$$

$ZC_1[L(SHC_n)] = 12(2)^2 + (12n-12)(3)^2 + (12n-12)(5)^2 + (18n^2 - 30n + 12)(6)^2$
After doing some calculations we get
$ZC_1[L(SHC_n)] = 648n^2 - 672n + 72 \ \forall \ n \geq 2$ □

**Theorem 2.8.** *The Second Zägreb connection index of Line graph of sub-division of Honeycomb Network for all $n \geq 2$ is:-*
$ZC_2[L(SHC_n)] = 972n^2 - 1092n + 162 \ \forall \ n \geq 2$

*Proof.* Let G= $L[S(HC_n)]$ is a line graph of Sub-Division of Honeycomb Network for all $n \geq 2$. The size of $[L(SHC_n)]$ is $27n^2 - 15n$. Every edges contains two end points which are its vertices. Those edges which has same connection number vertices are known as sigma-type edges. Those edges in which one vertex have odd connection number and other vertex have even connection number are known as gamma-type edges with respect to connection numbers.

Table 7: Connection Numbers of a Graph G based on edges

| $(\tau_u, \tau_v)$,where $uv \in E(G)$ | No. of E(G) |
|---|---|
| $(2, 2)$ | $6$ |
| $(3, 3)$ | $6n - 12$ |
| $(5, 5)$ | $6n - 6$ |
| $(6, 6)$ | $27n^2 - 51n + 24$ |
| $(2, 3)$ | $12$ |
| $(3, 5)$ | $12n - 12$ |
| $(5, 6)$ | $12n - 12$ |

"The connection numbers of $[L(SHC_n)]$ of different edges are given in table."

"The second *Zägreb* connection index is define as"

$$ZC_2(G) = \sum_{(u,v)} (\tau_u.\tau_v)$$

$ZC_2[L(SHC_n)]$=$6(2 \times 2) + (6n - 12)(3 \times 3) + (6n - 6)(5 \times 5) + (27n^2 - 51n + 24)(6 \times 6) + 12(2 \times 3) + (12n - 12)(3 \times 5) + (12n - 12)(5 \times 6)$

After simplification we get the result

$ZC_2[L(SHC_n)]$=$972n^2 - 1092n + 162 \; \forall \; n \geq 2$ □

**Theorem 2.9.** *The First modified Zägreb connection index of Line graph of sub-division of Honeycomb Network for all $n \geq 2$ is:-*

$ZC_1^*[L(SHC_n)] = 324n^2 - 288n + 12 \; \forall \; n \geq 2$

*Proof.* Let G= $L[S(HC_n)]$ is a line graph of Sub-Division of Honeycomb Network for all $n \geq 2$.

The connection numbers of $[L(SHC_n)]$ of different edges are given in previous table

"The first modified *Zägreb* connection index is define as"

$$ZC_1^*(G) = \sum_{(u,v)} (\tau_u + \tau_v)$$

$ZC_1^*[L(SHC_n)]$=$6(2 + 2) + (6n - 12)(3 + 3) + (6n - 6)(5 + 5) + (27n^2 - 51n + 24)(6 + 6) + 12(2 + 3) + (3 + 5)(12n - 12) + (5 + 6)(12n - 12)$

After simplification we get

$ZC_1^*[L(SHC_n)]$=$324n^2 - 288n + 12 \; \forall \; n \geq 2$ □

*2.5. Results for Triangular Benezoid Network*

**Theorem 2.10.** *The first Zägreb connection index of Triangular Benezoid Network $TB_n$ for all $n \geq 2$ is:-*

$ZC_1(TB_n)$= $36n^2 + 24n - 42 \; \forall \; n \geq 2$

*Proof.* Let G= $TB_n$ is a graph of triangular benezoid network for all $n \geq 2$. The order of $TB_n$ is $n^2 + 4n + 1$

Table 8: Connection Numbers of a graph G based on edge.

| $(\tau_u)$, | No. of V(G) |
|---|---|
| 2 | 3 |
| 3 | 6 |
| 4 | $6n-9$ |
| 6 | $n^2-2n+1$ |

"The connection numbers of $TB_n$ for different vertices are given below in table."

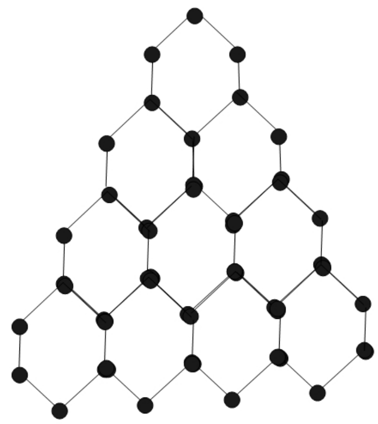

Figure 4: Triangular Benezoid for $n = 4$

"The first *Zägreb* connection index is define as"

$$ZC_1(G) = \sum_{u \in V(G)} (\tau_u)^2$$

$ZC_1(TB_n) = 3(2)^2 + 6(3)^2 + (6n-9)(4)^2 + (n^2-2n+1)(6)^2$
After doing some calculations we get
$ZC_1(TB_n) = 36n^2 + 24n - 42 \ \forall \ n \geq 2$ □

**Theorem 2.11.** *The Second Zägreb connection index of Triangular Benezoid Network $TB_n$ for all $n \geq 2$ is:-*
$ZC_2(TB_n) = 54n^2 + 6n - 48 \ \forall \ n \geq 2$

*Proof.* Let G= $TB_n$ is a graph of triangular benezoid network for all $n \geq 2$. The size of $TB_n$ is $\frac{3}{2}n(n+3)$
The connection numbers of $TB_n$ for different vertices are given below in table

Table 9: Connection Numbers of a Graph G based on edges

| $(\tau_u, \tau_v)$,where $uv \in E(G)$ | No. of E(G) |
|---|---|
| $(4,4)$ | $6n-12$ |
| $(6,6)$ | $\frac{1}{2}(3n^2-9n+6)$ |
| $(2,3)$ | 6 |
| $(6,6)$ | $27n^2-51n+24$ |
| $(3,4)$ | 6 |
| $(4,6)$ | $3n-3$ |

"The second *Zägreb* connection index is define as"

$$ZC_2(G) = \sum_{(u,v)} (\tau_u.\tau_v)$$

$ZC_2(TB_n)$= $(4 \times 4)(6n-12) + (6 \times 6)\frac{1}{2}(3n^2 - 9n + 6) + (2 \times 3)(6) + (3 \times 4)(6) + (4 \times 6)(3n-3)$
After doing some calculation we get
$ZC_2(TB_n)$ =$54n^2 + 6n - 48 \ \forall \ n \geq 2$ □

**Theorem 2.12.** *The First modified Zägreb connection index of Triangular Benezoid Network for all $n \geq 2$ is:-*
$ZC_1^*(TB_n) \ = 18n^2 + 24n - 18 \ \forall \ n \geq 2$

*Proof.* Let G= $(TB_n)$ is a graph of Triangular Benezoid Network for all $n \geq 2$.
The connection numbers of $(TB_n)$ of different edges are given in previous table
"The first modified *Zägreb* connection index is define as"

$$ZC_1^*(G) = \sum_{(u,v)} (\tau_u + \tau_v)$$

$ZC_1^*(TB_n)$=$(4+4)(6n-12) + (6+6)\frac{1}{2}(3n^2 - 9n + 6) + (2+3)(6) + (3+4)(6) + (4+6)(3n-3)$
After simplification we get
$ZC_1^*(TB_n)$=$18n^2 + 6n - 48 \ \forall \ n \geq 2$ □

*2.6. Results for Line Graph of Sub-Division of Triangular Benezoid*

**Theorem 2.13.** *The first Zägreb connection index of line graph of sub-division of Triangular Benezoid Network $[L(STB_n)]$ for all $n \geq 2$ is:-*
$ZC_1[L(STB_n)]$= $108n^2 + 96n - 156 \ \forall \ n \geq 2$

*Proof.* Let G= $[L(STB_n)]$ is a line graph of sub-division of triangular benezoid network for all $n \geq 2$. The order of $[L(STB_n)]$ is $3n(n+3)$
"The connection numbers of $[L(STB_n)]$ for different vertices are given below in table."

Table 10: Connection Numbers of a graph G based on edge.

| $(\tau_u)$, | No. of V(G) |
|---|---|
| 2 | 12 |
| 3 | $6n-6$ |
| 5 | $6n-6$ |
| 6 | $3n^2 - 3n$ |

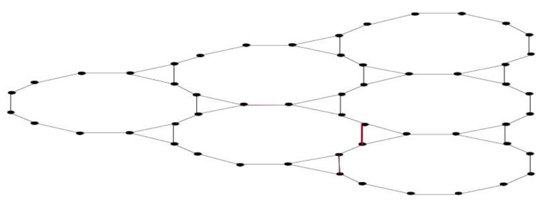

Figure 5: Line graph of Sub-Division of Triangular Benezoid for $n = 3$

"The first *Zägreb* connection index is define as"

$$ZC_1(G) = \sum_{u \in V(G)} (\tau_u)^2$$

$ZC_1[L(STB_n)]= 12(2)^2 + (6n-6)(3)^2 + (6n-6)(5)^2 + (3n^2-3n)(6)^2$
After doing some calculations we get
$ZC_1[L(STB_n)]= 108n^2 + 96n - 156 \; \forall \; n \geq 2$ □

**Theorem 2.14.** *The Second Zägreb connection index of line graph of sub-division of Triangular Benezoid Network $TB_n$ for all $n \geq 2$ is:-*
$ZC_2[L(STB_n)]$ $=162n^2 + 102n - 219 \; \forall \; n \geq 2$

*Proof.* Let G= $[L(STB_n)]$ is a line graph of sub-division of triangular benezoid network for all $n \geq 2$. The size of $TB_n$ is $\frac{3}{2}(3n^2 + 7n - 2)$
"The connection numbers of $[L(STB_n)]$ for different vertices are given below in table."

Table 11: Connection Numbers of a Graph G based on edges

| $(\tau_u, \tau_v)$,where $uv \in E(G)$ | No. of E(G) |
|---|---|
| $(2, 2)$ | 9 |
| $(3, 3)$ | $3n - 6$ |
| $(5, 5)$ | $3n - 3$ |
| $(6, 6)$ | $\frac{1}{2}(9n^2 - 15n + 6)$ |
| $(2, 3)$ | 6 |
| $(3, 5)$ | $6n - 6$ |
| $(5, 6)$ | $6n - 6$ |

"The second *Zägreb* connection index is define as"

$$ZC_2(G) = \sum_{(u,v)} (\tau_u . \tau_v)$$

$ZC_2[L(STB_n)] = (2 \times 2)(9) + (3 \times 3)(3n-6) + (5 \times 5)(3n-3) + (6 \times 6)\frac{1}{2}(9n^2 - 15n + 6) + (2 \times 3)(6) + (3 \times 5)(6n-6) + (5 \times 6)(6n-6)$
After doing some calculation we get
$ZC_2[L(STB_n)] = 162n^2 + 102n - 219 \; \forall \; n \geq 2$ □

**Theorem 2.15.** *The First modified Zägreb connection index of line graph of sub-division of Triangular Benezoid Network for all $n \geq 2$ is:-*
$ZC_1^*[L(STB_n)]$ $= 54n^2 + 72n - 78 \; \forall \; n \geq 2$

*Proof.* Let G= $[L(STB_n)]$ is a line graph of sub-division of Triangular Benezoid Network for all $n \geq 2$.
The connection numbers of of different edges are given in previous table
"The first modified *Zägreb* connection index is define as"

$$ZC_1^*(G) = \sum_{(u,v)} (\tau_u + \tau_v)$$

$ZC_1^*[L(STB_n)]$=$(2+2)(9) + (6+6)\frac{1}{2}(9n^2 - 15n + 6) + (3+3)(3n-6) + (5+5)(3n-3) + (2+3)(6) + (3+5)(6n-6) + (5+6)(6n-6)$
After simplification we get

$ZC_1^*[L(STB_n)] = 54n^2 + 72n - 78 \ \forall \ n \geq 2$ □

## 3. Conclusion

In this paper, we have discussed topological indices, which are particular based upon degree based connection index like first Zagreb connection index, second Zagreb connection index, first modified Zagreb connection index for subdivision of different networks like hexagonal, honey comb and some others molecular structural networks. The study of new design architectural structure always has open problem of above networks and art sciences. Here, new constructed networks of topological connection indices, construction of these networks, properties of good topological indices have been discussed. In upcoming research papers, our interest to sketch some new graphs as well as networks and we will discuss their topological connection indices that will be helpful to understand their topologies. We will work for different values of connection numbers for different and unique graphs within the diameter of the graph.

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